

Harmonic Mean: The harmonic mean is defined as:-

$$HM = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}}$$

$$= \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} \quad (2)$$

Example: Let $S = \{26, 12, 14, 15, 22, 17, 9, 11, 18, 16, 28, 20, 22, 8\}$

Find

- ① Mode
- ② Median
- ③ GM
- ④ HM

Solution:- Mode = 22

$$\text{Median} = (X_7 + X_8) / 2$$

8, 9, 11, 12, 14, 15, 16, 17, 18, 20, 22, 22, 26, 28

$$\begin{aligned} \text{Median} &= \frac{16 + 17}{2} \\ &= 16.5 \end{aligned}$$

$$\begin{aligned} GM &= \sqrt[14]{(26)(12)(14)(15)(22)(17)(9)(11)(18)(16)(28) \\ &\quad (20)(22)(8)} \\ &= 15.95 \end{aligned}$$

8

or

$$GM = \log_{10}^{-1} \left[\frac{1}{n} \sum_{i=1}^{14} \log_{10} x_i \right]$$

$$= \log_{10}^{-1} \left[\frac{16.8379}{14} \right]$$

$$= \log_{10}^{-1} [1.2027]$$

$$= 15.95$$

$$HM = \frac{14}{\sum \frac{1}{x_i}}$$

$$= \frac{14}{\left(\frac{1}{26} + \frac{1}{12} + \frac{1}{14} + \frac{1}{15} + \frac{1}{22} + \frac{1}{17} + \frac{1}{9} + \frac{1}{11} + \frac{1}{18} + \frac{1}{16} + \frac{1}{28} \right. \\ \left. + \frac{1}{20} + \frac{1}{22} + \frac{1}{8} \right)}$$

$$= \frac{14}{0.9409}$$

$$= 14.879$$

1/12/2024 (9)

Variance and Standard Deviation:

The variance or (mean square) for a set of values is defined as the sum of squared deviations of individual values from the mean, divided by the sample size reduced by one

$$\text{Var} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad n \leq 30$$

$$= \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$$= \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad n > 30$$

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{n-1}$$

where $\text{cov}(x, y)$ represents the covariance between two vectors x, y

If $\text{cov}(X, Y) = 0$ Then we said that X and Y are independent

σ^2 , Σ are the symbols of Variance where σ^2 is the variance of population and Σ is the symbol of Variance matrix

S^2 is the symbol of Variance of Sample

Standard Deviation: standard deviation is the square root of variance and it denotes as

$$\text{S.D} = \sqrt{S^2}$$

$$\text{standard Error} = \frac{\sqrt{SD^2}}{n} = \frac{SD}{n}$$

Coefficient of variation

$$CV = \frac{SD}{\text{Mean}} \times 100$$

Example: The marks of 10 students in statistics are as follows

Name	Mark	Name	Mark	Name	Mark
A	20	D	24	G	21
B	25	E	19	H	24
C	18	F	20	I	20
				J	20

(3) (11)

Find

- ① Mean ② Mode ③ Median ④ GM
⑤ HM ⑥ Variance ⑦ C.V

Solution

$$\bar{X} = \frac{\sum X_i}{n}$$

$$\sum X_i = (20 + 25 + 18 + 24 + 19 + 20 + 21 + 24 + 40)$$
$$= 211$$

$$\bar{X} = \frac{211}{10}$$
$$= 21.1$$

$$\text{Mode} = 20$$

$$\text{Median} = \frac{X_5 + X_6}{2}$$

$$18, 19, 20, 20, 20, 20, 21, 21, 24, 25$$

$$\therefore \text{Median} = \frac{20 + 20}{2}$$
$$= 20$$

$$\text{GM} = \sqrt[10]{18 \cdot 19 \cdot 160000 \cdot 21 \cdot 576 \cdot 25}$$

$$= 20.983$$