2- $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$

Proof/

Let $x \in A \cap (B \cup C) \quad \rightarrow \quad x \in A \quad and \;\; x \in (B \cup C)$

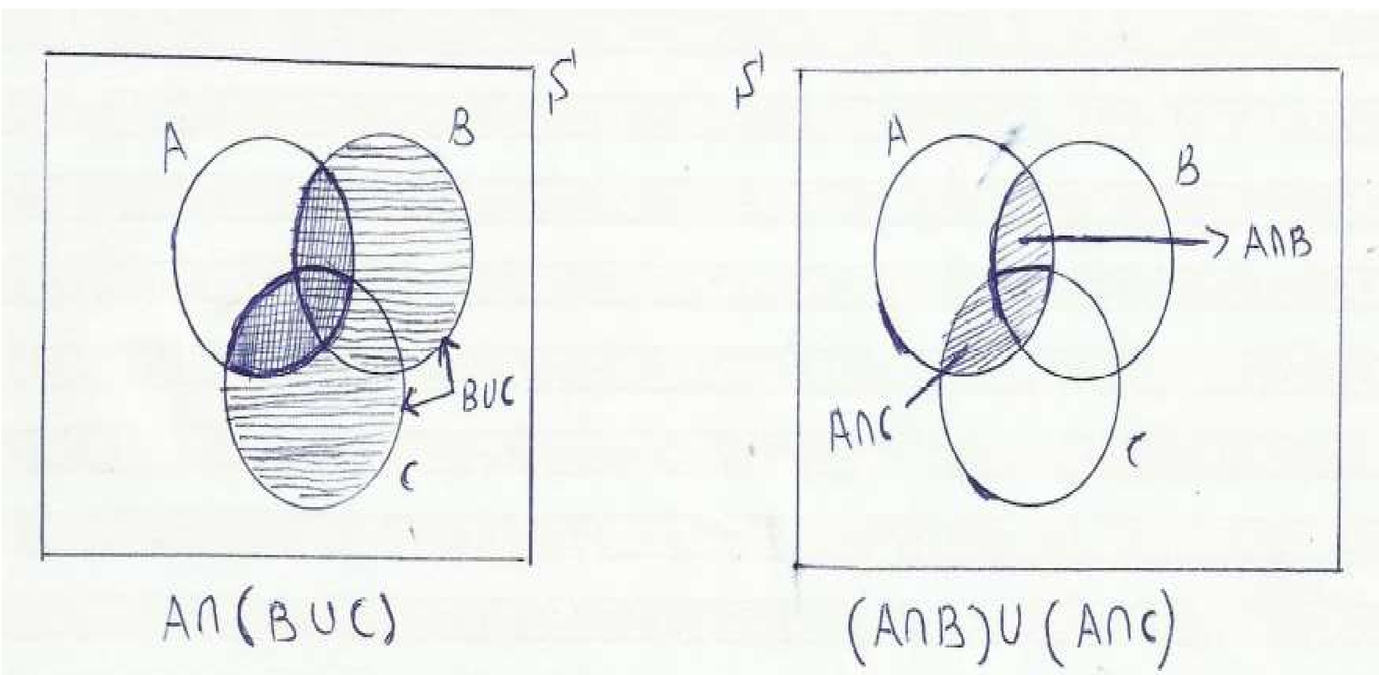
$\rightarrow \quad x \in A \quad and \;\; (x \in B \;\; or \;\; x \in C)$

$\rightarrow \quad (\, x \in A \quad and \;\; x \in B)\, or\, (\, x \in A \;\; and \;\; x \in C)$

$\rightarrow \quad \big(x \in (A \cap B) \;\; or \;\; x \in (A \cap C)\big)$

$\rightarrow \quad x \in (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$

$\therefore \qquad \boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$



** To prove the De Morgan's Laws :

1- $(A \cup B)^c = A^c \cap B^c$

Proof/

Let $x \in (A \cup B)^c \rightarrow x \in S \; and \; x \notin (A \cup B)$

$\rightarrow x \in S \; and \; (x \notin A \; and \; x \notin B)$

$\rightarrow (x \in S \; and \; x \notin A) \; and \; (x \in S \; and \; x \notin B)$

