Since $\sum y_i = \sum \hat{y}_i$ then:

$$S_{y\hat{y}}^2 = \left[\sum (y_i - \bar{y}) \left(\bar{y} - \hat{\beta}_1 \bar{X} + \hat{\beta}_1 X_i - \bar{y}\right)\right]^2$$

Taking $\hat{\beta}_1$ as a common factor, we get:

$$S_{y\hat{y}}^2 = \left[\sum (y_i - \bar{y}) \left(\hat{\beta}_1 (X_i - \bar{X}\right)\right]^2$$

That is:

$$S_{y\hat{y}}^2 = \left[\hat{\beta}_1 \sum (y_i - \bar{y}) (X_i - \bar{X})\right]^2$$

$$S_{y\hat{y}}^2 = \left[\hat{\beta}_1 S_{Xy}\right]^2 = [SS\ due\ to\ Regression]^2$$

$$\therefore S_{y\hat{y}} = SSR(X_1)$$

Through the denominator:

$$S_{\hat{y}\hat{y}} = \sum (\hat{y}_i - \bar{\hat{y}})^2 = \sum (\bar{y} - \hat{\beta}_1 \bar{X} + \hat{\beta}_1 X_i - \bar{\hat{y}})^2$$

In the same manner and after compensation $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{X}$ , and taking $\hat{\beta}_1$ as a common factor, we get:

$$S_{\hat{y}\hat{y}} = \sum (\bar{y} - \hat{\beta}_1 \bar{X} + \hat{\beta}_1 X_i - \bar{\hat{y}})^2$$

$$S_{\hat{y}\hat{y}} = \sum (\hat{\beta}_1 (X_i - \bar{X}))^2 = \hat{\beta}_1^2 \sum (X_i - \bar{X})^2 = \hat{\beta}_1^2 S_{XX}$$
$$= SS\ due\ to\ Regression$$

$$S_{y\hat{y}} = S_{\hat{y}\hat{y}} = SSR = SS\ due\ to\ Regression$$

$$\therefore r_{y\hat{y}}^2 = \left[\frac{S_{y\hat{y}}^2}{\sqrt{S_{yy}S_{\hat{y}\hat{y}}}}\right]^2 = \frac{S_{y\hat{y}}^2}{S_{yy}S_{\hat{y}\hat{y}}} = \frac{[SS\ due\ to\ Regression]^2}{SS\ Total \qquad SS\ due\ to\ Regression}$$

$$\therefore r_{y\hat{y}}^2 = \frac{SS\ due\ to\ Regression}{SS\ Total} = R^2$$

$$\therefore R^2 = r_{y\hat{y}}^2$$

$$\therefore R = r_{y\hat{y}}$$

$$\therefore \quad 0 \leq R^2 = r_{y\hat{y}}^2 \leq 1 \rightarrow 0 \leq r_{y\hat{y}} \leq 1$$

That is, the multiple correlation coefficient is the square root of the coefficient of determination $R^2$. Therefore, when the model matches the data, the value of $R^2$ approaches the correct one, meaning that the observed values y and expected values $\hat{y}$ are very close.

he interpretation of the coefficient of determination is what explains the importance of the mathematical model in describing the relationship between X and y To give the percentage of what this relationship explains through the model for the variables occurring in y.

In simple linear regression we have:

$$r_{y\hat{y}} = r_{Xy}$$

This can be proven as follows:

$$r_{y\hat{y}} = \frac{S_{y\hat{y}}}{\sqrt{S_{yy}S_{\hat{y}\hat{y}}}} = \frac{\sum(y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{(S_{yy})\sum(\hat{y}_i - \bar{\hat{y}})^2}}$$

In the same manner and after compensation $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{X}$ and take $\hat{\beta}_1$ factoring together the numerator and denominator we get:

$$r_{y\hat{y}} = \frac{\sum (y_i - \bar{y})(\cancel{\bar{y}} + \hat{\beta}_1 (X_i - \bar{X}) - \cancel{\bar{\hat{y}}})}{\sqrt{(S_{yy}) \sum (\cancel{\bar{y}} + \hat{\beta}_1 (X_i - \bar{X}) - \cancel{\bar{\hat{y}}})^2}}$$

$$r_{y\hat{y}} = \frac{\hat{\beta}_1 \sum (y_i - \bar{y})(X_i - \bar{X})}{\sqrt{\hat{\beta}_1^2 S_{yy} \sum (X_i - \bar{X})^2}} = \frac{\hat{\beta}_1 S_{Xy}}{\sqrt{\hat{\beta}_1^2 [S_{yy}\ S_{XX}]}}$$

$$r_{y\hat{y}} = \frac{\cancel{\hat{\beta}_1} S_{Xy}}{\cancel{\hat{\beta}_1} \sqrt{[S_{yy}\ S_{XX}]}}$$

$$\therefore r_{y\hat{y}} = \frac{S_{Xy}}{\sqrt{[S_{yy}\ S_{XX}]}} = r_{Xy}$$

$$\therefore R = r_{y\hat{y}} = |r_{Xy}|$$

$$\therefore R^2 = r_{y\hat{y}}^2 = r_{Xy}^2$$

**Partial correlation coefficient**

The partial correlation coefficient is defined as a measure of the linear relationship between two variables after fixing the effect of other variables.

The partial correlation coefficient between variables i and j after making variable k constant is:

$$r_{ij.k} = \frac{r_{ij} - r_{ik}r_{jk}}{\sqrt{(1 - r_{ik}^2)(1 - r_{jk}^2)}}$$

Where $r_{ij}$ is the simple correlation coefficient between variables i and j.

and $r_{ij.k}$, it is the first-order partial correlation coefficient.

The partial correlation coefficient (second order) between variables i and j after making the effect of the remaining variables L and K constant is:

$$r_{ij.kL} = \frac{r_{ij.k} - r_{iL.k}r_{jL.k}}{\sqrt{(1 - r_{iL.k}^2)(1 - r_{jL.k}^2)}}$$

Or it could be:

$$r_{ij.kL} = \frac{r_{ij.L} - r_{ik.L}r_{jk.L}}{\sqrt{(1 - r_{ik.L}^2)(1 - r_{jk.L}^2)}}$$

**Standard partial regression coefficient**

The standard partial regression coefficient, symbolized by $\hat{\beta}_i^*$, is the partial regression coefficient when it is of standard or standardized form.

When there is a correlation between the variables X1 and X2 that came from the variance of the values X1 and X2, we cannot judge which of them has a