Let $C = \{ one\ is\ defective\ and\ one\ is\ non-defective \}$

$$\therefore P(C) = \frac{h}{n} = \frac{n(C)}{n} = \frac{C_1^4 * C_1^8}{C_2^{12}} = \frac{4*8}{66} = \frac{32}{66} = \frac{16}{33}$$

Exp)

Two digits are chosen at random from the digits {1,2,3,4,5} ; List the 2-digits numbers that can be formed . What is the probability that the number is even?

Sol/

$\underline{5}\ \ \underline{4} = 5 \times 4 = 20$ ways or

$$P_r^n = P_2^5 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 * 4 = 20 \text{ ways}$$

{12 , 13 , 14 , 15 , 21 , 23 , 24 , 25 , 31 , 32 , 34 , 35 , 41 , 42 , 43 , 45 , 51 , 52 , 53 , 54}

Even number :

$\underline{4}\ \ \underline{2} = 4 \times 2 = 8$ ways

$$\therefore P(even) = \frac{h}{n} = \frac{8}{20} = 0.4$$

Exp) A Committee of 4 persons are chosen at random from a class containing 7 girls and 9 boys . What is the probability that the committee will contain :

1_ 3 boys .

2_ at least one girl .

Sol/

$$S = C_4^{16} = \frac{n!}{r!(n-r)!} = \frac{16!}{4!*12!} = 1820$$

1_ 3 boys : $C_3^9 C_1^7 = \frac{9!}{3!*6!} * \frac{7!}{1!*6!} = 84 * 7 = 588$

$$\therefore P(3\ boys) = \frac{C_3^9 C_1^7}{C_4^{16}} = \frac{588}{1820} = 0.3231$$

2_ $B = \{ at\ least\ one\ girl \}$

$$\therefore B = C_3^9 C_1^7 + C_2^9 C_2^7 + C_1^9 C_3^7 + C_0^9 C_4^7$$

$$= \frac{9!}{3!*6!} * \frac{7!}{1!*6!} + \frac{9!}{2!*7!} * \frac{7!}{2!*5!} + \frac{9!}{1!*8!} * \frac{7!}{3!*4!} + \frac{9!}{0!*9!} * \frac{7!}{4!*3!}$$

$$= 84 * 7 + 36 * 21 + 9 * 35 + 1 * 35 = 1694$$

$$\therefore P(B) = \frac{C_3^9 C_1^7 + C_2^9 C_2^7 + C_1^9 C_3^7 + C_0^9 C_4^7}{C_4^{16}} = \frac{1694}{1820} = 0.931$$

Or

$$P(B) = 1 - P(not\ girl) = 1 - \frac{C_4^9 C_0^7}{C_4^{16}} = 1 - \frac{126}{1820} = 1 - 0.06923 = 0.931$$

3_ Let $D = \{ exactly\ 3\ girl \}$

$$\therefore P(D) = \frac{n(D)}{n} = \frac{C_3^7 C_1^9}{C_4^{16}} = \frac{35 * 9}{1820} = \frac{315}{1820} = 0.1731$$

4_ Let $F = \{ at\ least\ one\ boy \}$

$$\therefore P(F) = \frac{h}{n} = \frac{n(F)}{n} = \frac{C_1^9 C_3^7 + C_2^9 C_2^7 + C_3^9 C_1^7 + C_4^9 C_0^7}{C_4^{16}} = 0.981$$

Or

$$P(F) = 1 - P(not\ boy) = 1 - \frac{C_0^9 C_4^7}{C_4^{16}} = 1 - \frac{35}{1820} = 1 - 0.0192 = 0.9808$$

Exp)

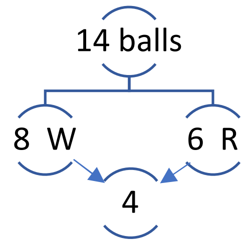
A bag contains 8 white and 6 red balls , 4 balls are drawn at random from this bag ; Finds the probability of the following events :

