- Find the probability that the family will have at least two boys .

$$\boldsymbol{P(X \geq 2)} = P(X=2) + P(X=3) + P(X=4) = \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/16 & 0 \leq x < 1 \\ 5/16 & 1 \leq x < 2 \\ 11/16 & 2 \leq x < 3 \\ 15/16 & 3 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$$

Ex) A machine produces a certain item with 0.05 defectives. random sample of 6 items is selected from the output of this machine. Let X the number of defectives in the sample. Find the probability mass function of X .

Sol) X = 0 , 1 , 2 , 3 , 4 , 5 , 6      X: تمثل عدد الوحدات التالفة او المعيبة

n = 6      حجم العينة

P = 0.05      نسبة المعيب

q = 1-P = 1 – 0.05 =      نسبة غير المعيب

$P(X=x) = C_x^n \ P^x q^{n-x} = C_x^6 \ (0.05)^x (0.95)^{6-x} \ , \ X = 1,2,3,\dots,6$

The p.m.f. of X is given by:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{P(X)}$ | $(0.95)^6$ | $6(0.05)^1(0.95)^5$ | … | … | … | … | $(0.05)^6$ |

What is the probability that there will be 3 defective items in the sample?

$P(X=3) = C_3^6 \ (0.05)^3 (0.95)^{6-3}$

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مثال/ واجب : لو كان لدينا صندوق يحتوي على 30 كرة 10 حمراء و 20 بيضاء، سحبت خمسة كرات من الصندوق بشكل عشوائي، جد دالة الكتلة الاحتمالية اذا علمت ان X يمثل سحب الكرة الحمراء.

Ex) Three coins are tossed once. Let the r.v. X denoted the number of heads appears. Find the probability.

Sol)

$$S = \{ HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

3 2 1 0

X = 0 , 1 , 2 , 3 عدد مرات ظهور الصورة

*p.m.f.* is given by:

$$P(X) = \begin{cases} C_x^n \ P^x q^{n-x} & ; \quad X = 0,1,2,3 \\ 0 & \quad o.w. \end{cases}$$

& $P = \frac{1}{2} = H \quad ; \quad q = \frac{1}{2} = T \quad ; \quad n = 3$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X)$ | 1/8 | 3/8 | 3/8 | 1/8 |

* What is the probability that two head appear.

$P(X = 2) = C_2^3 \ (\frac{1}{2})^2 (\frac{1}{2})^{3-2} = \frac{3}{8}$

* What is the probability that at least one head appears.

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \frac{1}{8} = \frac{7}{8}$

* What is the probability that at most two head appears.

$P(X \leq 2) = P(X = 2) + P(X = 1) + P(X = 0) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$

Ex) Team A has probability 2/3 of wining when ever it plays, if A plays 4 games; Find the probability function that wins:

i) exactly 2 games .

ii) at least one game .

iii) more than half of the game .

Sol) $P = \frac{2}{3}$ ; $q = 1 - P = 1 - \frac{2}{3} = \frac{1}{3}$ ; $n = 4$

$$P(X) = \begin{cases} C_x^n \ P^x q^{n-x} & ; \quad X = 0,1,2,3,4 \\ 0 & \quad o.w. \end{cases} \quad ; \quad X \sim b(4, \tfrac{2}{3})$$

i) $P(X = 2) = C_2^4 \ (\frac{2}{3})^2 (\frac{1}{3})^{4-2} = \frac{24}{81}$

ii) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - C_0^4 \ \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{4-0} = \frac{80}{81}$

iii) $P(X > 2) = P(X = 3) + P(X = 4) = \frac{48}{81}$

Ex) A study has shown that in Baghdad university 70 % of all staff own two television sets, find the probability that among 8 of each staff:-

a) 2 will have 2 Tv. Sets .

b) at least 6 will have 2 Tv. Sets .

c) at most 2 will have 2 Tv. Sets .

Sol) P = 0.70 ; q = 0.30 ; n = 8 ; $X \sim b(8, 0.70)$

$$P(X) = \begin{cases} C_x^n \ P^x q^{n-x} & ; \quad X = 0,1,2,3,\dots,8 \\ 0 & \quad o.w. \end{cases}$$

a) $P(X = 2) = C_2^8 \ (0.7)^2 (0.3)^{8-2} = 0.01$

b) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) = 0.5517$

c) $P(X \leq 2) = P(X = 2) + P(X = 1) + P(X = 0) = 0.0113$

## 2.4- Poisson Distribution:

The Poisson distribution appears in many natural and physical phenomena suck as:

1 - The number of misprints per page in large text.

2- The number of accidents per unit of time {hour, day, week or month} on a highway.

الجسيمات المنبعثة          مشعة          مادة

3- The number of $\alpha -$ particles emitted by a radioactive substance per unit of time.

4- The number of telephone calls per unit of time received at some switchboard.

5- The number of customers entering a post office or any (government department) on a given period of time.

Always $n \to \infty$ كبيرة جدا

$$P(X) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & ; \quad X = 0,1,2,3,\dots \\ 0 & \quad o.w. \end{cases}$$

Discrete Poisson distribution $x \sim Po(\lambda)$

$\lambda$ : it is a parameter of distribution.

*p.m.f.* $\sum_{x=0}^{\infty} P(X) = 1$

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda}[1 + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots] = e^{-\lambda}e^{\lambda} = e^0 = 1$$

ويتم تحويل توزيع Binomial الى توزيع Poisson عندما $n \to \infty$ وان $P \to 0$ لتصبح معلمة التوزيع بالشكل التالي:-

$\lambda = nP$

وهو قانون يستخدم لاستخراج قيمة $\lambda$ عند عدم توفرها في السؤال.

Ex) Suppose that 3% of the items made by a factory are defective. A sample of 100 items is drawn at random. What is the probability that it will contain exactly 2 defective items? Using

a) Binomial Distribution ; b) Poisson distribution

Sol) $P = 0.03$ , $q = 1 - P = 0.97$ , $n = 100$

a) $P(X = 2) = C_2^{100} (0.03)^2(0.97)^{98} = 0.2251$