$$\therefore P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{2}{4} \quad or \quad \frac{C_1^2}{C_1^4}$$

$C = \{exactly\ two\ head\ appear\} = \{HH\} = 1$

$$\therefore P(C) = \frac{h}{n} = \frac{n(C)}{n} = \frac{1}{4} \quad or \quad \frac{C_1^1}{C_1^4}$$

$(A \cap B) = \{HT, TH\} = 2$

$$\therefore P(A \cap B) = \frac{h}{n} = \frac{n(A \cap B)}{n} = \frac{2}{4} \quad or \quad \frac{C_1^2}{C_1^4}$$

EX/

Two dice are thrown once, let $A$ & $B$ be two events defined by:

$A = \{ the\ first\ dice\ shows\ the\ number\ 1\}$

$B = \{ the\ sum\ of\ the\ two\ numbers\ appearing\ is\ less\ than\ 6\}$

Find $P(A)$ , $P(B)$ , $P(A \cup B)$ & $P(A \cup B^c)$

SoL/

$A = \{ (1,1); (1,2); (1,3); (1,4); (1,5); (1,6)\}$

$B = \{ (1,1); (1,2); (2,1); (1,3); (3,1); (1,4); (4,1); (2,2); (2,3); (3,2)\}$

$$\therefore P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{6}{36} \quad or \quad \frac{C_1^6}{C_1^{36}}$$

$$\therefore P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{10}{36} \quad or \quad \frac{C_1^{10}}{C_1^{36}}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore (A \cap B) = \{ (1,1); (1,2); (1,3); (1,4)\}$

$$\therefore P(A \cap B) = \frac{h}{n} = \frac{n(A \cap B)}{n} = \frac{4}{36}$$

$$\therefore P(A \cup B) = \frac{6}{36} + \frac{10}{36} - \frac{4}{36} = \frac{12}{36}$$

$$A^c = \{(2,1);(2,2);(2,3);\dots;(6,4);(6,5);(6,6)\}$$

$$\therefore P(A^c) = \frac{h}{n} = \frac{n(A^c)}{n} = \frac{30}{36} \quad or \quad P(A^c) = 1 - P(A) = 1 - \frac{6}{36} = \frac{30}{36}$$

$$\therefore P(B^c) = 1 - P(B) = 1 - \frac{10}{36} = \frac{26}{36}$$

$$\therefore P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$$

And we have

$$(A \cap B^c) = (A/B)$$

$$\therefore P(A \cap B^c) = P(A/B) = P(A) - P(A \cap B)$$

$$= \frac{6}{36} - \frac{4}{36} = \frac{2}{36}$$

$$\therefore P(A \cup B^c) = \frac{6}{36} + \frac{26}{36} - \frac{2}{36} = \frac{30}{36}$$

$$P(A^c \cup B) = H.W.$$

$$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$$ ……H.W

Where $(A^c \cap B^c) = (A \cap B)^c$

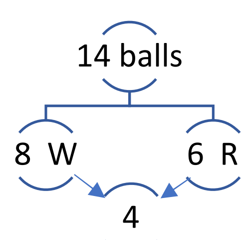
# EX/

A bag contains 8 white and 6 red balls; 4 balls are drawn at random from this bag ; Find the probability of the following events?

A= {There will be 2 red balls}.

B= {There will be at least 2 white balls}.

Find $P(A \cup B)$




SoL/

$$S = C_4^{14} = \frac{14!}{4! * (10)!} = 1001$$

$$A = C_2^6 C_2^8 = \frac{6!}{2! * 4!} \times \frac{8!}{2! * 6!} = 15 \times 28 = 420$$

$$\therefore P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{420}{1001} = 0.4196$$

$$P(B) = \frac{C_2^8 C_2^6 + C_3^8 C_1^6 + C_4^8 C_0^6}{C_4^{14}} = 0.8252$$

$$P(A \cap B) = \frac{C_2^8 C_2^6}{C_4^{14}} = 0.4196$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4196 + 0.8252 - 0.4196 = 0.8252$$

Example – A class contains 10 men and 20 women of which half the men and half the women have brown eyes, find the probability that a person chosen at random is a man or has brown eyes ?

