Chapter one

Set theory

1.1 - Basic set theory قواعد نظرية المعلومات

Definitions:

1- ***Set*** **:** A set is collection of objects, the objects are called element (Events) of the set .

المجموعة : هي عبارة عن أشياء مجمعة ، وهذه الأشياء تسمى (عناصر او حوادث) في مجموعة .

A set is usually denoted by capital letter e.g. A,B,C,... ; If X is an event belonging to the set $\boldsymbol{A}$, we shall write $x \in \boldsymbol{A}$ if x is not an event or element of $\boldsymbol{A}$, we shall $x \notin \boldsymbol{A}$.

Exp) ***A***={0,1,2,3,4} and ***B***={ X : X < 8 } where (X : positive integer عدد صحيح موجب)

∴ **B** = { 1,2,3,4,5,6,7 }

2- ***Sample space*** $(\boldsymbol{S}, \boldsymbol{\Omega})$ **:** The all possible outcomes of a random experiment are called the sample space .

Exp) The experiment of tossing a coin is :

$$\boldsymbol{S} = \{ \boldsymbol{H}, \boldsymbol{T} \}$$

3- ***Random Experiment :*** All outcomes of the experiment known . وهي كل المخرجات من التجربة المعلومة

4- ***Events*** : An event is any collection of possible outcomes of an experiment that is any subset of $\boldsymbol{S}$

الحدث : أي ترتيب او مجموعة محتمل ظهوره في التجربة وهو مجموعة جزئية من فضاء العينة

Exp) $\boldsymbol{S} = \{ \mathbf{1, 2, 3, 4, 5, 6} \}$

$\boldsymbol{X} = \{ \mathbf{3} \}$

$\boldsymbol{Y} = \{ \mathbf{2, 3} \}$

$\boldsymbol{Z} = \{ \mathbf{1, 2, 3, 4, 5, 6} \} = \boldsymbol{S}$

5- ***Subset*** : The set ***A*** is said to be a subset of $\boldsymbol{S}$ **if** every element of ***A*** is also an element of $\boldsymbol{S}$.

يقال ان ***A*** مجموعة جزئية من فضاء الحالة او العينة ، اذا كانت كل العناصر الموجودة في ***A*** أيضا موجودة في فضاء العينة

i.e. A is contained in $\boldsymbol{S}$; that is mean :

$\boldsymbol{x} \in (\boldsymbol{A} \subset \boldsymbol{S})$

$\boldsymbol{x} \in \boldsymbol{A}$ $\boldsymbol{and}$ $\boldsymbol{x} \in \boldsymbol{S}$

6-***Empty set*** : A set is said to be an empty set or null set if it has no element , and is denoted by $\emptyset$;

$\emptyset \subset \boldsymbol{S}$

7-***Equal set***: Two set ***A*** and ***B*** are said to be equal if every elements in ***A*** are contained in ***B*** also .

$\boldsymbol{A} \subset \boldsymbol{B}$ and $\boldsymbol{B} \subset \boldsymbol{A}$

Then **A = B**

8-***Universal set***: All set under investigation are assumed to be subset of the space, we shall call it the space and denote it by $\boldsymbol{S}$ or $\boldsymbol{\Omega}$ **.**

9-***The complement***: The complement of the set A with respect to the $\boldsymbol{S}$, denoted by ($\overline{\boldsymbol{A}}$, $\boldsymbol{A^c}$, $\boldsymbol{A'}$) it is the set of all the element that are in $\boldsymbol{S}$ but not in ***A*** .

That is mean : $\boldsymbol{A^c} = \{ x : x \in \boldsymbol{s} \quad and \quad x \notin \boldsymbol{A} \}$.

10-***set difference***: Let ***A*** and ***B*** be any two subset of $\boldsymbol{S}$, the set of all points in ***A*** that are not in ***B*** , will denoted by $(\boldsymbol{A/B} \quad \boldsymbol{or} \quad \boldsymbol{A - B})$

That is mean :- $\boldsymbol{A/B} = \{ \boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{A} \quad \boldsymbol{and} \quad \boldsymbol{x} \notin \boldsymbol{B} \}$

$$\boldsymbol{A^c} = \boldsymbol{S/A}$$

$$\boldsymbol{A^c} = \{ \boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{S} \quad \boldsymbol{and} \quad \boldsymbol{x} \notin \boldsymbol{A} \}$$

- If ***A*** is a subset of $\boldsymbol{B}$ then $\boldsymbol{A} \subset \boldsymbol{B}$ and read ***A*** contained ***B*** or $\boldsymbol{B} \supset \boldsymbol{A}$ and read ***B*** contains ***A*** .
- If $\boldsymbol{A} \subset \boldsymbol{B}$ and $\boldsymbol{B} \subset \boldsymbol{A}$ then ***A*** = ***B*** .

