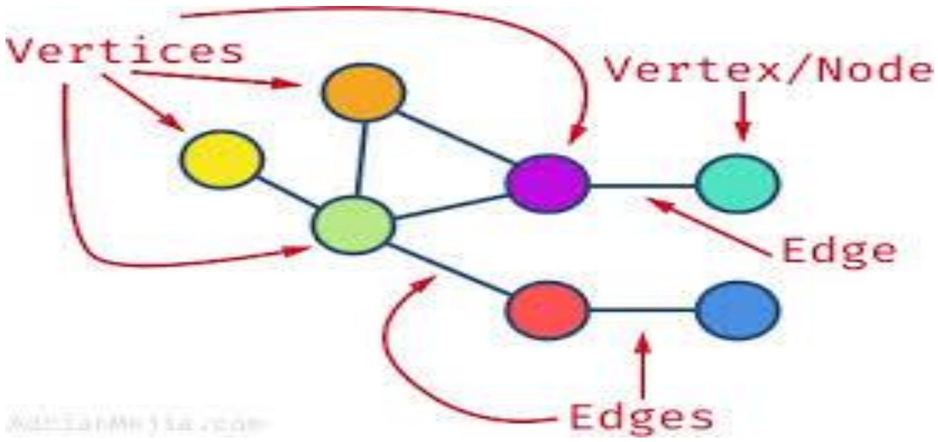
Graph Data Structure



by

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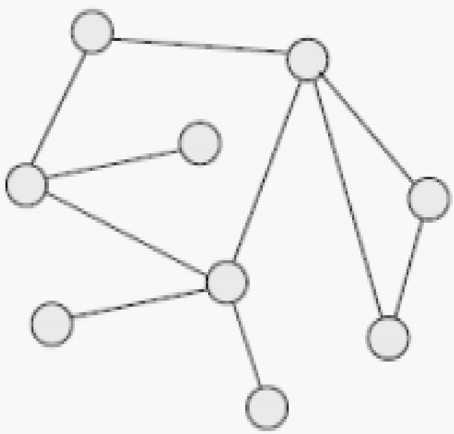
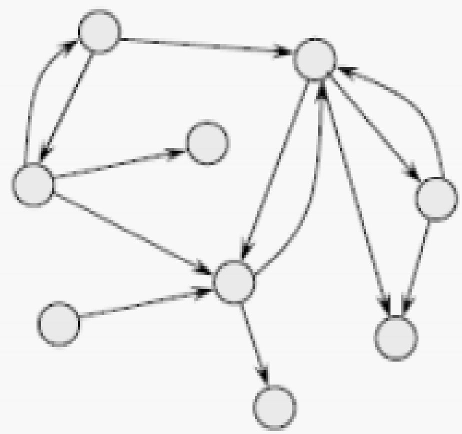
Graph Data Structure

A graph G consists of a set V of vertices and a set E of pairs of distinct vertices from V. These pairs of vertices are called edges.

If the pairs of vertices are unordered, G is an undirected graph. If the pairs of vertices are ordered, G is a directed graph or digraph.



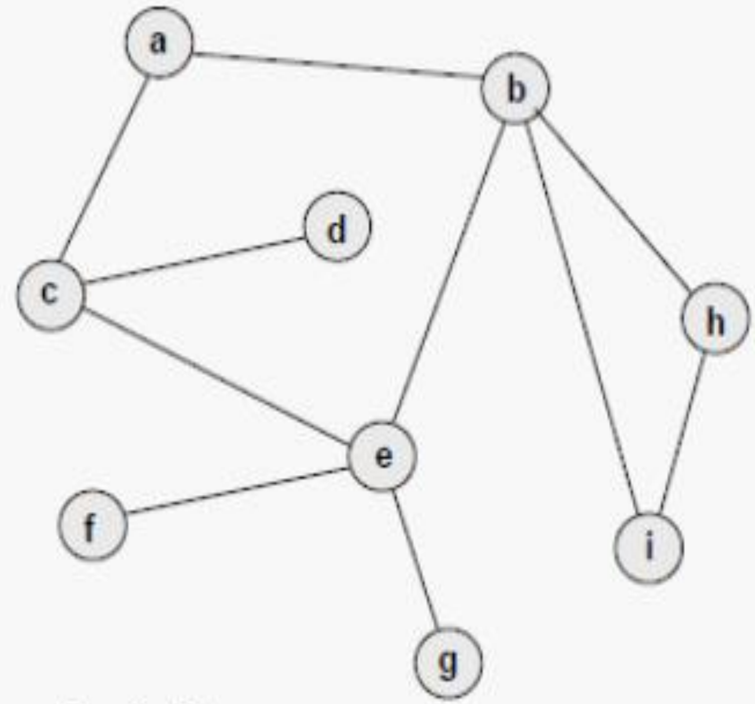
An undirected graph.



