Example: The following data represents the independent variable (X) which represents the age variable in years, and the dependent variable or response variable (y) which represents blood pressure, for ten people who were randomly selected. The data are shown in the table below:

y_i^2	X_i^2	$X_i y_i$	X_i	y_i	ت
12544	1225	3920	35	112	1
16384	1600	5120	40	128	2
16900	1444	4940	38	130	3
19044	1936	6072	44	138	4
24964	4487	10586	67	158	5
26244	4096	10368	64	162	6
19600	3481	8260	59	140	7
30625	4761	12075	69	175	8
15625	625	3125	25	125	9
20164	2500	7100	50	142	10
202094	**26157**	**71566**	**491**	**1410**	**المجموع**

It is required to create an analysis of variance table to test the following hypothesis:

$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

Solution: The hypothesis is written first as follows:

$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

Finding the sum of squares of the regression function, and using the values obtained in solving the example in the previous lecture, we can substitute in the following formulas:

$$SSR(X_1) = \hat{\beta}_1^2 S_{XX}$$
$$SSR(X_1) = (1.1396)^2 (2048.9)$$
$$SSR(X_1) = 2660.9$$

Find the sum of the total squares as follows:

$$SST = S_{yy}$$
$$SST = 3284$$

Calculate the sum of squared errors:

$$SSe = SST - SSR(X_1)$$
$$SSe = S_{yy} - \hat{\beta}_1^2 S_{XX}$$
$$SSe = 3284 - 2660.9$$
$$SSe = 623.1$$

S.O.V	d.f	S.S	M.S	Fcal.
R(X1)	1	$SSR(X_1) = 2660.9$	$M.S.R(X_1) = \frac{2660.9}{1} = 2660.9$	$F_{cal.} = \frac{2660.9}{77.9} = 34.16$
Error	8	$SSe = 623.1$	$M.S.e = \frac{623.1}{8} = 77.9$	
Total	9	$SST = 3284$		

As for the tabular F value, and through the F distribution tables, it will be:

$$F_{tab.} = F(0.05,1,8)$$
$$F_{tab.} = 5.32$$

By comparing the calculated value with the tabular value:

$F_{cal.} = 34.16 > F_{tab.} = 5.32$

Decision: Since the calculated value is greater than the table value, we reject the null hypothesis and accept the alternative hypothesis, i.e. $H_1 : \beta_1 \neq 0$.

Note:

Using the t-test has an advantage over using the F-test for the following reasons:

1- The t-test can be used to test a one-tailed hypothesis, i.e. $t(\alpha, n-1)$.

2- The t-test can be used to test the null hypothesis that β_1 equals a certain value, such as:

$H_0 : \beta_1 = 10$

$H_1 : \beta_1 \neq 10$

3- The t-test can be used to form confidence limits for β_1.

Confidence intervals$(\boldsymbol{\alpha - 1})\%$

Confidence intervals $(\boldsymbol{\alpha - 1})\%$ حدود الثقة

Confidence intervals are set for the parameter and not for the statistic, for example, they are set for μ and not for $\bar{X}$. Estimating a confidence interval or limit is the process of finding the true value of the parameter between an upper and lower confidence limit so that you can be confident of a value of $(\boldsymbol{\alpha - 1})\%$. The true value of the parameter lies between two confidence limits that take the following form:

$$\hat{\theta} \mp t(\frac{\alpha}{2}, n-2) S(\hat{\theta})$$

If the limits to be found for the estimator X are then the confidence limits are as follows:

$$Lower\ bound = \bar{X} - t(\frac{\alpha}{2}, n-2) S(\bar{X})$$

$$Upper\ bound = \bar{X} + t(\frac{\alpha}{2}, n-2) S(\bar{X})$$

Note: Confidence limits are the limits within which the null hypothesis is accepted and between which lies the upper and lower limits.

1- Estimating the confidence interval (limits) for the parameter β_0.

The formula used to find confidence limits for parameter β_0 is as follows:

$$\hat{\beta}_0 - t(\frac{\alpha}{2}, n-2)S(\hat{\beta}_0) \le \beta_0 \le \hat{\beta}_0 + t(\frac{\alpha}{2}, n-2)S(\hat{\beta}_0)$$

2- Estimating the confidence interval (limits) for the parameter β_1.

