We now form the analysis of variance table:

S.O.V	d.f	S.S	M.S	Fcal.
$R(X_1, X_2, \ldots, X_m)$	m	$\hat{\beta}' X' y - n\bar{y}^2$	$M.S.R$	$\frac{M.S.R}{M.S.e}$
$Error(X_1, X_2, \ldots, X_m)$	$n-m-1$	$SST - SSR$	$M.S.e$	
$Total$	$n-1$	$y' y - n\bar{y}^2$		

The hypothesis to be tested is:

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4$
$H_1: \beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4$

Interpretation of the null hypothesis: There is no variable that explains the changes occurring in y.

Interpretation of the alternative hypothesis: At least one variable is important in explaining the changes occurring in y.

The sums of squares are calculated as in the previous example and included in the analysis of variance table as follows:

S.O.V	d.f	S.S	M.S	Fcal.
$R(X_1, X_2, X_3, X_4)$	4	32.83	8.209	1.263
$Error(X_1, X_2, X_3, X_4)$	1	6.496	6.496	
$Total$	5	39.33		

$F(0.05,4,1) = 224.58$

Comparing the two values we get:
$F_{cal.} = 1.263 < F(0.05,4,1) = 224.58$

Thus, the null hypothesis is accepted and the alternative hypothesis is rejected, that is, the variables X1, X2, X3, and X4 do not have a significant effect on the variable y.

2-**Determination coefficient.**

$$R^2 = \frac{SS\ due\ to\ Regression}{SS\ Total}$$

$$R^2 = \frac{32.83}{39.33} = 0.834$$

3-Multiple correlation coefficient.

$$r_{y\hat{y}} = \sqrt{R^2} = \sqrt{0.834} = 0.913$$

3-**Partial correlation coefficient between X1 and X3 with constant X2 (first-order partial correlation coefficient).**

$$r_{ij.k} = \frac{r_{ij} - r_{ik}r_{jk}}{\sqrt{(1-r_{ik}^2)(1-r_{jk}^2)}}$$

$$r_{13.2} = \frac{r_{13} - r_{12}r_{32}}{\sqrt{(1-r_{12}^2)(1-r_{32}^2)}}$$

We find the simple correlation coefficient between X1 and X3 as follows:

$$r_{X1X3} = \frac{S_{X1X3}}{\sqrt{S_{X1X1}S_{X3X3}}} = \frac{\sum(X_{i1}-\bar{X}_1)(X_{i3}-\bar{X}_3)}{\sqrt{(\sum X_{i1}^2 - \frac{(\sum X_{i1})^2}{n})(\sum X_{i3}^2 - \frac{(\sum X_{i3})^2}{n})}} = \frac{3}{\sqrt{(4)(47.5)}} = 0.217$$

In the same way, we find the rest of the correlation coefficients between the variables. For example, the correlation coefficient between the two variables X1 and X2 can be found as follows:

$$r_{X1X2} = \frac{S_{X1X2}}{\sqrt{S_{X1X1}S_{X2X2}}} = \frac{\sum(X_{i1}-\bar{X}_1)(X_{i2}-\bar{X}_2)}{\sqrt{(\sum X_{i1}^2 - \frac{(\sum X_{i1})^2}{n})(\sum X_{i2}^2 - \frac{(\sum X_{i2})^2}{n})}} = -0.1552$$

And so on for the rest of the simple correlation coefficients between the variables to form the correlation matrix, which is:

$$r = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{12} & r_{22} & r_{23} & r_{24} \\ r_{13} & r_{23} & r_{33} & r_{34} \\ r_{14} & r_{24} & r_{34} & r_{44} \end{bmatrix}$$

Thus, the correlation matrix is as follows:

$$r = \begin{bmatrix} 1 & -0.1552 & 0.217 & 0.1519 \\ -0.1552 & 1 & 0.236 & -0.448 \\ 0.217 & 0.236 & 1 & -0.242 \\ 0.1519 & -0.448 & -0.242 & 1 \end{bmatrix}$$

Thus, after finding the correlation coefficients we need to find the partial correlation coefficient between X1 and X3 with X2 constant (partial correlation coefficient of the first order), we apply the formula as follows:

$$r_{13.2} = \frac{r_{13} - r_{12}r_{32}}{\sqrt{(1 - r_{12}^2)(1 - r_{32}^2)}}$$

$$r_{13.2} = \frac{0.217 - (-0.1522)(0.236)}{\sqrt{(1 - (-0.1522)^2)(1 - (0.236)^2)}} = \frac{0.235}{\sqrt{0.9206}} = \frac{0.253}{0.959} = 0.263$$

5-**Partial correlation coefficient between X1 and X3 with X2 and X4 constant (second-order partial correlation coefficient).**

$$r_{ij.kL} = \frac{r_{ij.k} - r_{iL.k}r_{jL.k}}{\sqrt{(1 - r_{iL.k}^2)(1 - r_{jL.k}^2)}}$$

$$r_{13.24} = \frac{r_{13.2} - r_{14.2}r_{34.2}}{\sqrt{(1 - r_{14.2}^2)(1 - r_{34.2}^2)}} = \frac{0.263 - (0.0933)(-0.157)}{\sqrt{(1 - (0.0933)^2)(1 - (-0.157)^2)}}$$