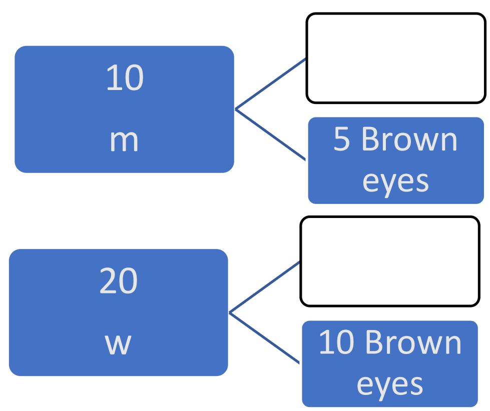
Solution/

let A = person is a man

& B = person has brown eyes

we seek $P(A \cup B)$

Then :




$$P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{10}{30}$$

$$P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{15}{30}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n} = \frac{5}{30}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{10}{30} + \frac{15}{30} - \frac{5}{30} = \frac{20}{30} = \frac{2}{3}$$

H.W.

From the above example find the probability that a person chosen at random is a woman or with non-brown eyes

A = a person is a woman

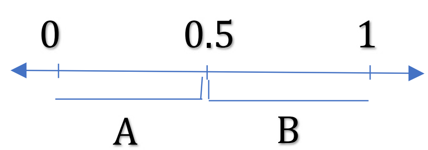
B = a person with non-brown eyes

Hint / we also seek about $P(A \cup B)$

Problems [ Axiomatic Approach of Probability]

1- Let the sample space $S = \{ x: 0 < x < 1\}$ $if$ $A = \{x: 0 < x < 0.5\}$ $and$ $B = \{x: 0.5 \leq x < 1\}$ , Find $P(B)$ $if$ $P(A) = 1/4$

SoL/ $A$ & $B$ are m.e. events



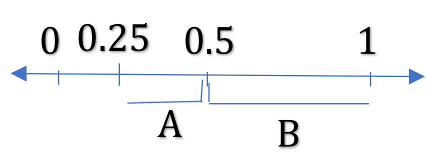

The $P(A \cup B) = P(A) + P(B)$

$$1 = \frac{1}{4} + P(B)$$

$$\therefore P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

H.W. $P(A^c)$ , $P(B^c)$ , $P(A \cap B^c)$ & $P(A^c \cup B^c)$

2- Let the subset $A = \{x: 0.25 < x < 0.5\}$ & $B = \{x: 0.5 \leq x < 1\}$ the sample space $S = \{ x: 0 < x < 1\}$ be such that $P(A) = \frac{1}{8}$ $and$ $P(B) = \frac{1}{2}$ , Find $P(A \cap B)$ , $P(A \cup B)$ , $P(A^c)$ & $P(A^c \cap B^c)$ H.W.



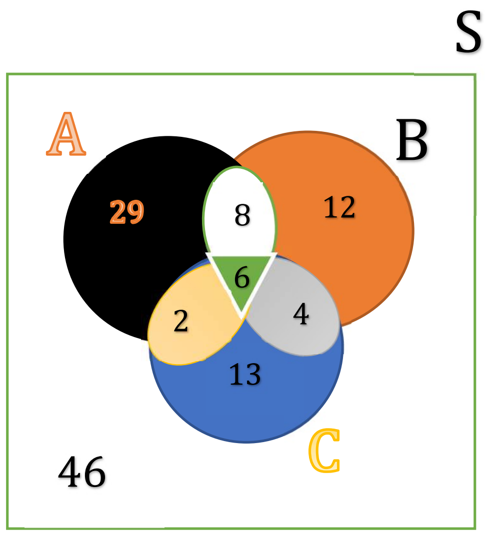
Example: the Venn diagram be of 120 students, 45 are studying French, 30 are studying Spanish, 25 are studying Italian, 14 are studying French and Spanish; 10 are studying Spanish and Italian; 8 are studying French and Italian; and 6 are studying the three languages. If a student is chosen at random, find the probability that the student

