Chapter Three

Probability

3.1_ Introduction

Probability is the study of random or nondeterministic (غير محدد) experiment (that is ; we wish to consider some experiment results and the repetition of the experiment dose not always produce the same result).

For example , suppose one tossed a fair coin repeatedly , keeping a record of the number of heads h in the first n tossed ($n = 1,2,3,\dots$), then it has bean empirically (تجريبيا) observed (مشاهدة) that the ratio $f = h/n$, called the relative frequency becomes stable in the long run , i.e. approaches a limit (يقترب من الحد) . This stability is the basis of probability theory .

3.2_ Random Experiment , Sample Space and Events :

_ Definition :

_ Random Experiment : Is a procedure which results in some nondeterministic outcomes in a particular situation(حالة خاصة او معينة) .

_ Outcome : Is a single realization (مفهوم وحيد) of phenomenon under consideration (للظاهرة قيد البحث) , the outcome need not always be number or quantities which are representable in term of numbers .

_ Sample Space : The set of all possible outcomes of a random experiment denoted by $S \text{ or } \Omega$.

_ Events : It's a subset of sample space denoted by E .

_ Elementary Event (حدث ابتدائي او اولي) : Is an event consisting of only one element of the sample space S .

These points are illustrated (موضحة) with some examples :

1_ If the experiment consists of tossing a coin once , then

$$S = \{H, T\} \text{ where } H = head \text{ and } T = tail$$

When 2 tossing a coin :

$$S = \{HH, HT, TH, TT\}$$

When 3 tossing a coin :

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\} .$$

2_ If a couple is planning to have three children , then S consists of 8 elements ; i.e.

$$S = \{BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG\}$$

Where B means that the outcome of birth is boy and G is a girl .

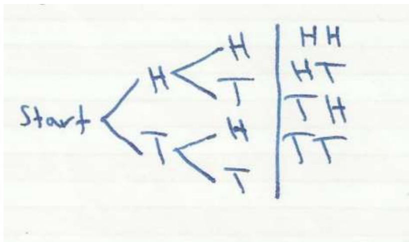
If E is the event that the couple will have 2 boys , then :

$$E = \{BBG, BGB, GBB\}$$

Also ; if F is the event that the couple will have at most on boy , then :

$$F = \{BGG, GBG, GGB, GGG\}$$

3_ Toss a two coins :



$$S = \{HH, HT, TH, TT\}$$

Let $A =$ one head appear

$\therefore\ A = \{HT, TH\}$

Let $B =$ no head appear or two tail appear

$\therefore\ B = \{TT\}$

Let $C =$ at must two head appear

$\therefore\ C = \{HH, HT, TH\}$

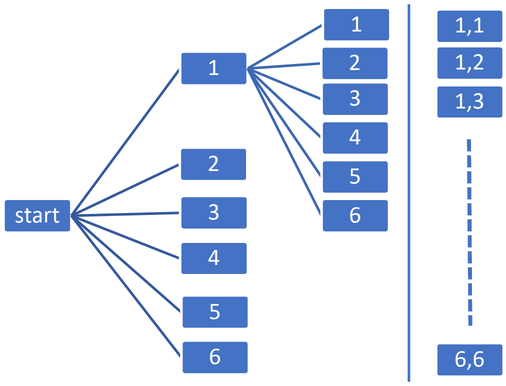
Let $D =$ at least two head appear

$\therefore\ D = \{HH\}$

4_ If the experiment consists of tossing a pair of dice, then the sample space consists of 36 elements; i.e.

$$S = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

or



Define the following events as :

$A = \{the\ sum\ on\ the\ two\ dice\ is\ 6\ \}$

$B = \{both\ dice\ show\ the\ same\ number\ \}$

$C = \{at\ least\ one\ of\ them\ is\ 4\ \}$

Then :

$A = \{(1,5)\ ;\ (2,4)\ ;(3,3)\ ;(4,2)\ ;(5,1)\}$

$B = \{(1,1)\ ;\ (2,2)\ ;(3,3)\ ;(4,4)\ ;(5,5)\ ;(6,6)\}$

$C = \{(1,4)\ ;\ (2,4)\ ;(3,4)\ ;(4,4)\ ;(5,4)\ ;(6,4)\ ;\ (4,1)\ ;\ (4,2)\ ;(4,3)\ ;(4,5)\ ;(4,6)\}$

5_ Consider the experiment involving the toss of single dice ; then :