Exp) Let $\boldsymbol{S}$ be the set of all natural numbers; $\boldsymbol{S} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \boldsymbol{4}, \boldsymbol{5}, \dots \dots\}$

Define : $\boldsymbol{A} = \{ \boldsymbol{x}: \boldsymbol{x} \quad is\ an\ even\ number, \quad \boldsymbol{x} \in \boldsymbol{S}\}$

And $\boldsymbol{B} = \{ \boldsymbol{y}: \boldsymbol{y} \quad is\ a\ multiple\ of\ 3\ , \quad \boldsymbol{y} \in \boldsymbol{S}\}$

Find : $(\boldsymbol{A^c}\ , \ \boldsymbol{B^c}\ , \ \boldsymbol{A/B}\ \ , \ \boldsymbol{B/A}\ )$

Solution:

$$\boldsymbol{A} = \{ \boldsymbol{2}, \boldsymbol{4}, \boldsymbol{6}, \boldsymbol{8}, \boldsymbol{10}, \boldsymbol{12}, \boldsymbol{14}, \boldsymbol{16}, \dots \}$$

$$\boldsymbol{B} = \{ \boldsymbol{3}, \boldsymbol{6}, \boldsymbol{9}, \boldsymbol{12}, \boldsymbol{15}, \boldsymbol{18}, \boldsymbol{21}, \dots \}$$

$$\boldsymbol{A^c} = \{ \boldsymbol{1}, \boldsymbol{3}, \boldsymbol{5}, \boldsymbol{7}, \boldsymbol{9}, \dots \} \ ; \ \boldsymbol{A^c} = \{ \boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{S} \quad \boldsymbol{and} \quad \boldsymbol{x} \notin \boldsymbol{A} \}$$

$$\boldsymbol{B^c} = \{ \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{4}, \boldsymbol{5}, \boldsymbol{7}, \boldsymbol{8}, \boldsymbol{10}, \boldsymbol{11}, \dots \}$$

$$\boldsymbol{A/B} = \{ \boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{A} \quad \boldsymbol{and} \quad \boldsymbol{x} \notin \boldsymbol{B} \}$$

$$\boldsymbol{A/B} = \{\boldsymbol{2}, \boldsymbol{4}, \boldsymbol{8}, \boldsymbol{10}, \boldsymbol{14}, \boldsymbol{16}, \boldsymbol{20}, \dots \}$$

$$\boldsymbol{B/A} = \{ \boldsymbol{3}, \boldsymbol{9}, \boldsymbol{15}, \boldsymbol{21}, \boldsymbol{27}, \dots \}$$

11-***Union of set***: Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be any two subset of $\boldsymbol{S}$, the union of $\boldsymbol{A}$ and $\boldsymbol{B}$, denoted by $(\boldsymbol{A} \cup \boldsymbol{B})$; is the set of all elements belonging either to $\boldsymbol{A}$ or to $\boldsymbol{B}$ or both.

$$(\boldsymbol{A} \cup \boldsymbol{B}) = \{\boldsymbol{x}: either\ \boldsymbol{x} \in \boldsymbol{A}\ or\ \boldsymbol{x} \in \boldsymbol{B}\ or\ both\ \}$$

$$\boldsymbol{x} \in (\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{x} \in \boldsymbol{A}\ or\ \boldsymbol{x} \in \boldsymbol{B}$$

$$\boldsymbol{x} \notin (\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{x} \notin \boldsymbol{A}\ and\ \boldsymbol{x} \notin \boldsymbol{B}$$

12-***Intersection of set***: Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be any two subset of $\boldsymbol{S}$, The intersection denoted by $(\boldsymbol{A} \cap \boldsymbol{B})$ is the set of all elements that are in both $\boldsymbol{A}$ and $\boldsymbol{B}$.

$$(\boldsymbol{A} \cap \boldsymbol{B}) = \{\boldsymbol{x}: \boldsymbol{x} \in \boldsymbol{A}\ and\ \boldsymbol{x} \in \boldsymbol{B}\ \}$$

$$\boldsymbol{x} \in (\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{x} \in \boldsymbol{A}\ and\ \boldsymbol{x} \in \boldsymbol{B}$$

$$\boldsymbol{x} \notin (\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{x} \notin \boldsymbol{A}\ or\ \boldsymbol{x} \notin \boldsymbol{B}$$

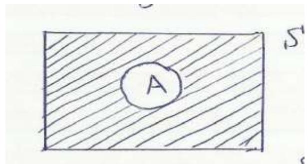
- Two sets $\boldsymbol{A}$ and $\boldsymbol{B}$ are said to be disjoint or mutually exclusive if they have no common elements i.e. :

$$\boldsymbol{A} \cap \boldsymbol{B} = \emptyset$$

- **Venn-Diagrams :**

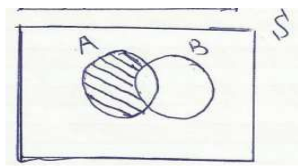
The set operations can be represented by diagrams which are called Venn diagrams .