$\rightarrow x \in A^c \; and \; x \in B^c$

$\rightarrow x \in (A^c \cap B^c)$

$\therefore (A \cup B)^c \subset A^c \cap B^c$

برهان الطرف الثاني

$A^c \cap B^c = (A \cup B)^c$

Proof/

Let $x \in A^c \cap B^c \rightarrow x \in A^c \; and \; x \in B^c$

$\rightarrow (x \in S \; and \; x \notin A) \; and \; (x \in S \; and \; x \notin B)$

$\rightarrow x \in S \; and \; (x \notin A \; and \; x \notin B)$

$\rightarrow x \in S \; and \; x \notin (A \cup B)$

$\rightarrow x \in (A \cup B)^c$

$\therefore A^c \cap B^c \subset (A \cup B)^c$

2- $(A \cap B)^c = A^c \cup B^c$

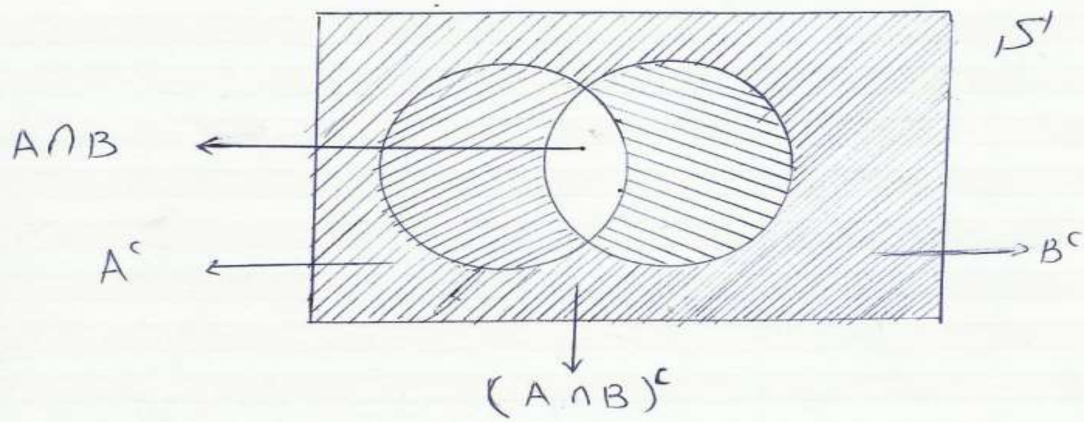
Proof/ Let $x \in (A \cap B)^c \rightarrow x \in S'$ and $x \notin (A \cap B)$

