$$P_0(t+h) = P_0(t) - \lambda h P_0(t) + o(h)$$

$$\frac{P_0(t+h) - P_0(t)}{h} = -\lambda P_0(t) + \frac{o(h)}{h}$$

as $h \to 0$ we have:-

$$P_0'(t) = -\lambda P_0(t) \qquad , \text{where } \lim_{h \to 0} \frac{o(h)}{h} = 0$$

$$\frac{P_0'(t)}{P_0(t)} = -\lambda$$

$$\int \frac{P_0'(t)}{P_0(t)} dt = \int -\lambda \, dt$$ بأخذ التكامل للطرفين

$$L_n P_0(t) = -\lambda t + C$$ C هو ثابت التكامل

$$\overset{exp}{\Longrightarrow} \quad P_0(t) = e^{-\lambda t + c}$$ بأخذ exp للطرفين

$$= e^{-\lambda t} \cdot e^{c}$$

$$= k e^{-\lambda t} \qquad \text{where } k = e^{c}$$

then $P_0(0) = k e^{-\lambda(0)} = k$

$1 = k$ since $P_r\{X(0) = 0\} = 1$