

### 3. Irreducible chain

غير قابل للاختزال أو التجزئة

If every state can be reached from every other state (in any number of transitions), then the chain is said to be irreducible chain and the transition matrix is said to be irreducible matrix.

### 4. Closed set of states

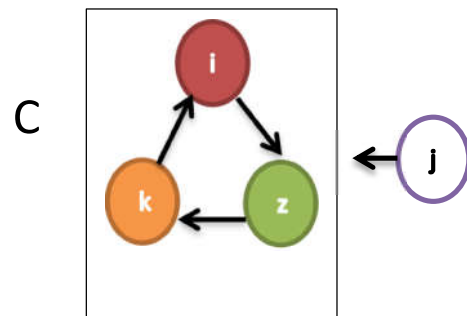
مجموعة مغلقة من الحالات

If  $(c)$  is a set of states such that no state outside  $(c)$  can be reached from any state in  $(c)$  then the set  $(c)$  is said to be closed set. If  $(c)$  is a closed set and  $(i \in c)$  while  $(j \notin c)$  then:

$$1. P_{kj} = 0 \quad (k \in c)$$

$$2. P_{ij} = \sum_{k=0}^{\infty} P_{ik} P_{kj} = 0$$

In general  $P_{ij}^n = 0$  for  $n \geq 1$



### 5. Absorbing State

حالة الاشباع أو الاستيعاب

If closed set  $(c)$  contains only one state  $(j)$  then the state  $(j)$  is called absorbing state. The state  $(j)$  is an absorbing state iff :

$$P_{jj} = 1, \quad P_{jk} = 0 \quad \forall \quad k \neq j$$

#### Remarks:

- The set of all states of a M.C will be a closed set.

- If a M.C does not contain any other closed set then the chain will be irreducible.
- The chain which are not irreducible are called reducible. In this case the number of closed set is two or more.

**Example(1):** A Markov chain  $X_1, X_2, X_3 \dots$  has the transition probability matrix:

	0	1	2
0	0	$1/3$	$2/3$
1	$2/3$	0	$1/3$
2	$1/3$	$2/3$	0

Sketch the transition diagram and classify this chain.

**Example(2):** A Markov chain  $X_1, X_2, X_3 \dots$  has the transition probability matrix:

	a	b	c	d	e
a	$1/2$	0	$1/2$	0	0
b	0	1	0	0	0
c	0	0	$1/2$	0	$1/2$
d	$1/3$	$1/3$	0	$1/3$	0
e	$1/2$	0	$1/2$	0	0

Sketch the transition diagram and classify this chain.