$\rightarrow x \in S'$ and $(x \notin A$ or $x \notin B)$

$\rightarrow (x \in S'$ and $x \notin A)$ or $(x \in S'$ and $x \notin B)$

$\rightarrow x \in A^c$ or $x \in B^c$

$\rightarrow x \in (A^c \cup B^c)$



3 - $x \in (A/B) \rightarrow x \in A$ and $x \notin B$

4- $x \in A^c \rightarrow x \in S$ and $x \notin A$

** Prove that :

a) $\boldsymbol{A/B} = \boldsymbol{A \cap B^c}$

b) $\boldsymbol{B/A} = \boldsymbol{B \cap A^c}$

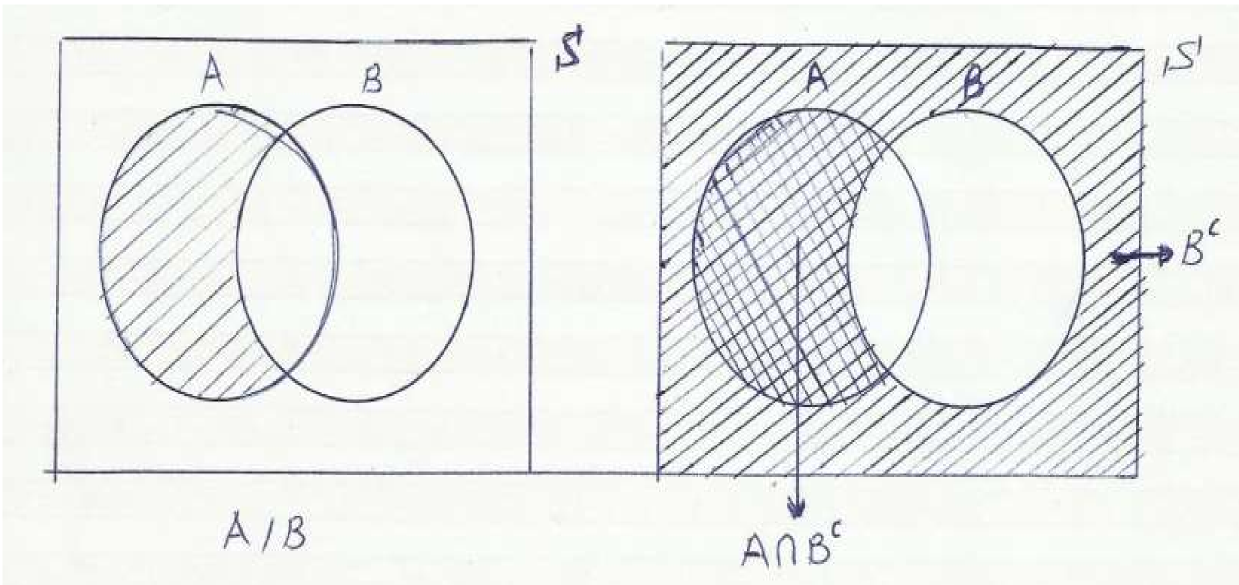
سيتم الاثبات بطريقتين

1- باستخدام الرسم .

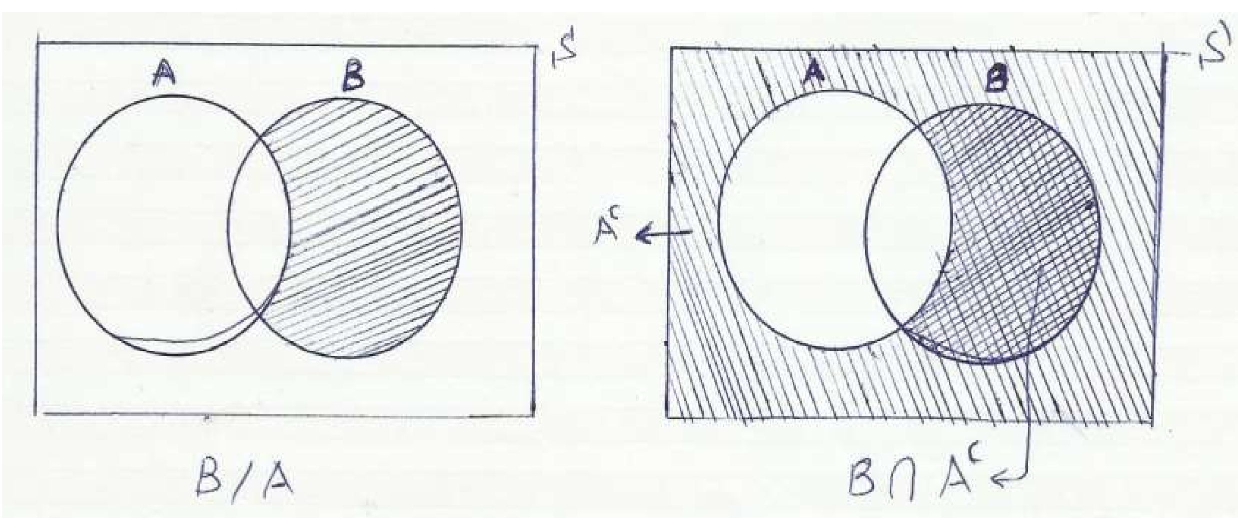
2- باستخدام البرهان .