1- A^c =



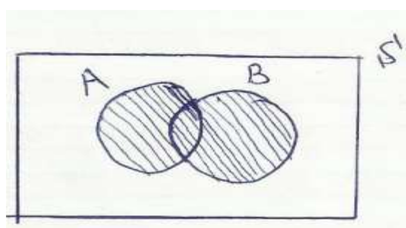
$$A^c = \{ x : x \in S \quad \textbf{and} \quad x \notin A \}$$

2- A/B =



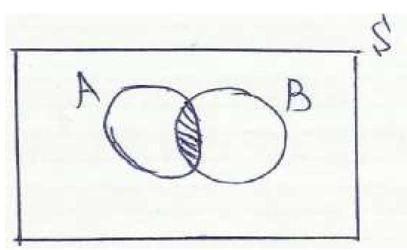
$$A/B = \{ x : x \in A \quad \textbf{and} \quad x \notin B \}$$

3- $A \cup B$ =



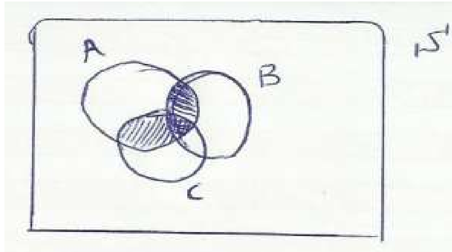
$$(A \cup B) = \{x : x \in A \text{ or } x \in B \text{ or both} \}$$

4- $A \cap B$ =

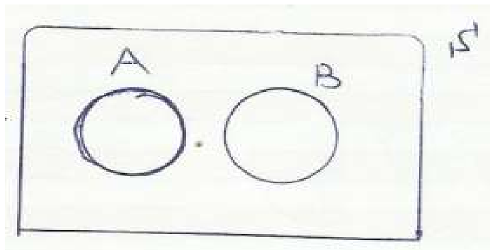


$$(A \cap B) = \{x : x \in A \text{ and } x \in B \}$$

5- $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) \quad =$



6- $\boldsymbol{A} \cap \boldsymbol{B} = \emptyset$



1.2- Some Fundamental Theorems بعض النظريات الأساسية

- Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be any subset of $\boldsymbol{S}$

1- Idempotent Laws قانون الثبات

$$\boldsymbol{A} \cup \boldsymbol{A} = \boldsymbol{A} \quad and \quad \boldsymbol{A} \cap \boldsymbol{A} = \boldsymbol{A}$$

2- Associative Laws قانون التبديل

$$\boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}$$

3- Commutative Laws قانون التساوي

$$\boldsymbol{A} \cup \boldsymbol{B} = \boldsymbol{B} \cup \boldsymbol{A} \quad and \quad \boldsymbol{A} \cap \boldsymbol{B} = \boldsymbol{B} \cap \boldsymbol{A}$$

4- Distributive Laws قانون التوزيع

$$\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$$

$$\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$$

5- Identity Laws قانون الأحادية

$$\boldsymbol{A} \cup \emptyset = \boldsymbol{A} \quad and \quad \boldsymbol{A} \cap \emptyset = \emptyset$$

$$\boldsymbol{A} \cup \boldsymbol{S} = \boldsymbol{S} \quad and \quad \boldsymbol{A} \cap \boldsymbol{S} = \boldsymbol{A}$$

6- Complement Laws قانون المتممة او المكمل

$$\boldsymbol{A} \cup \boldsymbol{A^c} = \boldsymbol{S} \quad and \quad A \cap \boldsymbol{A^c} = \emptyset$$

$$(\boldsymbol{A^c})^{\boldsymbol{c}} = \boldsymbol{A} \quad ; \quad \boldsymbol{S^c} = \emptyset \quad and \quad \emptyset^{\boldsymbol{c}} = \boldsymbol{S}$$

7- De Morgan's Laws قانون دي موركان

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

** To prove the distributive Laws :

1- $\boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

Proof/

Let $x \in A \cup (B \cap C) \ \rightarrow \ x \in A \ \ or \ \ x \in (B \cap C)$

$\rightarrow \ x \in A \ \ or \ \ (x \in B \ \ and \ x \in C)$

$\rightarrow \ (x \in A \ \ or \ \ x \in B) \ \ and \ (x \in A \ \ or \ \ x \in C)$

$\rightarrow \ \big(x \in (A \cup B) \ \ and \ x \in (A \cup C)\big)$

$\rightarrow \ x \in (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$

$\therefore \ \boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$ والعكس صحيح