

Distributions Related with Poisson Process

التوزيعات المرتبطة بالعملة البوانسونية

Consider a Poisson Process, and let Z_1 denote the time to the first event. The X_n denote the time between the $(n-1)$ th and n th event. The Process $\{Z_n, n \geq 1\}$ is called interval time Process.

Theorem (2): The interval time between two events occurrences of a Poisson Process $\{X(t)\}$ has an exponential distribution with mean $\frac{1}{\lambda}$.

Proof: Let Z be a r.v. representing the interval between two event occurrence of a Poisson Process $\{X(t)\}$, and let:-

$$F(Z) = P_r\{Z \leq z\} \quad \text{be the distribution function}$$

Then the event $\{Z_1 > t\}$ take place iff no event of the Poisson Process occur in the interval $[0, t]$, then:

$$P_r\{Z_1 > t\} = P_r\{X(t) = 0\} = e^{-\lambda t}$$

$$\begin{aligned} F(Z_1) &= P_r\{Z_1 \leq t\} = 1 - P_r\{Z_1 > t\} \\ &= 1 - e^{-\lambda t} \end{aligned}$$