Example(2): **S=**{A,B,C}

$$P=\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \end{bmatrix} \end{array}$$

A Markov process is completely defined once its transition probability matrix and initial state X_0 (or, more generally, the probability distribution of X_0) are specified. We shall now prove this fact.

Theorem(1): Let P_{ij} denote the transition probability in one step from state(i) to state(j),and let the initial distribution of X_0 as follows $P[X_0=i_0]$, i=0,1,2,… then :

$$P[X_0=i_0, X_1=i_1, X_2=i_2 ,\ldots\ldots, X_n=i_n]=P_{i0}*P_{i0,i1}*P_{i1,i2}*\ldots.*P_{in-1,in}$$

Proof: By the definition of the conditional Prob. ,we have :

$$P(A|B)=\frac{P(A,B)}{P(B)}$$

$$P(A,B)=P(A|B).P(B)$$

$$P[X_0=i_0, X_1=i_1, X_2=i_2 ,\ldots\ldots, X_{n-1}=i_{n-1}, X_n=i_n]=$$

$$P[X_n=i_n \mid X_{n-1}= i_{n-1}, X_{n-2}= i_{n-2},\ldots, X_1=i_1, X_0=i_0]* P[X_{n-1}= i_{n-1}, X_{n-2}= i_{n-2},\ldots, X_1=i_1, X_0=i_0] \qquad \ldots(1)$$

Now by the definition of a Markov process:

$$P[X_n=i_n \mid X_{n-1}= i_{n-1}, X_{n-2}= i_{n-2},\ldots, X_1=i_1, X_0=i_0]$$

$$= P[X_n = i_n \mid X_{n-1} = i_{n-1}]$$

$$= P_{i_{n-1},i_n} \qquad \ldots(2)$$

Substituting (2) in (1) gives :

$$= P[X_n = i_n \mid X_{n-1} = i_{n-1}] * P[X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, \ldots, X_1 = i_1, X_0 = i_0]$$

$$= P[X_n = i_n \mid X_{n-1} = i_{n-1}] * P[X_{n-1} = i_{n-1} \mid X_{n-2} = i_{n-2}, \ldots, X_1 = i_1, X_0 = i_0] * P[[X_{n-2} = i_{n-2}, X_{n-3} = i_{n-3}, \ldots, X_1 = i_1, X_0 = i_0]$$

$$= P[X_n = i_n \mid X_{n-1} = i_{n-1}] * P[X_{n-1} = i_{n-1} \mid X_{n-2} = i_{n-2}] * P[[X_{n-2} = i_{n-2}, X_{n-3} = i_{n-3}, \ldots, X_1 = i_1, X_0 = i_0]$$

$$= P_{i_{n-1},i_n} * P_{i_{n-2},i_{n-1}} * P[[X_{n-2} = i_{n-2}, X_{n-3} = i_{n-3}, \ldots, X_1 = i_1, X_0 = i_0]$$

$$= P_{i_{n-1},i_n} * P_{i_{n-2},i_{n-1}} * P_{i_{n-3},i_{n-2}} * \ldots * P_{i_0}$$

In the same way we can deduce that:

$$\therefore P[X_0 = i_0, X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n] = P_{i_0} * P_{i_0,i_1} * P_{i_1,i_2} * \ldots * P_{i_{n-1},i_n}$$

3.3 Some Examples of Transition Probability Matrix

Example(1): A Markov chain X_0, X_1, X_2 ... on states 0, 1, 2 has the transition probability matrix:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \end{bmatrix} \end{array}$$