A = { there will be 2 red balls }

B = { there will be at least 2 white balls }

D = { at most 2 red balls }

Sol/



$$S = C_4^{14} = \frac{n!}{r!(n-r)!} = 1001$$

$$A = C_2^6 C_2^8 = \frac{6!}{2!*4!} * \frac{8!}{2!*6!} = 15 * 28 = 420$$

$$\therefore P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{420}{1001} = 0.4196$$

$$B = C_2^8 C_2^6 + C_3^8 C_1^6 + C_4^8 C_0^6$$

$$= \frac{8!}{2!*6!} * \frac{6!}{2!*4!} + \frac{8!}{3!*5!} * \frac{6!}{1!*5!} + \frac{8!}{4!*4!} * \frac{6!}{0!*6!}$$

$$= 28 * 15 + 56 * 6 + 70 * 1 = 826$$

$$\therefore P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{826}{1001} = 0.8252$$

$$D = C_2^6 C_2^8 + C_1^6 C_3^8 + C_0^6 C_4^8$$

$$\therefore P(D) = \frac{C_2^6 C_2^8 + C_1^6 C_3^8 + C_0^6 C_4^8}{C_4^{14}} = \frac{15*28 + 6*56 + 1*70}{1001} = \frac{826}{1001} = 0.8252$$

Exp)

Three cards are drawn at random from a deck of 52 cards, let the events A, B, C, D and E be defined as:

$A = an\ ace$

$B = 2\ cards\ of\ daimond$

$C = 2\ picture\ cards\ and\ one\ numberd\ card\ of\ spad$

$D = one\ numbered\ card, one\ Jack\ and\ the\ Queen\ of\ heart$

$E = the\ first\ card\ is\ an\ ace\ and\ the\ others\ one\ picture\ cards$

Find the probability of the above events.

Sol/

$$S = C_3^{52} = \frac{52!}{3!*49!} = 22100$$

$$A = C_1^4 C_2^{48} = \frac{4!}{1!*3!} * \frac{48!}{2!*46!} = 4 * 1128 = 4512$$

$$\therefore P(A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{4512}{22100} = 0.20416$$

$$B = C_2^{13} C_1^{39} = \frac{13!}{2!*11!} * \frac{39!}{1!*38!} = 78 * 39 = 3042$$

$$\therefore P(B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{3042}{22100} = 0.13765$$

$$C = C_1^{10} C_2^{12} = \frac{10!}{1!*9!} * \frac{12!}{2!*10!} = 10 * 66 = 660$$

$$\therefore P(C) = \frac{h}{n} = \frac{n(C)}{n} = \frac{660}{22100} = 0.002986$$

$$D = C_1^{40} C_1^4 C_1^1 = \frac{40!}{1!*39!} * \frac{4!}{1!*3!} * \frac{1!}{1!*0!} = 40 * 4 * 1 = 160$$

$$\therefore P(D) = \frac{h}{n} = \frac{n(D)}{n} = \frac{160}{22100} = 0.00224$$

ملاحظة مهمة : عندما يذكر ترتيب معين لعملية الاختيار كما هو الحال في الحدث (E) حيث ذكر ان الورقة الأولى التي سحبت هي من (الآس " الواحد في ورق اللعب") والورقتان الثانية والثالثة من الصور ، عندئذٍ نستخدم في الحل التباديل بدلاً من التوافيق :

$$E = P_1^4 * P_2^{12}$$

$$\therefore P(E) = \frac{P_1^4 * P_2^{12}}{P_3^{52}} = \frac{\frac{4!}{3!} * \frac{12!}{10!}}{\frac{52!}{49!}} = \frac{528}{22100} = 0.003982$$

Exp)

Three light bulbs are chosen at random from 15 bulbs which are 5 of them are defective ; Find the probability that :

