$\therefore\ P(B) = \frac{5}{12} * \frac{1}{2} + \frac{6}{10} * \frac{1}{2} = \frac{\mathbf{61}}{\mathbf{120}} = \mathbf{0.51}$

Example: Three different machines $M_1, M_2\ and\ M_3$ were used for producing a large batch (وجبة او دفعة كبيرة) of similar manufactured items, suppose that 20% of items were produced by machine M_1 , 30% by machine M_2 and 50% by machine M_3 . suppose further that 1% of the items produced by machine M_1 are defective, 2% of the items produced by machine M_2 are defective and 3% of the items produced by machine M_3 are defective; Finally: Suppose that one item is selected at random from the entire batch and it is found to be defective.

We shall determine the probability that this item was produced by machine M_2 . If we selected one item from this batch, what is the probability that is

1- Defective
2- Non-defective

Sol/

1- $P(D) = \sum_{i=1}^{n} P(D|M_i)\, P(M_i)$

$$= P(D|M_1)\, P(M_1) + P(D|M_2)\, P(M_2) + P(D|M_3)\, P(M_3)$$

$$= (0.01)\,(0.20) + (0.02)\,(0.30) + (0.03)\,(0.50)$$

$$= 0.023$$

2- $P(ND) = \sum_{i=1}^{n} P(ND|M_i)\, P(M_i)$

$$= P(ND|M_1)\, P(M_1) + P(ND|M_2)\, P(M_2) + P(ND|M_3)\, P(M_3)$$

$$= (0.99)\,(0.20) + (0.98)\,(0.30) + (0.97)\,(0.50)$$

$$= 0.977$$

Or $P(ND) = 1 - P(D) = 1 - 0.023 = 0.977$

3- If the selected item found defective, what is the probability that it came from machine M_2 ?

$$P(M_2|D) = \frac{P(M_2 \cap D)}{P(D)} = \frac{P(D|M_2)\, P(M_2)}{\sum_{i=1}^{n} P(D|M_i)\, P(M_i)} = \frac{(0.02)*(0.30)}{0.023} = 0.261$$

4- If the selected item found defective, what is the probability that it came from machine M_1 ?

$$P(M_1|D) = \frac{P(M_1 \cap D)}{P(D)} = \frac{P(D|M_1)\,P(M_1)}{\sum_{i=1}^{n} P(D|M_i)\,P(M_i)} = \frac{(0.01)*(0.20)}{0.023} = 0.087$$

5- If the selected item found non-defective, what is the probability that it came from machine M_1 ?

$$P(M_1|ND) = \frac{P(M_1 \cap ND)}{P(ND)} = \frac{P(ND|M_1)\,P(M_1)}{\sum_{i=1}^{n} P(ND|M_i)\,P(M_i)} = \frac{(0.99)*(0.20)}{0.977} = 0.203$$

Example: In a certain college 4% of the men and 1% of the women taller than 180 c.m., furthermore, 60% of the students are women. Now if a student is selected at random and is taller than 180 c.m.; What is the probability that the student is a woman?

Sol/

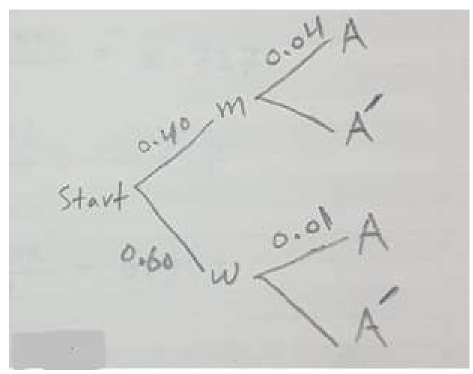
$P(w) = 0.60$

$P(m) = 0.40$

Let A is the student taller than 180 c.m.

$A =$ taller than 180 c.m.

$A^{-} =$ less than 180 c.m.



$$P(w|A) = \frac{P(w \cap A)}{P(A)} = \frac{P(A|w)\,P(w)}{P(A|w)\,P(w) + P(A|m)\,P(m)}$$

$$= \frac{(0.01)*(0.60)}{(0.01)*(0.60) + (0.04)*(0.40)} = 0.273$$

1- what is the probability that he, she will taller than 180 c.m.

2- what is the probability that he, she will less than 180 c.m.

3- If the selected student found taller than 180 c.m. ; what is the probability that she is a woman?

4- If the selected student found taller than 180 c.m. ; what is the probability that he is a man?

5- If the selected student found less than 180 c.m. ; what is the probability that she is a woman?

