Then $P_0(t) = e^{-\lambda t} = \dfrac{e^{-\lambda t}(\lambda t)^0}{0!}$ --- ①

And for $n \geq 1$

$$P_n'(t) = -\lambda\left[P_n(t) - P_{n-1}(t)\right]$$

$$P_n'(t) + \lambda P_n(t) = \lambda P_{n-1}(t)$$

Multiplying by $e^{\lambda t}$, we get:

$$e^{\lambda t}\left[P_n'(t) + \lambda P_n(t)\right] = \lambda e^{\lambda t} P_{n-1}(t)$$

If $n = 1$, then:

$$e^{\lambda t}\left[P_1'(t) + \lambda P_1(t)\right] = \lambda e^{\lambda t} P_0(t)$$

$$e^{\lambda t} P_1'(t) + \lambda e^{\lambda t} P_1(t) = \lambda e^{\lambda t} e^{-\lambda t}$$ From eq(1)

$$\frac{d}{dt}\left(e^{\lambda t} \cdot P_1(t)\right) = \lambda$$

$$d\left(e^{\lambda t} \cdot P_1(t)\right) = \lambda dt$$