b) $\lambda = nP = 100 * 0.03 = 3$

$$P(X = 2) = \frac{e^{-3}3^x}{x!} = \frac{e^{-3}3^2}{2!} = 0.22404$$

Ex) The average of claims filed against an insurance company is 2 claims per day. What is the probability that on any given day:

1- Exactly one claim is filed against the insurance company?

2- No claim is filed against the insurance company?

3- Exactly three claims are filed against the insurance company?

Sol)

1- $\lambda = 2$ { average number of claims per day }

X = 1 { exact number of claims filed }

According to the Poisson distribution:

$$P(X) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & ; \quad X = 0,1,2,3,\ldots \\ 0 & o.w. \end{cases}$$

$$P(X = 1) = \frac{e^{-2}2^1}{1!} = \frac{(2.71828)^{-2} * 2}{1} = 0.27067$$

2- $$P(X = 0) = \frac{e^{-2}2^0}{0!} = (2.71828)^{-2} = 0.13534$$

3- $$P(X = 3) = \frac{e^{-2}2^3}{3!} = \frac{8}{6 * (2.71828)^2} = 0.18045$$

Ex) A student makes, on the average, one typing error on each page, what is the probability that the student would make exactly 2 typing error in a 3-page term paper?

$n * P = \lambda = 3$ (Average number of typing errors per 3 pages)

$X = 2$ (Exact number of typing errors in a 3-page term paper)

According to the Poisson distribution:

$n * P = 3 * 1 = 3$

$\therefore \; x \sim Po(\lambda = 3)$

$$P(X) = \begin{cases} \frac{e^{-3}3^x}{x!} & ; \quad X = 0,1,2,3,\ldots \\ 0 & o.w. \end{cases}$$

$$P(X=2) = \frac{e^{-3}3^2}{2!} = \frac{9}{2*(2.71828)^3} = 0.22404$$

2.5- Geometric Distribution: التوزيع الهندسي

Suppose that independent trials, each one having a probability (P) of being a success, are performed until the first success occurs. If X denotes the number of trials required, then:

يمثل حالات الفشل الى حين وصول اول حالة نجاح من عدة محاولات للفشل.

$$P(X) = \begin{cases} Pq^x & ; \quad X = 0,1,2,3,\ldots \\ 0 & o.w. \end{cases} \quad ; \quad P + q = 1$$

Discrete Geometric distribution $x \sim Ge(P)$

P : it is a parameter of distribution.

To show that $P(X)$ is $P.m.f.$

متوالية هندسية غير منتهية

$$\sum_{x=0}^{\infty} P(X) = \sum_{x=0}^{\infty} Pq^x = P\sum_{x=0}^{\infty} q^x = P[1 + q + q^2 + q^3 + \cdots] = P\left[\frac{1}{1-q}\right]$$

$$= P\left[\frac{1}{P}\right] = 1$$

ملاحظة :

أحيانا تكتب الدالة بدلالة جميع المحاولات

عدد المحاولات الكلية

$$Y = X + 1$$

النجاح الوحيد

عدد المحاولات الفشل

$$\rightarrow X = Y - 1 \quad \text{where} \quad Y \sim Ge(P)$$

$$\therefore \; P(Y) = \begin{cases} Pq^{y-1} & ; \quad Y = 1,2,3,\ldots \\ 0 & o.w. \end{cases}$$

Ex) If the probability that a person will believe a rumor (اشاعة) about the retirement (تقاعد) of a certain politician is 1/3 , then the probability that the fifth person to hear the rumor will be the first one to believe it is.

Sol)

$$P(X=5) = Pq^{x-1} = \frac{1}{3}(\frac{2}{3})^{5-1} = \frac{16}{243}$$

Ex) If the probability of the birth of a child with a defective heart in a special hospital is 0.07, then the probability that the 10^{th} child born in the hospital is the first one which has a defective heart is:

Sol)

$$P(X=10) = Pq^{x-1} = (0.07)(0.93)^{9} = 0.036428$$

Ex) The probability that a man hit the target is 5/6.

I) If he fires 5 times, what is the probability that he hits the target 4 time?

II) What is the probability that he will hit the target for the first time at his sixth shot?

Sol)

I) $X \sim b(n=5\,, P=\frac{5}{6})$

$$P(X=x) = C_x^n\ P^x q^{n-x} = C_x^5\ (\frac{5}{6})^x(\frac{1}{6})^{5-x} \qquad X=0,1,2,3,4,5$$

$$P(X=4) = C_4^5\ (\frac{5}{6})^4(\frac{1}{6})^1 =$$

II) $X \sim Ge(P=\frac{5}{6})$

$$P(X=x) = Pq^x \qquad X=0,1,2,3,\ldots$$

$$P(X=5) = (\frac{5}{6})(\frac{1}{6})^5$$

Or

$P(X=6) = (\frac{5}{6})(\frac{1}{6})^{6-1}$ او بدلالة الدالة الكلية

Ex) Assume that you toss a coin until you get the first head. What is the *p.m.f.* of this experiment.

Sol) $X \sim Ge(P = \frac{1}{2})$

$$P(X) = Pq^x = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^x \quad ; \quad X = 0,1,2,3,\dots$$

What is the probability that you will get the first head at the third trial?

$X \sim Ge\left(P = \frac{1}{2}\right) \quad ; \quad X = 0,1,2,3$ تمثل محاولات الفشل

$P(X) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^x \; ; \; X = 0,1,2,3$

$P(X = 2) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$ عدد محاولات الفشل

$P(Y) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{3-1}$ تمثل عدد المحاولات الكلية

<u>ملاحظه:</u>

في حالة احتواء السؤال على كلمة (first) او (until) في اغلب الأحيان فإن التوزيع هو هندسي.

Ex) Let *X,Y* two random variable with binomial distribution with parameter (2,P) and (3,P) respectively, if $P(X \geq 1) = 0.75$, find $P(Y \geq 1)$

Sol) $X \sim b(2\,,P) \quad , \; Y \sim b(3\,,P)$

$$P(X \geq 1) = 1 - P(X < 1)$$
$$= 1 - P(X = 0)$$
$$\because \; P(X \geq 1) = 0.75$$
$$\rightarrow \; P(X = 0) = 1 - \; 0.75 = 0.25$$
$$\rightarrow \; P(X = 0) = C_0^2 \; P^0 q^{2-0} = \; q^2$$
$$\therefore \; q^2 = 0.25 \quad \rightarrow \quad q = \sqrt[2]{0.25} = 0.5$$
$$\because \; p + q = 1 \quad \rightarrow \quad P + 0.5 = 1 \; \Longrightarrow \; P = 0.5$$
$$\therefore \; P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0)$$
$$= 1 - \; C_0^3 \, (0.5)^0 (0.5)^3 \quad = 1 - \; (0.5)^3$$
$$\Longrightarrow \; P(Y \geq 1) = 0.875$$