A = {non defective}

B = {exactly one is defective}

C = {at least one is defective}

Sol/

$$S = C_3^{15} = \frac{15!}{3!*12!} = 455$$

$$A = C_3^{10} = \frac{10!}{3!*7!} = 120$$

$$\therefore P\ (A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{120}{455} = 0.2637$$

$$B = C_1^5 * C_2^{10} = \frac{5!}{1!*4!} * \frac{10!}{2!*8!} = 5 * 45 = 225$$

$$\therefore P\ (B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{225}{455} = 0.4945$$

$$C = C_1^5 * C_2^{10} + C_2^5 * C_1^{10} + C_3^5 * C_0^{10} =$$

$$= \frac{5!}{1!*4!} * \frac{10!}{2!*8!} + \frac{5!}{2!*3!} * \frac{10!}{1!*9!} + \frac{5!}{3!*1!} * \frac{10!}{0!*10!} = 335$$

$$\therefore P\ (C) = \frac{h}{n} = \frac{n(C)}{n} = \frac{335}{455} = 0.7363$$

Or

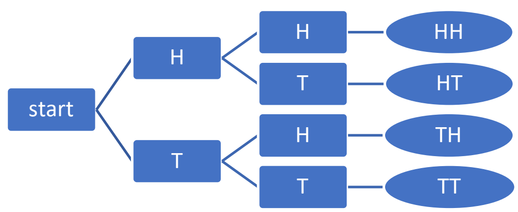
$$P\ (C) = 1 - P\ (A) = 1 - \frac{C_3^{10} * C_0^5}{C_3^{15}} = 1 - 0.2637 = 0.7363$$

Exp)

Two coins are to be tossed once , Find the possible outcomes of this experiments ?

Sol/

H : head ; T : tail



$\therefore S = \{HH, HT, TH, TT\}$

Exp)

A pair of dice is to be rolled once, Find the following events?

A _ The event that the sum of the two faces is 7.

B _ The event that the sum of the two faces is 11.

C _ The event that the sum of the two faces is either 7 or 11.

Sol/

A= {(1,6) ;(2,5) ;(3,4) ;(4,3) ;(5,2) ;(6,1)}

B= {(5,6) ;(6,5)}

C= $A \cup B$ = {(1,6) ;(2,5) ;(3,4) ;(4,3) ;(5,2) ;(6,1) ;(5,6) ;(6,5)} probability of these events لو كان المطلوب هو إيجاد

$$\therefore P\ (A) = \frac{h}{n} = \frac{n(A)}{n} = \frac{6}{36}$$

$$\therefore P\ (B) = \frac{h}{n} = \frac{n(B)}{n} = \frac{2}{36}$$

$$\therefore P\ (C) = \frac{h}{n} = \frac{n(C)}{n} = \frac{8}{36}$$

$E_1 = \{ The\ first\ face\ is\ greater\ than\ second\ face \}$

$E_1 = \{(2,1); (3,1); (3,2); (4,1); (4,2); (4,3); (5,1); (5,2); (5,3); (5,4);$
$(6,1);\ (6,2); (6,3); (6,4); (6,5) \}$

$$\therefore P\ (E_1) = \frac{h}{n} = \frac{n(E_1)}{n} = \frac{15}{36}$$

$E_2 = \{ The\ difference\ between\ two\ faces\ zero \}$

$E_2 = \{(1,1) ; (2,2) ; (3,3) ; (4,4) ; (5,5) ; (6,6)\}$

$$\therefore P\ (E_2) = \frac{h}{n} = \frac{n(E_2)}{n} = \frac{6}{36}$$

$E_3 = \{ The\ second\ face\ shows\ even\ number \}$

$E_4 = \{ The\ first\ face\ is\ an\ even\ and\ the\ second\ is\ 5 \}$

$E_5 = \{ The\ second\ face\ shows\ a\ number\ 3\}$

$E_6 = \{ The\ second\ face\ shows\ number\ 1\ or\ 5\ and\ the\ sum\ of$
$two\ faces\ (number) is\ less\ than\ 8\ and\ more\ than\ 3\ \}$

____(H.W)