Chapter two

"Some Discrete Distribution"

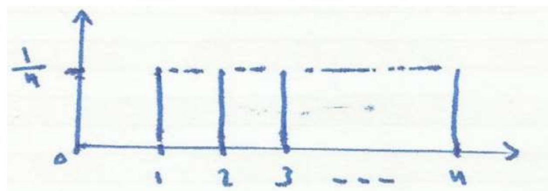
2.1- Uniform Distribution : التوزيع المنتظم

Suppose that a r.v. X takes on a finite set of real number [a , a+1 , a+2 , a+3 , … , n+a-1 = b] and have the probability mass function (p.m.f.) as given bellows :

$$P(X) = \begin{cases} \frac{1}{n} & ; \quad X = 1,2,3,\dots,n \\ 0 & \quad o.w. \end{cases}$$

Discrete uniform distribution $x \sim Ud(n)$

n : it is a parameter of distribution



Discrete Uniform Distribution also has satisfied the properties of the (p.m.f.) :

$$\sum_{x=a}^{b} P(X) = \sum_{x=a}^{b} \frac{1}{n} = 1$$

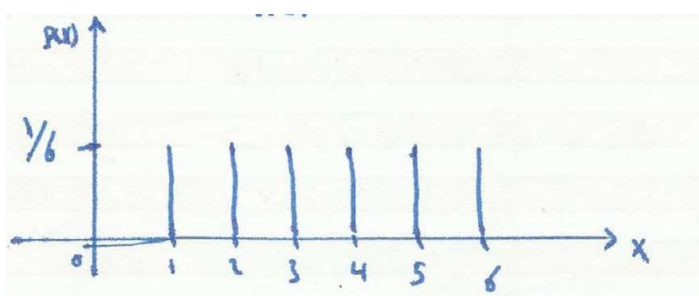
Where $\sum_{x=a}^{b} \frac{1}{n} = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} = \frac{n}{n} = 1$

Ex) A dice is throw once , define a random variable X is the number shown by the dice . Find the probability mass function of X . In this case the r.v. X takes the values 1 , 2 , 3 , 4 , 5 , & 6 ; and the corresponding probability function of X will be given as :

$$P(X) = \begin{cases} \frac{1}{6} & ; \quad X = 1,2,3,4,5,6 \\ 0 & \quad o.w. \end{cases}$$

Which is clearly that $0 \le P(x) \le 1$ and $\sum_{x=1}^{n} P(x) = 1$; therefore

$P(X)$ is a p.m.f. of X .



2.2- Bernoulli Distribution :

Suppose that a trial whose outcome can be classified as either a “success” or as “ failure” is performed . Let X b a r.v. taking value (1) if the outcome is success and (0) if it is failure , then X is said to be a Bernoulli r.v.

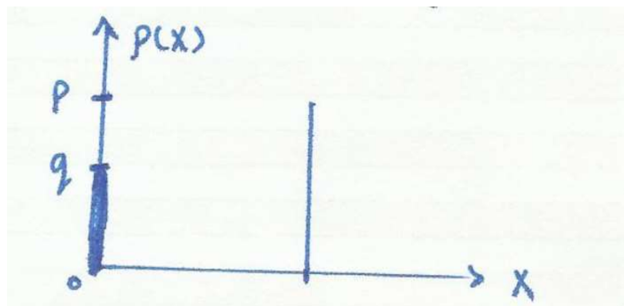
$$P(X) = \begin{cases} P^x(1-P)^{1-x} & ; \quad X = 0,1 \\ 0 & \quad o.w. \end{cases}$$

$P(X=0) = P^0 q^{1-0} = q$ فشل

$P(X=1) = P^1 q^{1-1} = P$ نجاح

Discrete Bernoulli distribution $x \sim Ber(P)$

P : it is a parameter of distribution



Ex) tossing the coins , $S = \{ H, T \}$, $\therefore P(X) = \frac{1}{2}$

$P(X) = P^x q^{1-x}$

$P(X=0) = (1/2)^0 (1/2)^{1-0} = 1/2$

$P(X=1) = (1/2)^1 (1/2)^{1-1} = 1/2$

Ex) An urn contains 10 red and 20 white balls , Draw five balls at random from the urn , one at a time and with replacement , let the draw of red ball be considered success , and the trails are independent .