$S = \{1,2,3,4,5,6\}$

$A = \{be\ an\ event\ of\ odd\ number\ \}$

$B = \{be\ an\ event\ of\ even\ number\ \}$

$C = \{divisible\ by\ 3\ \}$

$D = \{less\ than\ 4\ \}$

Then :

$A = \{1,3,5\}$; $B = \{2,4,6\}$

$C = \{3,6\}$; $D = \{1,2,3\}$

Remarks :

1_ $A \cup B$: Is a subset of sample space contain the element in A or B or Both

2_ $A \cap B$: Is a subset of sample space contain the element in A and B

3_ A^c : Is a subset of sample space contain the element in S but not in A

4_ A/B : The event (A) appear and event (B) not appear

5_ $A \cap B = \emptyset$: Event (A) and (B) are mutually exclusive (m.e.)

3.3 _ Kinds of Probability أنواع الاحتمال

There are three kinds of probability and these are :

1_ The Classical Approach . الطريقة الكلاسيكية

2_ The relative Frequency Approach . طريقة التردد النسبي

3_ The Subjective Approach . طريقة النظرة الشخصية

سوف نتعرف على الطريقة الكلاسيكية (التقليدية) ويعتبر القانون الأول في الاحتمالية :

3.3.1_ The Classical Approach :

Suppose an event E can happen in h ways out of total of n possible equally likely ways . Then the probability of occurrence (ظهور) of the event { called its Success} is denoted by :

$$P = P(E) = \frac{h}{n}$$

الجزء ← h

الكل ← n

$$0r = \frac{n(E)}{n}$$

That's mean : the probability of event E equals the number of outcomes favorable to E divided by the total number of outcomes .

i.e. : $P(E) = \frac{favorable\ outcomes}{total\ outcomes}$

The Probability of non-occurrence of the event , { called its failure } is denoted by :

$$q = P(not\ E) = \frac{n-h}{n}$$

$$or\ \ q = 1 - \frac{h}{n} = 1 - P = 1 - P(E)$$

Thus :

$$P + q = 1$$

or

$$P(E) + P(not\ E) = 1$$

Exp)

1_ A fair coin is tossed once , then $S = \{H, T\}$:

Thus, the probability of getting a head is $P = \frac{1}{2}$

and the probability of getting a tail is $P = \frac{1}{2}$

2_ When an unbiased dice is thrown once , then $S = \{1,2,3,4,5,6\}$

The six outcomes are mutually exclusive because two or more faces cannot turn up simultaneously (يظهر في وقت واحد) then :

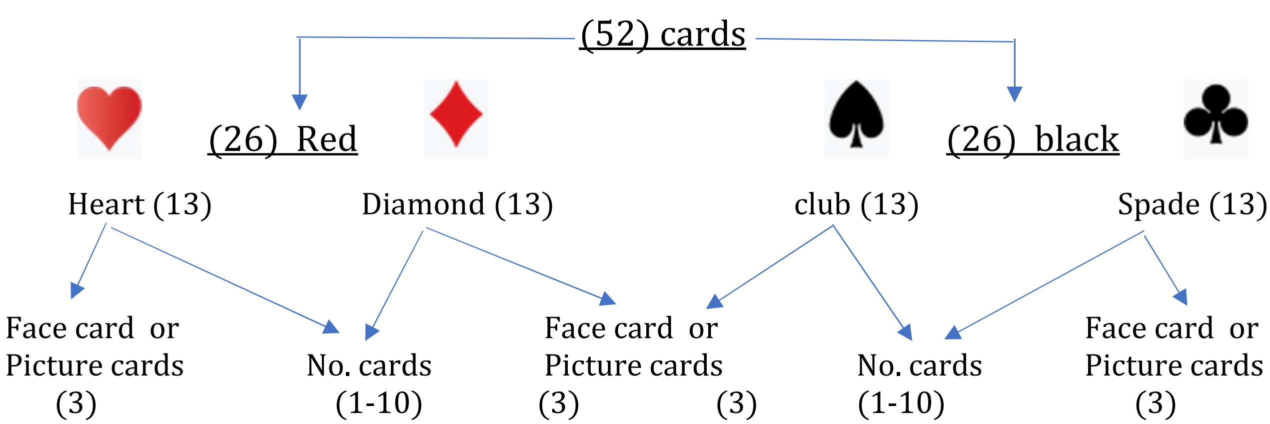
$$P = (1) = P = (2) = P = (3) = P = (4) = P = (5) = P = (6) = \frac{1}{6}$$