A directed graph.

Undirected Graph Terminology

An undirected graph G, where:



V = {a, b, c, d, e, f, g, h, i}

E = { {a, b}, {a, c}, {b, e}, {b, h}, {b, i} ,
{c, d} , {c, e} , {e, f} , {e, g} , {h, i} }

e = {c, d} is an edge, incident upon the vertices c and d

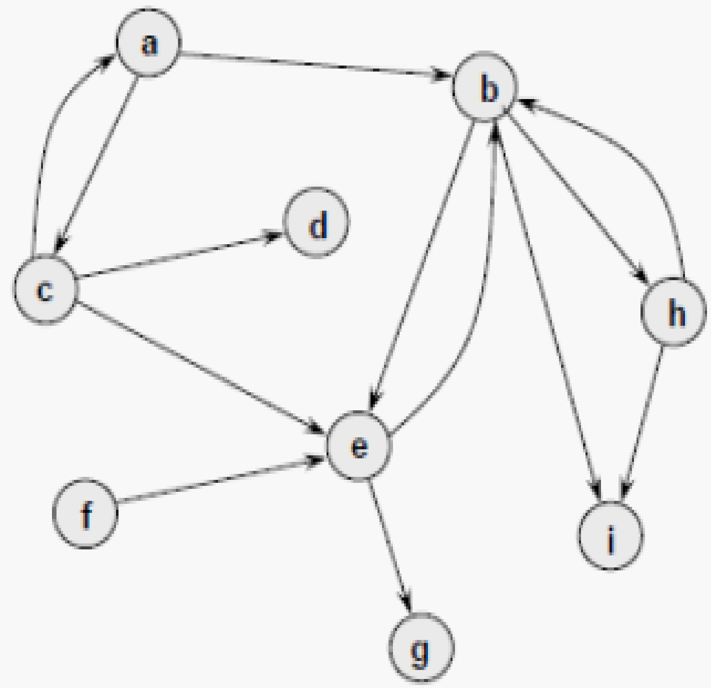
Two vertices, x and y, are adjacent if {x, y} is an edge (in E).

A path in G is a sequence of distinct vertices, each adjacent to the next.

A path is simple if no vertex occurs twice in the path.

Directed Graph Terminology

The terminology for directed graphs is only slightly different…



e = (c, d) is an edge, from c to d

A directed path in a directed graph G is a sequence of distinct vertices, such that there is an edge from each vertex in the sequence to the next.

A directed graph G is weakly connected if, the undirected graph obtained by suppressing the directions on the edges of G is connected (according to the previous definition).

A directed graph G is strongly connected if, given any two vertices x and y in G, there is a directed path in G from x to y.

Represent Graph Data Structure

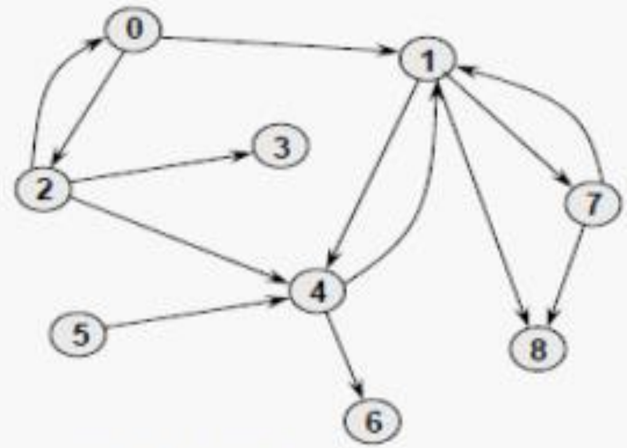
1- Adjacency Matrix Representation

2- Adjacency Table Representation

3- Adjacency List Representation

1. Adjacency Matrix Representation

A graph may be represented by a two-dimensional adjacency matrix:



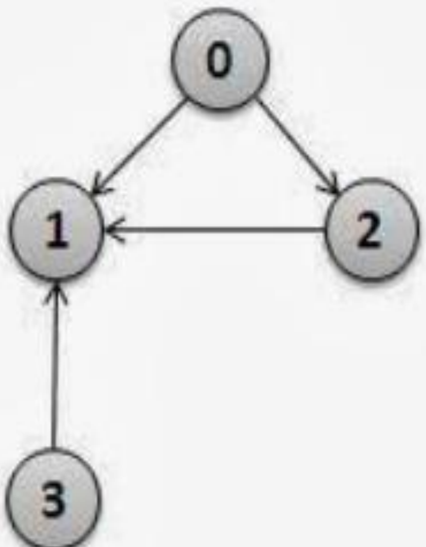
If G has n = |V| vertices, let M be an n by n matrix whose entries are defined by

$$m_{ij} = \begin{cases} 1 & \text{if (i, j) is an edge} \\ 0 & \text{otherwise} \end{cases}$$

$M(G) =$

	0	1	2	3	4	5	6	7	8
0	0	1	1	0	0	0	0	0	0
1	0	0	0	0	1	0	0	1	1
2	1	0	0	1	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	1	0	0
5	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Example For Adjacency Matrix Representation

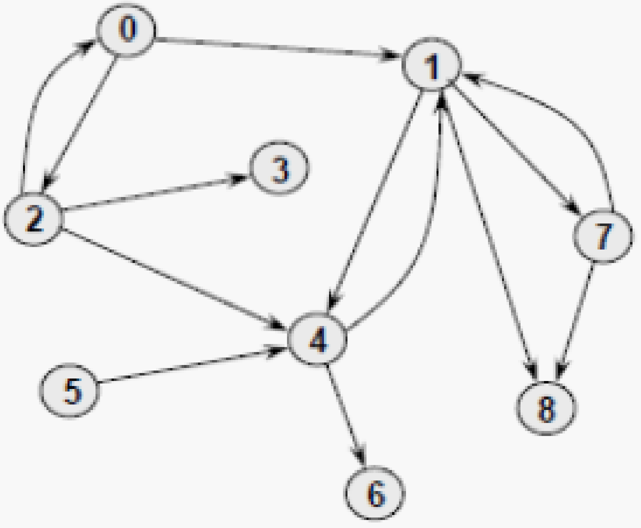


	0	1	2	3
0	0	1	1	0
1	0	0	0	0
2	0	1	0	0
3	0	1	0	0

Adjacency Matrix Representation of Directed Graph

2. Adjacency Table Representation

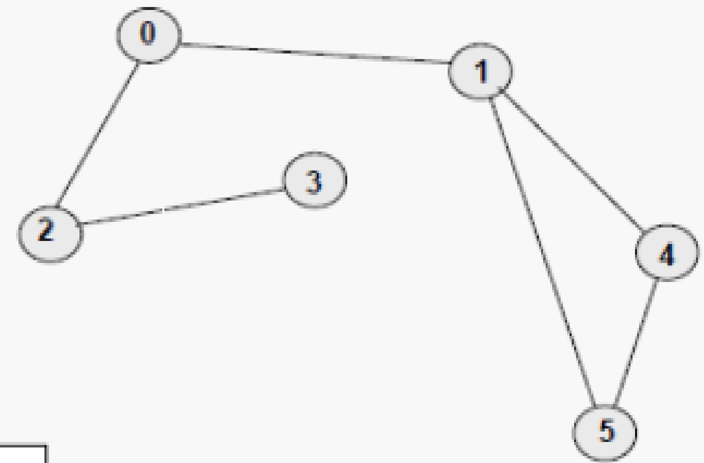
A slightly different approach is to represent only the adjacent nodes in the structure:

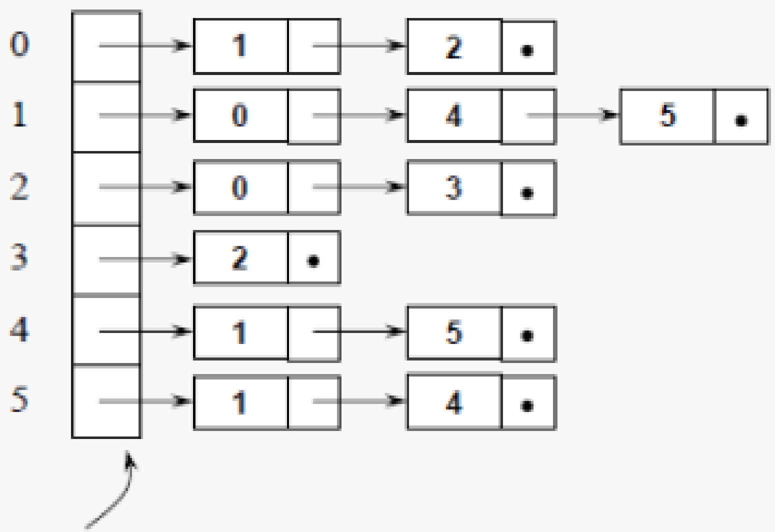


0	|	1	2	
1	|	4	7	8
2	|	0	3	4
3	|			
4	|	1	6	
5	|	4		
6	|			
7	|	1	8	
8	|			

3. Adjacency List Representation

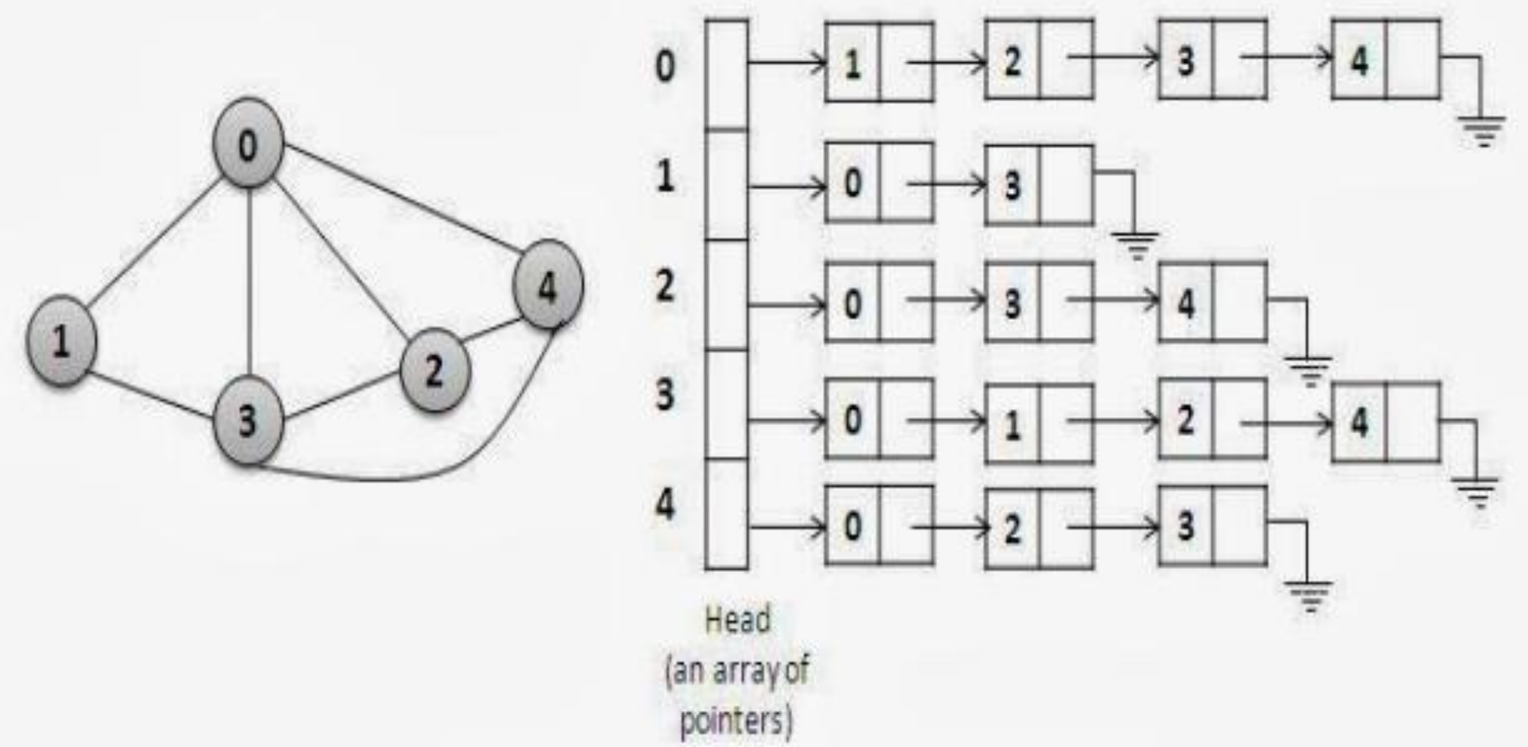
The adjacency list structure is simply a linked version of the adjacency table:





Array of linked lists, where list nodes store node labels for neighbors.

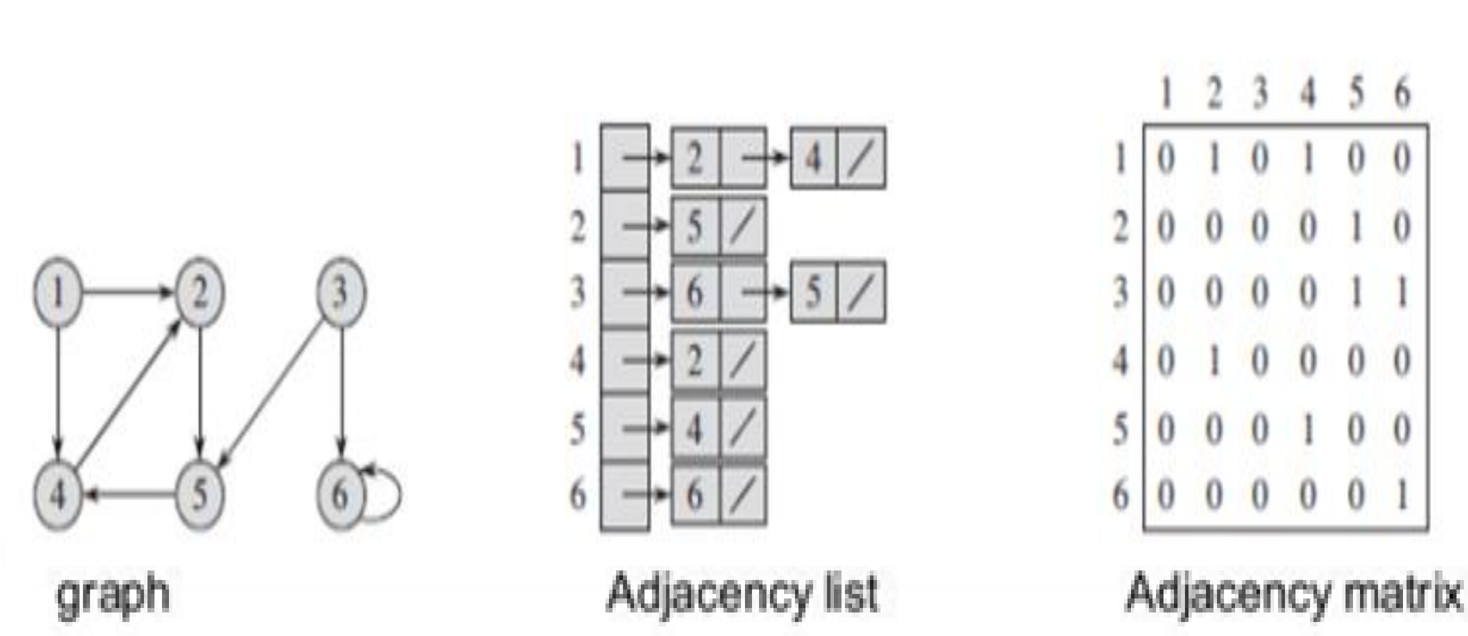
Example for Adjacency List Representation



Adjacency List Representation of Graph

Example for Adjacency List & Adjacency Matrix Representation

Graph Representation- directed



Example for Adjacency List & Adjacency Matrix Representation

Graph Representation- undirected