1- is studying French or Spanish

2- is studying Spanish or Italian

3- is studying one of the three languages

4- is not studying anyone of the three languages




SoL/

A={ the student Is studying French}

B={ the student is studying Spanish}

C={ the student is studying Italian}

$$\mathbf{1 - P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= \frac{45}{120} + \frac{30}{120} - \frac{14}{120} = \frac{61}{120}$$

$$\mathbf{2 - P(B\ or\ C) = P(B \cup C) = P(B) + P(C) - P(B \cap c)}$$

$$= \frac{30}{120} + \frac{25}{120} - \frac{10}{120} = \frac{45}{120}$$

$$\mathbf{3 - P(A\ or\ B\ or\ C) = P(A \cup B \cup C)}$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= \frac{45}{120} + \frac{30}{120} + \frac{25}{120} - \frac{14}{120} - \frac{8}{120} - \frac{10}{120} + \frac{6}{120} = \frac{74}{120}$$

$$\mathbf{4 - P(A \cup B \cup C)^c = 1 - P(A \cup B \cup C)}$$

$$= 1 - \frac{74}{120} = \frac{46}{120}$$

Example : Let two cards be selected from an ordinary deck of cards(52). find the probability of the following:

1- each card have one color

2- the two cards from the same group

3- at least one of them diamond

SoL/

$$1-\boldsymbol{P(A)}=\frac{C_1^{26}C_1^{26}}{C_2^{52}}=\frac{676}{1325}=\mathbf{0.51}$$

$$2-\boldsymbol{P(B)}=\frac{C_2^{13}+C_2^{13}+C_2^{13}+C_2^{13}}{C_2^{52}}=\frac{312}{1325}=\mathbf{0.236}$$

$$3-\boldsymbol{P(C)}=\frac{C_1^{13}C_1^{39}+C_2^{13}C_0^{39}}{C_2^{52}}=\frac{507+78}{1325}=\mathbf{0.442}$$

Example: let a card be selected from an ordinary deck of cards(52)

Let A = { the card is a spade}

B = {the card is face card}

Compute $\boldsymbol{P(A), P(B)}$ & $\boldsymbol{P(A\cap B)}$

SoL/

$$1-\boldsymbol{P(A)}=\frac{C_1^{13}}{C_1^{52}}=\frac{13}{52}=\mathbf{0.25}$$

$$2-\boldsymbol{P(B)}=\frac{C_1^{12}}{C_1^{52}}=\frac{12}{52}=\mathbf{0.23}$$

$$\mathbf{3}-\boldsymbol{P(A\cap B)}=\frac{n(A\cap B)}{n}=\frac{C_1^{3}}{C_1^{52}}=\frac{3}{52}=\mathbf{0.058}$$

$\boldsymbol{P(A\cup B)}$ ? $\boldsymbol{H.W.}$

Example: Let $\boldsymbol{P}$ be a probability function on $\boldsymbol{S} = \{\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}\}$, Find $\boldsymbol{P(a_1)}$ If :

1_ $P(a_2) = \frac{1}{3}$ & $P(a_3) = \frac{1}{4}$

2_ $P(a_2) = 3P(a_1)$ & $P(a_3) = \frac{1}{5}$

3_ $P(\{a_2, a_3\}) = 2P(a_1)$

4_ $P(a_3) = 2P(a_2)$ & $P(a_2) = 3P(a_1)$

Sol/

1_ $P(a_1) + P(a_2) + P(a_3) = 1$

$\rightarrow P(a_1) + \frac{1}{3} + \frac{1}{4} = 1$

$\rightarrow P(a_1) = \frac{5}{12}$

2_ $P(a_1) + P(a_2) + P(a_3) = 1$

$P(a_1) + 3P(a_1) + \frac{1}{5} = 1$

$\rightarrow 4P(a_1) = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore P(a_1) = \frac{1}{5}$

3_ $P(a_1) + P(a_2) + P(a_3) = 1$

$P(a_1) + 2P(a_1) + 2P(a_1) = 1$

$\rightarrow 5\,P(a_1) = 1$

$\rightarrow P(a_1) = \frac{1}{5} = 0.2$

4_ $P(a_1) + P(a_2) + P(a_3) = 1$

$P(a_1) + 3P(a_1) + 2[3P(a_1)] = 1$

$\rightarrow 10\,P(a_1) = 1$

$\rightarrow P(a_1) = \frac{1}{10} = 0.1$