To fined the probability of getting an odd number , define

$A = \{1,3,5\}$

$$\rightarrow P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{3}{6} = \frac{1}{2}$$

3_ What is the probability of getting a cards :



Number Cards (40)		Faced or Picture cards (12)
Diamond :	(1-10)	Jake ; Queen ; King
Heart :	(1-10)	Jake ; Queen ; King
Club :	(1-10)	Jake ; Queen ; King
Spade :	(1-10)	Jake ; Queen ; King

1_ one Picture ; $P(A) = \frac{h}{n} = \frac{12}{52}$ or $\frac{C_1^{12}}{C_1^{52}}$

2_ one Number ; $P(B) = \frac{h}{n} = \frac{40}{52}$ or $\frac{C_1^{40}}{C_1^{52}}$

From the classical definition of probability of an event A we have :

1_ $0 \leq P(A) \leq 1$; $0 \leq \frac{h}{n}$ or $\frac{n(A)}{n} \leq 1$ for every n .

2_ If A is a certain event , then $P(A) = 1$ because $h = n$ 0r $n(A) = n$ and if A is an impossible event (null event); then :

$P(A) = 0$ because $h = n(A) = 0$

مثل رمي زهر النرد والمطلوب احتمال ظهور رقم اكبر من 6 .

3_ If A and B are two mutually exclusive events , then :

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$ because

$$P(A \cup B) = \frac{h}{n} = \frac{n(A \cup B)}{n} = \frac{n(A)+n(B)}{n} = \frac{n(A)}{n} + \frac{n(B)}{n} = P(A) + P(B)$$

Where $n(A) , n(B)$ and $n(A \cup B)$ are the number of occurrences of the events A , B and $(A \text{ or } B)$ respectively .

The result can be extended to the case of K mutually exclusive (m.e.) events $A_1 , A_2 , A_3 , A_4 , \ldots , A_k$ i.e.

$$P(A_1 \text{ or } A_2 \text{ or } A_3 \text{ or } \ldots \text{or } A_k = P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k) =$$

$$P(\cup_{i=1}^{k} A_i) = \sum_{i=1}^{k} P(A_i)$$

Exp) Let a card be selected at random from an ordinary deck of 52 cards ;

$A = \{ \text{the card is spade} \}$ and $B = \{ \text{the card is a faced card} \}$, i.e. a Jake , Queen or King ; Compute :

$P(A)$, $P(B)$ and $P(A \cap B)$

Sol/

Since we have an equiprobable space

$$P(A) = \frac{number\ of\ spades}{number\ of\ cards} = \frac{h}{n} = \frac{n(A)}{n} = \frac{C_1^{13}}{C_1^{52}} = \frac{13}{52}$$

$$P(B) = \frac{number\ of\ face\ cards}{number\ of\ cards} = \frac{h}{n} = \frac{n(B)}{n} = \frac{C_1^{12}}{C_1^{52}} = \frac{12}{52}$$

$$P(A \cap B) = \frac{number\ of\ spade\ face\ cards}{number\ of\ cards} = \frac{h}{n} = \frac{n(A \cap B)}{n}$$
$$= \frac{C_1^{3}}{C_1^{52}} = \frac{3}{52}$$

Exp) Let 2 items be chosen at random from vessel (وعاء) containing 12 items of which 4 are defective .

Let $A = \{\ both\ items\ are\ defective\ \}$

Let $B = \{\ both\ items\ are\ non - defective\ \}$

Find $P(A)\ \ and\ \ P(B)$.

Sol/

$$P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{n(A)}{S} = \frac{C_2^{4}}{C_2^{12}} = \frac{\frac{4!}{2!*2!}}{\frac{12!}{2!*10!}}$$

$$\therefore P(A) = \frac{6}{66} = \frac{1}{11}$$

$$P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{n(B)}{S} = \frac{C_2^{8}}{C_2^{12}} = \frac{\frac{8!}{2!*6!}}{\frac{12!}{2!*10!}}$$

$$\therefore P(A) = \frac{28}{66} = \frac{14}{33}$$