6- If the selected student found less than 180 c.m. ; what is the probability that he is a man?

Sol/

1- $P(A) = P(A|w)\,P(w) + P(A|m)\,P(m)$

$$= (0.01) * (0.60) + (0.04) * (0.40) = 0.022$$

2- $P(A^{-}) = P(A^{-}|w)\,P(w) + P(A^{-}|m)\,P(m)$

$$= (0.99) * (0.60) + (0.96) * (0.40) = 0.978$$

Or $P(A^{-}) = 1 - P(A) = 1 - 0.022 = 0.978$

3- $P(w|A) = \frac{P(A|w)\,P(w)}{P(A)} = \frac{(0.01)*(0.60)}{0.022} = 0.273$

4- $P(m|A) = \frac{P(A|m)\,P(w)}{P(A)} = \frac{(0.04)*(0.40)}{0.022} = 0.727$

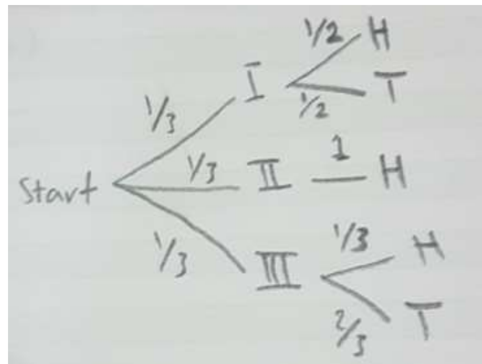
5- $P(w|A^{-}) = \frac{P(A^{-}|w)\,P(w)}{P(A^{-})} = \frac{(0.99)*(0.60)}{0.978} = 0.607$

6- $P(m|A^{-}) = \frac{P(A^{-}|m)\,P(w)}{P(A^{-})} = \frac{(0.96)*(0.40)}{0.978} = 0.393$

Example: A box contains three coins, one is fair, one is two headed and one coin is weighted so that the probability of head appearing is 1/3 . A coin is selected at random and toss once.

1- Given that the result is heads, find the probability that the two-headed coin was chosen.
2- Given that the result is tails, find the probability that the weighted coin was chosen.

Sol/



I- $P(H) = P(T) = \frac{1}{2}$

II- $P(H) = 1$

III- $P(H) = \frac{1}{3}$, $P(T) = \frac{2}{3}$

1- $P(II|H) = \frac{P(II \cap H)}{P(H)}$

$$= \frac{P(H|II)\,P(II)}{P(H|I)\,P(I) + P(H|II)\,P(II) + P(H|III)\,P(III)}$$

$$= \frac{1 * \frac{1}{3}}{\frac{1}{2} * \frac{1}{3} + 1 * \frac{1}{3} + \frac{1}{3} * \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{11}{8}} = \frac{6}{11}$$

2- $P(III|T) = \frac{P(III \cap T)}{P(T)}$

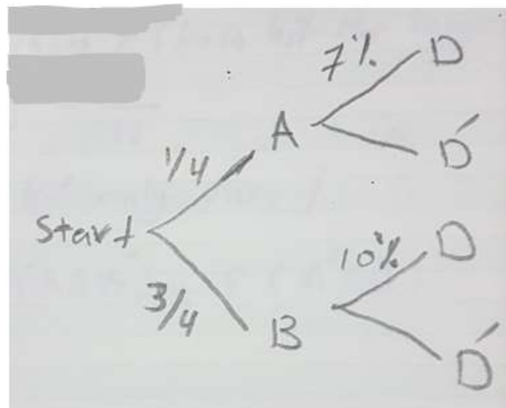
$$= \frac{P(T|III)\,P(III)}{P(T|I)\,P(I) + P(T|II)\,P(II) + P(T|III)\,P(III)}$$

$$= \frac{\frac{2}{3} * \frac{1}{3}}{\frac{7}{18}} = \frac{4}{7}$$

Example: Two factories manufacture electric bulbs, 7% of the bulbs from factory A and 10% from factory B are defective. Factory B produce three times as many bulbs as factory A each week. A bulbs is chosen at random from a week's production.

1- What is the probability that the bulbs is satisfactory?
2- If the bulbs is defective, what is the probability that is came from factory A ?

