

Example(3): A Markov chain $X_1, X_2, X_3 \dots$ has the transition probability matrix:

	1	2	3	4	5
1	0.1	0.6	0.3	0	0
2	0.7	0.3	0	0	0
3	0	1	0	0	0
4	0	0	0	0.5	0.5
5	0	0	0	1	0

Sketch the transition diagram and classify this chain.

B. Classification of States

Definitions:

1. First Passage: Suppose that a system start with state(j). Let $f_{jk}^{(n)}$ be the probability that it reaches the state k for the first time at nth step (or after n transition), and let $p_{jk}^{(n)}$ be the probability that it reaches the state k not necessary for the first time after n transitions. Then the probability of the first passage is:

$$f_{jk}^{(n)} = \text{pro}\{X_n = k ; X_r \neq k, r = 1, \dots, n - 1 | X_0 = j\}$$

Let F_{jk} denote the probability that the system starting with state(j) will ever reaches to state(k), i.e:

$$F_{jk} = \sum_{n=1}^{\infty} f_{jk}^{(n)}$$

The mean of the first passage from state(j) to state (k) is given by:

$$M_{jk} = \sum_{n=1}^{\infty} n f_{jk}^{(n)}$$