Exp) If repetition are not allowed :

1_ How many 3_digit numbers can be formed from the six digits {2,3,4,5,6,7}.

2_ How many of these are less than 400 .

3_ how many are even .

4_ how many are odd .

5_ how many are multiple of 5 .

Sol/

1_ <u> 6 </u> <u> 5 </u> <u> 4 </u> $=$ $6 \times 5 \times 4$ $= 120$ ways

0r

$$P_r^n = \frac{n!}{(n-r)!} \rightarrow P_3^6 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120$$

ملاحظة : اذا ذكر بالسؤال شرط (إيجاد عدد فردي او زوجي) او عبارة (اقل من او اكبر من عدد معين)

لا يمكن حل السؤال إلا بطريقة المواقع .

2_ <u> 2 </u> <u> 5 </u> <u> 4 </u> $=$ $2 \times 5 \times 4$ $= 40$ ways

3_ <u> 5 </u> <u> 4 </u> <u> 3 </u> $=$ $5 \times 4 \times 3$ $= 60$ ways

4_ <u> 5 </u> <u> 4 </u> <u> 3 </u> $=$ $5 \times 4 \times 3$ $= 60$ ways

5_ <u> 5 </u> <u> 4 </u> <u> 1 </u> $=$ $5 \times 4 \times 1$ $= 20$ ways

Exp) In how many ways can a party of 7 persons arrange them selves .

1_ In a row of chairs .

2_ A round a circular table .

Sol/

1_ $n! = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

2_ $(n - 1)! = (7 - 1)! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Exp)

1_ In how many ways can 3 boys and 2 girls sit in a row .

2_ In how many ways can they sit in a row if just the boys are sit together .

3_ In how many ways can they sit in a row if just the girls are sit together .

Sol/

1_ $n! = (3 + 2)! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \ \ ways$

2_ $3! \times 3! = 6 \times 6 = 36 \ \ ways$.

3_ $2! \times 4! = 2 \times 1 \times 4 \times 3 \times 2 \times 1 = 48 \ ways$.

Exp) Find the number of ways in which three of ten new movies are be ranked first , second and third by movies critics (لجنة تحكيمية).

Sol/ Let $n = 10 ; \ r = 3$

$$P_r^n = \frac{n!}{(n-r)!} \rightarrow P_3^{10} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = 720 \ ways$$

Exp) what is the number of ways in which 6 persons can seated in a row if a certain of them must sit side by side .

Sol/

  1_ $n! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \ ways$

  2_ 2 sit side by side $\rightarrow 2! \times 5! = \ 240 \ ways$

  3_ 3 sit side by side $\rightarrow 3! \times 4! = \ 144 \ ways$

Exp) In how many ways can eight teaching assistant be assigned to eight classes of a course in probability ?

Sol/ Let $n = 8 \ ; \ r = n = 8$

  $P_r^n = n! = P_8^8 = 8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320 \ \ ways$

Exp) How many different words can be formed from the 8 letter is (REMEMBER) ?

Sol/

$$P^n_{n_1,n_2,n_3,\ldots,n_k} \quad ; \quad R = 2 \; ; \; E = 3 \; ; \; M = 2 \; ; \; B = 1$$

$$P^8_{2,3,2,1} = \frac{n!}{n_1! * n_2! * n_3! * n_4!} = \frac{8!}{2! * 3! * 2! * 1!} = 1680 \; ways$$

Exp) How many different words can be formed from (MISSISSIPPI) ?

Sol/

$$P^n_{n_1,n_2,n_3,\ldots,n_k} \quad ; \quad M = 1 \; ; \; S = 4 \; ; \; P = 2 \; ; \; I = 4$$

$$P^{11}_{1,4,2,4} = \frac{n!}{n_1! * n_2! * n_3! * n_4!} = \frac{11!}{1! * 4! * 2! * 4!} = 34650 \; ways$$

2_3) Combinations   طريقة التوافيق

 Definition :-

    A combination is a selection of objects considered <mark>without regard to there</mark> <mark>order</mark> . الترتيب غير مهم

 The number of combinations of a set of n different objects , taken r at a time is :

$$C_r^n \;=\; \frac{n!}{r!(n-r)!} \;\; ; \quad 0 \le r \le n$$

1_ If $r > n$; $\rightarrow C_r^n = 0$

2_ If $r = n$ and if $r = 0$; $\rightarrow C_r^n = \dfrac{n!}{n! \times 0!} = 1$

3_ $C_n^n = C_{n-r}^n$

4_ If $n = n_1 + n_2 + n_3 + \cdots + n_k$ $\rightarrow C_{n_1,n_2,n_3,\ldots,n_k}^n = \dfrac{n!}{n_1! * n_2! * n_3! * \ldots * n_k!}$

Exp) In how many ways can a reader select 3 books without regard to their order from a set of 4 different books denoted by <mark>A , B , C , and D</mark> ?

Sol/   <mark>ABC , ABD , ACD , BCD</mark>

 So that $C_r^n = \dfrac{n!}{r!(n-r)!} \rightarrow C_3^4 = \dfrac{4!}{3!(4-3)!} = \dfrac{4!}{3! \times 1!} = \dfrac{4 \times 3!}{3!} = $ <mark>4</mark> $ways$

Exp) The <mark>3</mark> men can be chosen from the <mark>7</mark> men and the <mark>2</mark> women can be chosen from the <mark>5</mark> women, Hence the <mark>committee</mark> can be chosen in ?

