

A fuzzy set A in S is convex if and only if:

$$A(\lambda s_1 + (1 - \lambda)s_2) \geq \min[A(s_1), A(s_2)] \quad (1)$$

For all $s_1, s_2 \in R, \lambda \in [0, 1]$

Proof:

1 - Assume that the fuzzy set A in S is convex :

Then α -cut is convex

$$A^\alpha \Rightarrow \text{convex}$$

$$\forall s_1, s_2 \in A^\alpha \Rightarrow \lambda s_1 + (1 - \lambda)s_2 \in A^\alpha \dots \lambda \in [0, 1]$$

To prove eq. (1)

$$\text{Let } A(s_1) \leq A(s_2) \quad \& \quad \alpha = A(s_1) \dots \dots \dots (2)$$

From def. α - cut $= A^\alpha = \{s : s \in S : \mu_A(s) \geq \alpha, \alpha \in [0, 1]\}$

$$\begin{aligned} A(\lambda s_1 + (1 - \lambda)s_2) &\geq \alpha = A(s_1) \\ \dots \dots \dots &= \min[A(s_1), A(s_2)] \end{aligned}$$

From assume (2) $A(s_1) \leq A(s_2)$

لان من الفرضة (2) فتكون الأصغر $A(s_1)$ وممكن نكتب بالصيغة السابقة لهذا تكون

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For all $s_1, s_2 \in R, \lambda \in [0, 1]$

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