The formula used to find confidence limits for parameter β_1 is as follows:

$$\hat{\beta}_1 - t(\frac{\alpha}{2}, n-2)S(\hat{\beta}_1) \le \beta_1 \le \hat{\beta}_1 + t(\frac{\alpha}{2}, n-2)S(\hat{\beta}_1)$$

1- Estimating the confidence interval (bounds) for $\boldsymbol{\sigma^2}$ the (population variance)

Confidence limits $\%(n-\alpha)$ for the population variance $\boldsymbol{\sigma^2}$ can be formed using the following formula:

$$\frac{(n-1)MSe}{\chi^2(\frac{\alpha}{2}, n-2)} \le \sigma^2 \le \frac{(n-1)MSe}{\chi^2(1-\frac{\alpha}{2}, n-2)}$$

Where $\frac{(n-1)MSe}{\sigma^2}$ is distributed χ^2 With (n-2) degrees of freedom.

4- The confidence limits for the regression line itself for the mean response $\hat{y}_i$, i.e. $\hat{y}_0$.

We assume that X_0 is one of the original values of the variable X for which we want to estimate a confidence interval for the average response, i.e.:

$$\hat{y}_0 - t(\frac{\alpha}{2}, n-2)S(\hat{y}_0) \le E(y / X_0) \le \hat{y}_0 + t(\frac{\alpha}{2}, n-2)S(\hat{y}_0)$$

Example: The following data represents the independent variable (X) which represents the age variable in years, and the dependent variable or response variable (y) which represents blood pressure, for ten people who were randomly selected. The data are shown in the table below:

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المطلوب:

1-Confidence limits for the parameter β_0 .
2-Confidence limits for the parameter β_1.
3-Confidence limits for the parameter σ^2.
4- Forming confidence limits for the mean response at $X_0 = 40$.

Sol:

1-Confidence limits for the parameter β_0 .

By solving the above example, the value of $\boldsymbol{\beta_0}$ and the value of $\boldsymbol{S_{\hat{\beta}_0}}$ are as follows:

$\hat{\beta}_0 = 85.043, S_{\hat{\beta}_0} = 9.97$

Thus, the confidence limits are as follows:

$$85.043 - t(0.025,8)(9.79) \le \beta_0 \le 85.043 + t(0.025,8)(9.79)$$
$$85.043 - (2.31)(9.79) \le \beta_0 \le 85.043 + (2.31)(9.79)$$
$$62.05 \le \beta_0 \le 108.03$$

2-Confidence limits for the parameter β_1.

By solving the above example, the value of $\boldsymbol{\beta_1}$ and the value of $\boldsymbol{S_{\hat{\beta}_1}}$ are as follows:

$\hat{\beta}_1 = 1.1396, S_{\hat{\beta}_1} = 0.194$

Thus, the confidence limits are as follows:

$$1.139 - t(0.025,8)(0.194) \le \beta_1 \le 1.139 + t(0.025,8)(0.194)$$
$$1.139 - (2.31)(0.194) \le \beta_1 \le 1.139 + (2.31)(0.194)$$
$$0.69 \le \beta_1 \le 1.587$$

3-Confidence limits for the parameter σ^2

By solving the above example, the required values for the confidence limits for the population variance are as follows:

$MSe = 77.9$

The tabular values were found from the chi-square $\boldsymbol{\chi^2}$ distribution tables, as follows:

$$\chi^2(\frac{\alpha}{2}, n-2) = \chi^2(0.025,8) = 17.53$$

$$\chi^2(1-\frac{\alpha}{2}, n-2) = \chi^2(0.975,8) = 2.18$$

Thus, the confidence limits are as follows:

$$\frac{(n-1)MSe}{\chi^2(\frac{\alpha}{2}, n-2)} \le \sigma^2 \le \frac{(n-1)MSe}{\chi^2(1-\frac{\alpha}{2}, n-2)}$$
$$\frac{(9)(77.9)}{\chi^2(0.025,8)} \le \sigma^2 \le \frac{(9)(77.9)}{\chi^2(0.975,8)}$$
$$39.99 \le \sigma^2 \le 321.61$$