$$\therefore P_{ij}^{n} = \sum_{k=0}^{\infty} P_{ik} P_{kj}^{n-1}$$

In general we have :

$$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^{n} P_{kj}^{m}$$

With:

$$P_{ij}^{0} = \begin{cases} 1 & if \quad i = j \\ 0 & if \quad i \neq j \end{cases}$$

And this is the chapman-kolmogrov equation.

Example(1): Consider a M.C with transition matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$$

Find:

1. Two step transition matrix
2. 4- step transition matrix
3. P_{12}^{2} , P_{21}^{2}, P_{12}^{4}, P_{22}^{4}

3.6 The Classification of States and Chains

A. Classification of Chains

1. Accessibility إمكانية الوصول

If state (j) of chain can be reached from state(i) in any number of transition i.e ($P_{ij}^{n} > 0$ for any n>0) , then the state (j) is said to be accessible from state(i) to and denoted by ($i \longrightarrow j$).

Ex:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0.3 & 0.7 \\ 0.4 & 0 & 0.6 \\ 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$$

$0 \longrightarrow 1$ $0 \longrightarrow 2$

$1 \longrightarrow 0$ $1 \longrightarrow 2$

$2 \longrightarrow 0$ $2 \longrightarrow 1$

2. Communication States خاصية المبادلة أو الاتصال

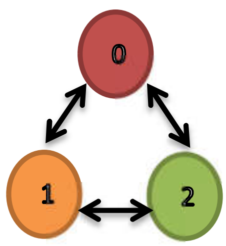
If two state (i) and (j) each accessible to each other then they are said to be communicated and denoted by ($i \longleftrightarrow j$).

The communication has the following properties:

a) Reflexivity
For any state(i) then ($i \longleftrightarrow i$).

b) Symmetry
If ($i \longleftrightarrow j$) then ($j \longleftrightarrow i$).

c) Transitivity



If ($i \longleftrightarrow j$) and ($j \longleftrightarrow k$) then ($i \longleftrightarrow k$).