Sol) $P(X=1) = {C_1^{10}}/{C_1^{30}} = \frac{10}{30} = \frac{1}{3}$ احتمال ظهور احدى الكرات حمراء (نجاح)

$$P(X) = \begin{cases} P^x q^{1-x} & ; \quad X = 0,1 \\ 0 & \quad o.w. \end{cases}$$

$P(X=0) = (1/3)^0 (2/3)^{1-0} = 2/3$

$P(X=1) = (1/3)^1 (2/3)^{1-1} = 1/3$

$\sum_{x=0}^{1} P(X) = \frac{2}{3} + \frac{1}{3} = 1 \quad \therefore \quad P.m.f.$

Ex) A fair dice is tossed one . call the outcome a success if a six is rolled , and all the other outcomes being considered failures .

This mean that we have a Bernoulli trial with probability :

$P(X) = \frac{1}{6}$; $S = \{ 1 , 2 , 3 , 4 , 5 , 6 \}$

Sol)

$$P(X) = P^x q^{1-x}$$

$$P(X = x) = (1/6)^x (5/6)^{1-x}$$

$$P(X = 0) = (1/6)^0 (5/6)^{1-0} = 5/6$$

$$P(X = 1) = (1/6)^1 (5/6)^{1-1} = 1/6$$

$$\sum_{x=0}^{1} P(X) = \frac{5}{6} + \frac{1}{6} = 1 \qquad \therefore it\ is\ a\ P.m.f.$$

Bernoulli

نجاح S F فشل

محاولة ظهور حالة نجاح او فشل مرة واحدة .

اما في حالة ظهور عدد من محاولات (النجاح او الفشل) فإن :-

S F F S F F S S F S

P q q P q q P P q P

فإن X تمثل عدد محاولات النجاح (اكثر من مرة) وبذلك يستخدم توزيع ثنائي الحدين Binomial

$P(X = x) = C_x^n \ P^x q^{n-x}$, $X = 0, 1 , 2 , 3 , \dots , n$ عدد محاولات النجاح

2.3- Binomial Distribution : توزيع ثنائي الحدين

Consider an experiment consisting of independent trials, each individual trial results in a success and failure with probability (P) and (1-P) respectively .

If the r.v. X represent he number of observed successes in the (n) trials , then X is Binomial random variable with the (p.m.f.) as given below .

$$P(X) = \begin{cases} C_x^n \ P^x q^{n-x} & ; \quad X = 0,1,2,\dots,n \\ 0 & o.w. \end{cases}$$

And

$$\sum_{x=0}^{n} P(X) = 1 \ \rightarrow \ \sum_{x=0}^{n} C_x^n \ P^x q^{n-x} = \ (P+q)^n = 1$$

نظرية ثنائي الحدين

Discrete Binomial distribution $x \sim b(n, P)$

n, P : are the parameters of distribution

Ex) A family has 4 children, assume that the birth of each sex is equally likely. Let X denote the number of boys in family. Find the p.m.f. of X .

Sol)

We have $P(B) = P(G) = \frac{1}{2} = P \ \ and \ \ 1-P = q \ \ , \ \ n = 4$

$$x \sim b\left(4 \, , \frac{1}{2}\right) \ \ and \ \ P(X) = C_x^n \ P^x q^{n-x} \, ; \ X = 0,1,2,3,4$$

$$= C_x^4 \ (\tfrac{1}{2})^x (\tfrac{1}{2})^{4-x} = C_x^4 (\tfrac{1}{2})^4$$

The p.m.f. of X is given by:

x	0	1	2	3	4
$P(X=x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

$$\boldsymbol{P(X=x)} = \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{16}{16} = 1 \ \text{ is p.m.f.}$$