greater impact on the model unless we eliminate this correlation by converting each variable to a standard variable. In order to convert the variable into a standard variable, we use the following formula:

$$X_i^* = \frac{X_i - \bar{X}}{S_{xi}}$$

The variable y has the following formula:

$$y_i^* = \frac{y_i - \bar{y}}{S_y}$$

were

$$S_{xi} = \sqrt{S_{xi}^2} = \sqrt{\frac{S_{xi\,xi}}{n-1}} = \sqrt{\frac{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}{n-1}}$$

$$S_{yi} = \sqrt{S_{yi}^2} = \sqrt{\frac{S_{yi\,yi}}{n-1}} = \sqrt{\frac{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}{n-1}}$$

Thus, the vector of standard parameters is:

$$\hat{\beta}^* = (X^{*'}X^*)^2 X^{*\prime} y^*$$

The standard partial regression coefficient can also be obtained from the partial regression coefficient through the following relationship:

$$\hat{\beta}_i^* = \hat{\beta}_i (\frac{S_{xi}}{S_{yi}})$$

The partial regression coefficient can also be obtained from the standard partial regression coefficient through the following relationship:

$$\hat{\beta}_i = \hat{\beta}_i^* (\frac{S_{yi}}{S_{xi}})$$

Example:

If you have the following data:

X_4	X_3	X_2	X_1	y	المشاهدة
1	10	6	4	3	1
4	8	7	5	6	2
2	2	8	3	8	3
5	4	3	4	9	4
4	6	2	3	4	5
3	3	1	5	2	6
19	33	27	24	32	المجموع

find the following:

1- Estimate the regression equation and then create an analysis of variance table.

2- Determination coefficient.

3- Multiple correlation coefficient.

4- Partial correlation coefficient between X1 and X3 with constant X2 (partial correlation coefficient of first order).

5- Partial correlation coefficient between X1 and X3 with X2 and X4 constant (second-order partial correlation coefficient).

6- Standard partial regression coefficient through:

i) Through the partial regression coefficient.

ii) Through data.

solution:

1- **Estimate the regression equation and then create an analysis of variance table.**

From the data, it is noted that there are four independent variables, namely X1, X2, X3, and X4. Therefore, we have a multiple regression model. To form the equation of the regression line, we use the following formula:

$\hat{\beta} = (X'X)^{-1}X'y$

We form the matrix (X'X) as follows:

$$(X'X) = \begin{bmatrix} n & \sum X_{i1} & \sum X_{i2} & \sum X_{i3} & \sum X_{i4} \\ \sum X_{i1} & \sum X_{i1}^2 & \sum X_{i1}X_{i2} & \sum X_{i1}X_{i3} & \sum X_{i1}X_{i4} \\ \sum X_{i2} & \sum X_{i2}X_{i1} & \sum X_{i2}^2 & \sum X_{i2}X_{i3} & \sum X_{i2}X_{i4} \\ \sum X_{i3} & \sum X_{i3}X_{i1} & \sum X_{i3}X_{i2} & \sum X_{i3}^2 & \sum X_{i3}X_{i4} \\ \sum X_{i4} & \sum X_{i4}X_{i1} & \sum X_{i4}X_{i2} & \sum X_{i4}X_{i3} & \sum X_{i4}^2 \end{bmatrix}$$

$$X'y = \begin{bmatrix} \sum y_i \\ \sum x_{i1}y_i \\ \sum x_{i2}y_i \\ \sum x_{i3}y_i \\ \sum x_{i4}y_i \end{bmatrix}$$

We prepare a table to calculate the values of the matrix (X'X) and the vector (X'y) as follows:

X_3X_4	X_2X_4	X_2X_3	X_1X_4	X_1X_3	X_1X_2	X_4	X_3	X_2	X_1	y	المشاهدة
10	6	60	4	40	24	1	10	6	4	3	1
32	28	56	20	40	35	4	8	7	5	6	2
4	16	16	6	6	24	2	2	8	3	8	3
20	15	12	20	16	12	5	4	3	4	9	4
24	8	12	12	18	6	4	6	2	3	4	5
9	3	3	15	15	5	3	3	1	5	2	6
99	76	159	77	135	106	19	33	27	24	32	المجموع

y^2	X_4y	X_3y	X_2y	X_1y	X_4^2	X_3^2	X_2^2	X_1^2
9	3	30	18	12	1	100	36	16
36	24	48	42	30	16	64	49	25
64	16	16	64	24	4	4	64	9
81	45	36	27	36	25	16	9	16
16	16	24	8	12	16	36	4	9
4	6	6	2	10	9	9	1	25
210	110	160	161	124	71	229	163	100

$$(X'X) = \begin{bmatrix} 6 & 24 & 27 & 33 & 19 \\ 24 & 100 & 106 & 135 & 77 \\ 27 & 106 & 163 & 159 & 76 \\ 33 & 135 & 159 & 229 & 99 \\ 19 & 77 & 76 & 99 & 71 \end{bmatrix} \quad X'y = \begin{bmatrix} 32 \\ 124 \\ 161 \\ 160 \\ 110 \end{bmatrix}$$

After finding the inverse of the matrix (X’X), we find the vector of estimated parameters as follows:

$$\hat{\beta} = (X'X)^{-1}X'y$$

$$\hat{\beta} = \begin{bmatrix} 6.836 & -0.972 & -0.243 & -0.0517 & -0.4423 \\ -0.972 & 0.2811 & 0.0135 & -0.0237 & -0.02611 \\ -0.243 & 0.0135 & 0.0314 & -0.005 & 0.023776 \\ -0.0517 & -0.0237 & -0.005 & 0.0248 & 0.01 \\ -0.4423 & -0.02611 & 0.023776 & 0.01 & 0.1208 \end{bmatrix} \begin{bmatrix} 32 \\ 124 \\ 161 \\ 160 \\ 110 \end{bmatrix}$$

$$\hat{\beta} = \begin{bmatrix} 2.078 \\ -0.739 \\ 0.767 \\ -0.298 \\ 1.389 \end{bmatrix}$$

That is, the estimated regression equation is:

$$\hat{y}_i = 2.078 - 0.739X_{i1} + 0.767X_{i2} - 0.298X_{i3} + 1.389X_{i4}$$