Sol/



Let $D =$ the event that the chosen bulb is defective

then

$D^{-} =$ the event that the chosen bulb is satisfactory

we have $P(B) = 3\,P(A)$

then $P(A) + P(B) = 1$

$P(A) + 3\,P(A) = 1$

$\therefore\ P(A) = \frac{1}{4} \quad ; \quad P(B) = \frac{3}{4}$

1- $P(D^{-}) = P(D^{-}|A)\,P(A) + P(D^{-}|B)\,P(B)$

$$= \frac{93}{100} * \frac{1}{4} + \frac{90}{100} * \frac{3}{4} = \frac{363}{400}$$

2- $P(A|D) = \dfrac{P(D|A)\,P(A)}{P(D|A)\,P(A) + P(AD|B)\,P(B)}$

$$= \frac{\frac{7}{100} * \frac{1}{4}}{\frac{37}{400}} = \frac{7}{37}$$

Example: The probability that A hits a target is 1/4 and the probability that B hits a target is 1/3 ;

1- If each fire once, what is the probability that both hit the target?
2- If each fire once, and the target is hit once, what is the probability that A hits the target?

Sol/

1- $P(A\ hit\ the\ target) = P(A) = \frac{1}{4}\ ; P(A^c) = \frac{3}{4}$

&

$P(B\ hit\ the\ target) = P(B) = \frac{1}{3}\ ; P(B^c) = \frac{2}{3}$

The two events are independent, then $P(both\ hit\ the\ target)$

$$= P(A \cap B) = P(A) * P(B) = \frac{1}{4} * \frac{1}{3} = \frac{1}{12}$$

2- $E = (\ the\ event\ that\ the\ target\ is\ hit\ only\ once\ )$

$$P(E) = P[(A \cap B^c) \cup (A^c \cap B)] = P(A \cap B^c) + P(A^c \cap B)$$

$$= P(A) * P(B^c) + P(A^c) * P(B)$$

$$\therefore\ P(E) = \frac{1}{4} * \frac{2}{3} + \frac{3}{4} * \frac{1}{3} = \frac{5}{12}$$

$$P(A|E) = \frac{P(A \cap E)}{P(E)} = \frac{\frac{1}{4} * \frac{2}{3}}{\frac{5}{12}} = \frac{2}{5}$$

Example: Two 4's and two 7's are arranged at random to form 4-digit numbers, we assume that all numbers are equally likely if the 4-digit number is odd, what is the probability that the two 4's are together?

Sol/

Here $S = \frac{4!}{2! * 2!} = 6$ elements that is :

$$S = \{\ 4477\ , 4774\ , 7744\ , 7447\ , 7474\ , 4747\ \}$$

Then :

$$A = \{\ 4477\ , 7744\ , 7447\ \} \Longrightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

$$B = \{ 4477 , 7447 , 4747 \} \Longrightarrow P(B) = \frac{3}{6} = \frac{1}{2}$$

$$(A \cap B) = \{ 4477 , 7447 \} \Longrightarrow P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Example: A problem in statistics is given to the three students A , B & C whose chances of solving it $\frac{2}{5}$, $\frac{3}{4}$ & $\frac{2}{3}$ respectively . What is the probability that the problem will be solve?

Sol/

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Since the three event A , B & C are independent:

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A) * P(B) - P(A) * P(C) - P(B) * P(C) + P(A) * P(B) * P(C)$$

$$= \frac{2}{5} + \frac{3}{4} + \frac{2}{3} - \left(\frac{2}{5}\right) * \left(\frac{3}{4}\right) - \left(\frac{2}{5}\right) * \left(\frac{2}{3}\right) - \left(\frac{3}{4}\right) * \left(\frac{2}{3}\right) + \left(\frac{2}{5}\right) * \left(\frac{3}{4}\right) * \left(\frac{2}{3}\right) = \frac{19}{20}$$

Or $P(A \cup B \cup C) = 1 - P(A \cup B \cup C)^c = 1 - P(A^c \cap B^c \cap C^c)$

$$= 1 - P(A^c)P(B^c)P(C^c) = 1 - \left[\frac{3}{5} * \frac{1}{4} * \frac{1}{3}\right] = \frac{19}{20}$$

Example: A fair coin is tossed three times, Find the probability of getting 2 heads given that the first shows a head?

$S = \{ HHH , HHT , HTH , THH , TTH , THT , HTT , TTT \} = 2^n = 2^3 = 8$

Let $\quad A = \{ \text{the event of getting } 2 \text{ heads} \}$

$A = \{ HHT , HTH , THH \}$

$\therefore P(A) = \frac{3}{8}$

Let $\quad B = \{ \text{the first toss shows a head} \}$

$B = \{ HHH , HHT , HTH , THH \}$

$\therefore P(B) = \frac{4}{8}$

$A \cap B = \{ HHT , HTH \}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$