By taking integration for both sides, we get:

$$e^{\lambda t} P_1(t) = \lambda t + c$$

$$P_1(t) = e^{-\lambda t} (\lambda t + c)$$

When $t = 0$, we have:

$$P_1(0) = (0 + c)$$

$$0 = 0 + c \quad \Rightarrow \boxed{c = 0}$$

$$\therefore P_1(t) = \lambda t \, e^{-\lambda t} = \frac{e^{-\lambda t} (\lambda t)^1}{1!} \quad \text{---} (2)$$

By using mathematical induction we have:

$$P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \qquad , \; n = 0, 1, 2, \ldots$$