 Sol/

$$C_3^7 \times C_2^5 \;=\; \frac{7!}{3!(7-3)!} \times \frac{5!}{2!(5-2)!} = \frac{7!}{3! \times 4!} \times \frac{5!}{2! \times 3!} = \;\text{<mark>350</mark>}\; ways$$

Exp) A student is to answer 8 out of 10 questions on an example .

1_ How many choice has he ?

2_ How many choice has if he must answer the first 3 questions ?

3_ How many choice has if he must answer at least 4 of the first 5 questions ?

4_ How many choice has if he must answer all the first 5 questions ?

Sol/

1_ The 8 questions can be selected in :

$$C_r^n = \frac{n!}{r!(n-r)!} \rightarrow C_8^{10} = \frac{10!}{8!(10-8)!} = \frac{10!}{8! \times 2!} = 45 \ ways$$

2_ if he must answer the first 3 questions ?

$$C_r^n = \frac{n!}{r!(n-r)!} \rightarrow C_5^7 = \frac{7!}{5!(7-5)!} = \frac{7!}{5! \times 2!} = 21 \ ways$$

3_ if he must answer at least 4 of the first 5 questions ?

$$C_4^5 \times C_4^5 + C_5^5 \times C_3^5$$

4_ if he must answer all the first 5 questions ?

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \times 2!} = 10 \ ways$$

Exp) In how many ways can a set of balls be selected from 8 white and 6 red balls such that there will be 3 white and 2 red balls ?

Sol/

8W balls , 6R balls

$$C_3^8 \times C_2^6 = \frac{8!}{3!(8-3)!} \times \frac{6!}{2!(6-2)!} = 56 \times 15 = 840 \ ways$$

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Exp)A class contain 9 boys and 3 girls :

1_ In how many ways can the teacher choose a committee of 4 .

2_ How many of them will contain at least one girl .

3_ How many of them will contain exactly one girl .

Sol/

$$1\_ \ C_4^{12} \ = \ \frac{12!}{4!(12-4)!} = \frac{12!}{4! \times 8!} = \ \boxed{495} \ ways$$

$$2\_ \ C_1^3 \times C_3^9 + \ C_2^3 \times C_2^9 + C_3^3 \times C_1^9 =$$

$$\frac{3!}{1! \times 2!} \times \frac{9!}{3! \times 6!} + \frac{3!}{2! \times 1!} \times \frac{9!}{2! \times 7!} + \frac{3!}{3! \times 0!} \times \frac{9!}{1! \times 8!} = \boxed{369} \ ways$$

$$3\_ C_1^3 \times C_3^9 = \boxed{252} \ ways$$

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ملاحظة :

</div>

1_ Less than	n	$\rightarrow$ $x < n$	اقل من
2_ more than	n	$\rightarrow$ $x > n$	اكثر من
3_ at must	n	$\rightarrow$ $x \geq n$	على الأكثر
4_ at least	n	$\rightarrow$ $x \leq n$	على الأقل

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من المثال السابق ليكن لدينا المطاليب التالية :

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4_ At least one boy ?

$$C_1^9 \times C_3^3 + \ C_2^9 \times C_2^3 + \ C_3^9 \times C_1^3 + \ C_4^9 \times C_0^3$$

5_ at must one girl ?

$$C_1^3 \times C_3^9 + \ C_0^3 \times C_4^9$$

ملاحظة مهمة :   اذا كان لدينا   a , b , c and d   المطلوب اختيار ثلاثة حروف من الحروف الأربعة
وفي حالتين وكما يلي :

الترتيب غير ضروري (غير مهم)	الترتيب ضروري (مهم)					
Combination	Permutation					
abc	abc	acb	bca	bac	cab	cba
abd	abd	adb	bda	bad	dab	dba
bcd	bcd	bdc	cdb	cbd	dbc	dcb
acd	acd	adc	cda	cad	dac	dca

$$C_3^4 = \frac{4!}{3!(4-3)!} = \boxed{4} \ ways \qquad\qquad P_3^4 = \frac{4!}{(4-3)!} = \boxed{24} \ ways$$

Exp) Assume that there are $n = 7$ students and that we wish to form 3 groups , 2 in the first , 3 in the sconed and 2 in the third .

Sol/ Let $n_1 = 2$, $n_2 = 3$ and $n_3 = 2$ then $n = n_1 + n_2 + n_3$

$$C_2^7 \times C_3^5 \times C_2^2 = \frac{7!}{2! * 5!} \times \frac{5!}{3! * 2!} \times \frac{2!}{2! * 0!} = \frac{7!}{2! * 3! * 2!}$$

In General :

$$C_{n_1}^n * C_{n_2}^{n-n_1} * C_{n_3}^{n-(n_1+n_2)} * \ldots * C_{n_k}^{n-(n_1+n_2+n_3+\cdots+n_k)}$$

$$= \frac{n!}{n_1!(n-n_1)!} * \frac{(n-n_1)!}{n_2!(n-(n_1+n_2))!} * \ldots * \frac{(n-(n_1+n_2+n_3+\cdots+n_{k-1}))!}{n_k!(n-(n_1+n_2+n_3+\cdots+n_k))!}$$

$$= \frac{n!}{n_1! * n_2! * n_3! * \ldots * n_k!} \quad where \quad n = n_1 + n_2 + n_3 + \cdots + n_k$$

$$\therefore C_{n_1,n_2,n_3}^n = \frac{n!}{n_1! * n_2! * n_3!} = \frac{7!}{2! * 3! * 2!}$$