Mathematically, if A is a fuzzy set in the universal set S, and s1 and s2 are membership functions of A, then a convex combination of s1 and s2 is defined as follows:

$$\lambda s_1 + (1-\lambda)s_2$$

When intersection is performed on two convex fuzzy sets, the intersected portion is also a convex fuzzy set.

This is shown in Fig. 4.4.

The shaded portions show that the intersected portion is also a convex fuzzy set. The membership functions can have different shapes like triangle, trapezoidal, Gaussian, etc.

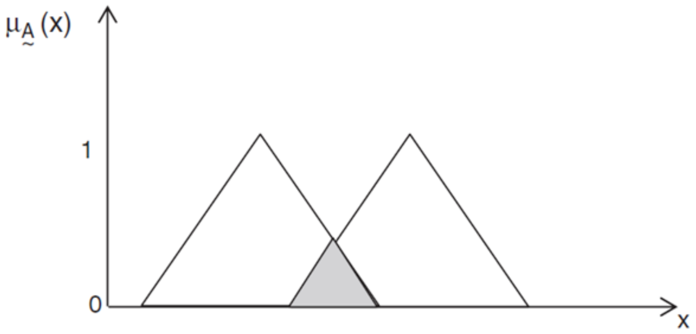

Fig. 4.4. Intersection of two convex sets

Convex fuzzy set:

If the fuzzy set is convex and α-cut is convex for every λ ϵ[0,1], then:

$$s_1, s_2 \in A^{\propto} \Rightarrow \lambda s_1 + (1-\lambda)s_2 \in A^{\alpha} \quad , \lambda \in [0,1]$$

Theorem

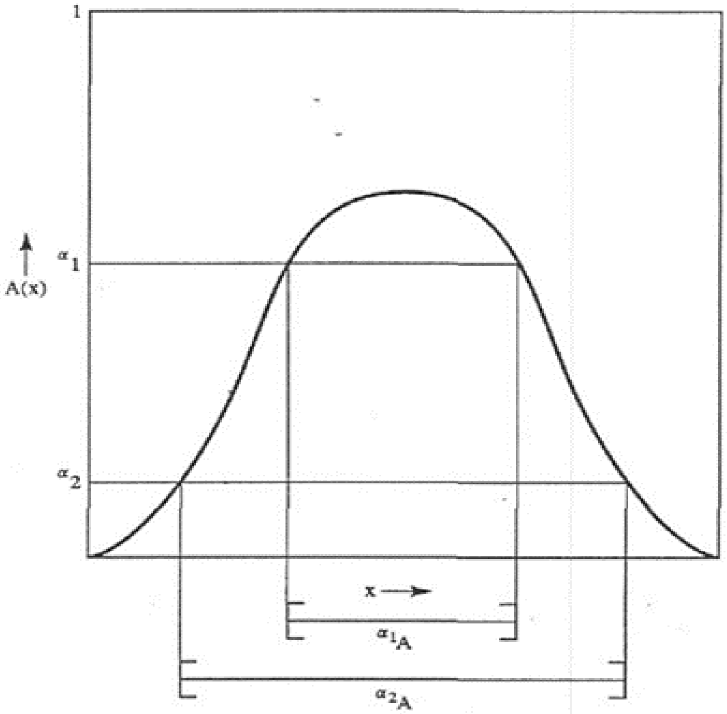

Figure 1.9 Subnormal fuzzy set that is convex.

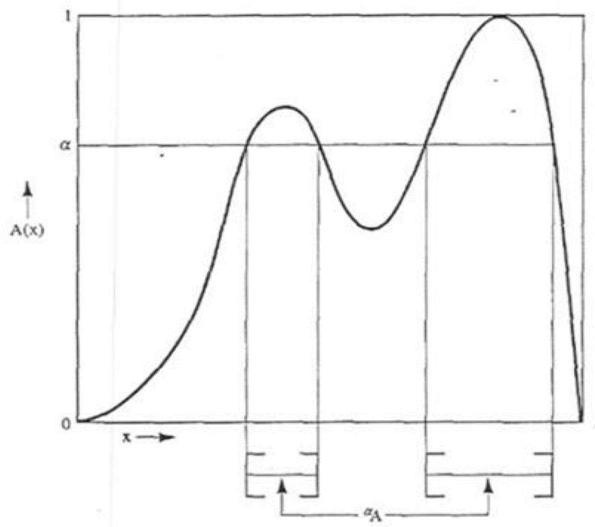

Figure 1.10 Normal fuzzy set that is not convex.

Convexity is an important property of fuzzy logic, and has many useful applications in decision making and control systems. For example, in optimization problems, convexity can be used to simplify the problem by ensuring that the objective function is easy to optimize, in control systems, and to ensure the stability and robustness of the system.