$$r_{13.24} = \frac{0.277}{0.983} = 0.282$$

Since the values in the above formula were found through the first-order partial correlation coefficient, for example, the partial correlation coefficient $r_{14.2}$ can be found as follows:

$$r_{14.2} = \frac{r_{14} - r_{12}r_{42}}{\sqrt{(1-r_{12}^2)(1-r_{42}^2)}} = \frac{0.1519 - (-0.1552)(-0.448)}{\sqrt{(1-(-0.155)^2)(1-(-0.448)^2)}}$$

$$r_{14.2} = 0.0933$$

The partial correlation coefficient $r_{34.2}$ can be found as follows:

$$r_{34.2} = \frac{r_{34} - r_{32}r_{42}}{\sqrt{(1-r_{32}^2)(1-r_{42}^2)}} = \frac{-0.242 - (0.236)(-0.448)}{\sqrt{(1-(0.236)^2)(1-(-0.448)^2)}}$$

$$r_{34.2} = -0.157$$

6- **Standard partial regression coefficient through**:

1-Through the partial regression coefficient.

Through the relationship between the standard partial regression coefficient and the partial regression coefficient, as follows:

$$\hat{\beta}_i^* = \hat{\beta}_i \left(\frac{S_{xi}}{S_{yi}}\right)$$

Where S_{x1} is the standard deviation of the variable X1, which can be found through the following formula:

$$S_{X1} = \frac{\sum(X_{i1} - \overline{X})^2}{n-1}$$

It applies to all variables and to the variable y as well. Thus, the standard deviation values for the rest of the variables will be as follows:

$S_y = 2.804\,, S_{X1} = 0.8944\,, S_{X2} = 2.8809\,, S_{X3} = 3.0822\,, S_{X4} = 1.4719$

Thus, the standard partial regression coefficient for the variables is as follows:

$$\hat{\beta}_1^* = \hat{\beta}_1 \left(\frac{S_{x1}}{S_y}\right) = (-0.739)\left[\frac{0.8944}{2.804}\right] = -0.235$$

$$\hat{\beta}_2^* = \hat{\beta}_2 \left(\frac{S_{x2}}{S_y}\right) = (0.767)\left[\frac{2.8809}{2.804}\right] = 0.788$$

$$\hat{\beta}_3^* = \hat{\beta}_3 \left(\frac{S_{x3}}{S_y}\right) = (-0.298)\left[\frac{3.0822}{2.804}\right] = -0.327$$

$$\hat{\beta}_4^* = \hat{\beta}_4 \left(\frac{S_{x4}}{S_y}\right) = (1.389)\left[\frac{1.4719}{2.804}\right] = 0.7291$$

2-Through data

The variables can be converted to standard form and then the standard partial regression coefficient can be calculated as follows:

$$X_i^* = \frac{X_i - \bar{X}_i}{S_i}$$

X_4	X_3	X_2	X_1	y	المشاهدة
-1.47196	1.459993	0.520658	0	-0.83192	1
0.566139	0.811107	0.867763	1.118034	0.237691	2
-0.79259	-1.13555	1.214868	-1.11803	0.950765	3
1.245505	-0.48666	-0.52066	0	1.307302	4
0.566139	0.162221	-0.86776	-1.11803	-0.47538	5
-0.11323	-0.81111	-1.21487	1.118034	-1.18846	6
					المجموع

Thus, using the same estimation method, the vector of estimated parameters is found using matrices, and thus the result will be as follows:

$$\hat{\beta}^{*} = \begin{bmatrix} -0.235 \\ 0.788 \\ -0.327 \\ 0.729 \end{bmatrix}$$

Estimating a confidence interval for a simple linear function with several partial coefficients

Assume that the regression equation is:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + e_i, i = 1,2,\dots,n$$

Suppose we have a linear function L, which takes the following form:

$$L = \beta_2 - \beta_3$$

To find a confidence interval for L, we do the following:

$$\hat{L} - t_{\frac{1}{2}\alpha,n-m-1} S_{\hat{L}} \leq L \leq \hat{L} + t_{\frac{1}{2}\alpha,n-m-1} S_{\hat{L}}$$

Since:

$$S_{\hat{L}} = \sqrt{S_{\hat{L}}^2} = \sqrt{S_{(\hat{\beta}_2-\hat{\beta}_3)}^2}$$

$$S_{(\hat{\beta}_2-\hat{\beta}_3)}^2 = S_{\hat{\beta}_2}^2 + S_{\hat{\beta}_3}^2 - 2Cov(\hat{\beta}_2, \hat{\beta}_3)$$

$$S_{(\hat{\beta}_2-\hat{\beta}_3)}^2 = MSe(C_{22} + C_{33} - 2C_{23})$$

Example: If you have the following data:

$$(X'X) = \begin{bmatrix} 7 & 14 & 17 & 18 \\ & 32 & 34 & 36 \\ & & 49 & 42 \\ & & & 48 \end{bmatrix}, \quad X'y = \begin{bmatrix} 56 \\ 116 \\ 156 \\ 142 \end{bmatrix}$$

Required: Find a confidence interval for $L = \beta_2 - \beta_3$ where $MSe = 1.22$ and n=7 and $t_{\frac{1}{2}0.05,7-3-1} = t_{0.025,3} = 3.182$