

2: let fuzzy set satisfy eq. (1) For all $s_1, s_2 \in R, \forall \lambda \in [0,1]$

To prove A is convex:

$$\Rightarrow A^\alpha \text{ is convex}$$

To prove $s_1, s_2 \in A^\alpha \Rightarrow \lambda s_1 + (1-\lambda)s_2 \in A^\alpha$

For all $s_1, s_2 \in A^\alpha$

$$A(s_1) \geq \alpha \ \& \ A(s_2) \geq \alpha$$

Form eq. (1)

$$\begin{aligned} A(\lambda s_1 + (1-\lambda)s_2) &\geq \min[A(s_1), A(s_2)] \\ &\dots\dots\dots \geq \min[\alpha, \alpha] = \alpha \\ A(\lambda s_1 + (1-\lambda)s_2) &\geq \alpha \end{aligned}$$

From $A(s_1) \geq \alpha$ and $s \in A^\alpha$ then

$$s_1, s_2 \in A^\alpha \Rightarrow A(\lambda s_1 + (1-\lambda)s_2) \in A^\alpha$$

Therefore A^α is convex for all $\alpha \in [0,1]$

Then A is convex

Concept Fuzzy number