الاثبات بالرسم : ....

a) $\boldsymbol{A/B} = \boldsymbol{A} \cap \boldsymbol{B^c}$



b) $\boldsymbol{B/A} = \boldsymbol{B} \cap \boldsymbol{A^c}$



الاثبات باستخدام البرهان

a) $\boldsymbol{A/B} = \boldsymbol{A} \cap \boldsymbol{B^c}$

- Let $x \in (A/B) \rightarrow x \in A$ and $x \notin B$
  $\rightarrow x \in A$ and $(x \in S'$ and $x \notin B)$
  $\rightarrow x \in A$ and $x \in B^c$
  $\rightarrow x \in (A \cap B^c)$
  $\rightarrow A/B \subset (A \cap B^c)$ 1
- Let $x \in (A \cap B^c) \longrightarrow x \in A$ and $x \in B^c$
  $\rightarrow x \in A$ and $(x \in S'$ and $x \notin B)$
  $\rightarrow x \in A$ and $x \notin B$
  $\rightarrow x \in (A/B)$
  $\rightarrow (A \cap B^c) \subset A/B$ 2

b) $\boldsymbol{B/A} = \boldsymbol{B} \cap \boldsymbol{A^c}$

let $x \epsilon (B / A) \quad \rightarrow \quad x \in B \; and \; x \notin A$.

$\rightarrow \quad x \in B \; and \, (x \in S \, and \; x \notin A)$

$\rightarrow \quad x \in B \; and \; x \in A^c$

$\rightarrow \quad x \in (B \cap A^c)$

$\rightarrow \quad (B / A) \subset (B \cap A^c)$

برهان الطرف الاخر

let $x \epsilon (B \cap A^c) \quad \rightarrow \quad x \in B \; and \; x \in A^c$.

$\rightarrow \quad x \in B \; and \, (x \in S \, and \; x \notin A)$

$\rightarrow \quad x \in B \; and \; x \notin A$

$\rightarrow \quad x \in (B / A)$

$\rightarrow \quad (B \cap A^c) \subset (B / A)$

$\therefore \quad (B / A) = (B \cap A^c)$

** Homework :-

1- $\boldsymbol{A} \cap (\boldsymbol{B} / \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) / (\boldsymbol{A} \cap \boldsymbol{C})$

2- $\boldsymbol{A} \cup \boldsymbol{B} = \boldsymbol{A} \cup (\boldsymbol{B} \cap \boldsymbol{A}^{c})$

1.3- Sequences and limits المتتابعات والغايات

We have already defined union and intersection of two sets ; these definitions extend to more than two sets .

To distinguish (تميز) between the sets in a collection of subsets of S , we assign names to the form of subscripts

لحد الان تم تعريف الاتحاد والتقاطع لمجموعتان فإذا كان لدينا اكثر من مجموعتان، يجب ان نتعرف على المتتابعات التي يمكن تعريفها بانها عبارة عن مجموعة من المجاميع الجزئية معرفة من مجموعة شاملة.

Let I be an index set and $\{A_r : r \in I\}$ be a collection of subsets of S indexed by I , which is called a Sequence of sets.

1- The Union of Sequences of sets:

$$\bigcup_{r \in I} A_r = \{ x : x \in S \,; x \in A_r \ \ for\ some\ \ r \in I\}$$

$$\bigcup_{r \in I} A_r = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \dots \cup A_n$$

$$\rightarrow x \in \bigcup_{r \in I} A_r \ \ if\ and\ only\ if\ x \in A_r \ \ for\ some\ \ r \in I$$

$$\rightarrow x \notin \bigcup_{r \in I} A_r \ \ if\ and\ only\ if\ x \notin A_r \ \ for\ all\ \ r \in I$$

2- The Intersection of Sequences of sets:

$$\bigcap_{r\in I} A_r = \{x : x \in S;\ x \in A_r \ \ for\ all \ \ r \in I\}$$

$$\bigcap_{r\in I} A_r = \ A_1 \cap A_2 \cap A_3 \cap A_4 \cap \dots \cap A_r$$

$$\rightarrow x \in \bigcap_{r\in I} A_r\ if\ and\ only\ if\ x \in A_r \ \ for\ all \ \ r \in I$$

$$\rightarrow x \notin \bigcap_{r\in I} A_r\ if\ and\ only\ if\ x \notin A_r \ \ for\ some \ \ r \in I$$

3- Increasing sequences المتتابعات المتزايدة

$A_1 < A_2 < A_3 < A_4 < \cdots;\ \bigcup_{r\in I} A_r = A_n\ ;\ \bigcap_{r\in I} A_r = A_1$