FUZZY LOGIC

Fuzzy logic is used to imitate human reasoning and cognition. Rather than strictly binary cases of truth, fuzzy logic includes 0 and 1 as extreme cases of truth but with various intermediate degrees of truth.

Fuzzy sets, on the other hand, allow elements to be partially in a set. Each element is given a degree of membership in a set. This membership value can range from 0 (not an element of the set) to 1 (a member of the set). It is clear that if one only allowed the extreme membership values of 0 and 1, that this would actually be equivalent to crisp sets. **A membership function** is the relationship between the values of an element and its degree of membership in a set. An example of membership functions. The sets (or classes) are numbers that are negative large, negative medium, negative small, near zero, positive small, positive medium, and positive large. The value, μ, is the amount of membership in the set.

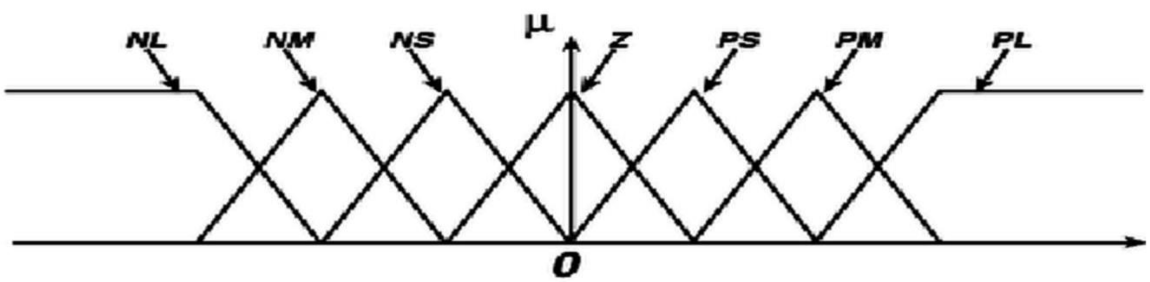
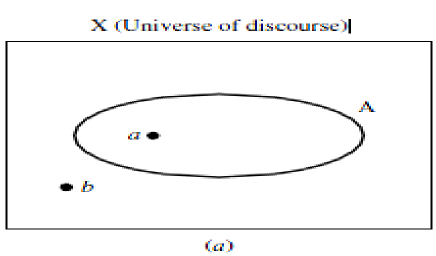


Fig: Membership Functions for the Set of All Numbers (N = Negative, P = Positive, L = Large,M = Medium, S = Small)

A **classical set** is defined by **crisp** boundaries

A fuzzy set is prescribed by **vague** or **ambiguous** properties; hence its boundaries are ambiguously specified



The universe of discourse : is the range of all possible values for an input to a fuzzy system. X, as a collection of objects all having the same characteristics.

The individual elements in the universe X will be denoted as x. The features of the elements in X can be discrete, countable integers or continuous valued quantities on the real line.

The total number of elements in a universe X is called its **cardinal number**, denoted n_x.Collections of elements within a universe are called **sets.**

Collections of elements within sets are called **subsets.** The collection of all possible sets in the universe is called the whole set (power set).

Why Fuzzy logic

1. Fuzzy logic is useful for commercial and practical purposes.

2. It can control machines and consumer product

3. It may be not give accurate reasoning but acceptable reasoning

4 fuzzy logic helps to deal with uncertainty in engineering

Examples of Fuzzy logic:

- The clock speeds of computer CPUs
- The operating currents of an electronic motor.
- The operating temperature of a heat pump (in degrees Celsius).
- The Richter magnitudes of an earthquake.
- The integers 1 to 10

Fuzzy Sets

Fuzzy Set Theory was formalized by Professor Lofti Zadeh at the University of California in 1965. What Zadeh proposed is very much a paradigm shift that first gained acceptance in the Far East and its successful application has ensured its adoption around the world.

A paradigm is a set of rules and regulations which defines boundaries and tells us what todo to be successful in solving problems within these boundaries. The boundaries of the fuzzy sets are vague and ambiguous. Hence, membership of an element from the universe in this set is measured by a function that attempts to describevagueness and ambiguity

Elements of a fuzzy set are mapped to a universe of **membership values** using a function-theoretic form. Fuzzy sets are denoted by a set symbol with a tilde under strike; $A_{\sim}$ would be the fuzzy set A.

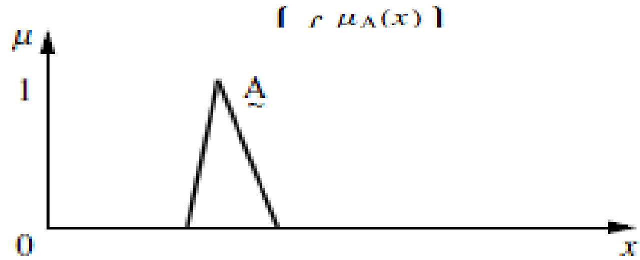
This function maps elements of a fuzzy set $A_{\sim}$ to a real numbered value on the interval 0 to 1. If an element in the universe, say x, is a member of fuzzy set A~, then this mapping is given by:

$$\mu_A(x) \in [0, 1]$$

When the universe of discourse, X, is discrete and finite, is as follows for a fuzzy setA~ :

$$\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \cdots \right\} = \left\{ \sum_i \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i} \right\}$$

When the universe, X, is continuous and infinite, the fuzzy setA~



Membership function for fuzzy set A~

Three fuzzy sets A , B, and C on the universe X

Fuzzy Set Operations

Union The membership function of the Union of two fuzzy sets A and B with membership $\boldsymbol{\mu_A}$ and $\boldsymbol{\mu_B}$ respectively is defined as the maximum of the two individual membership functions. This is called the maximum criterion.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B)$$

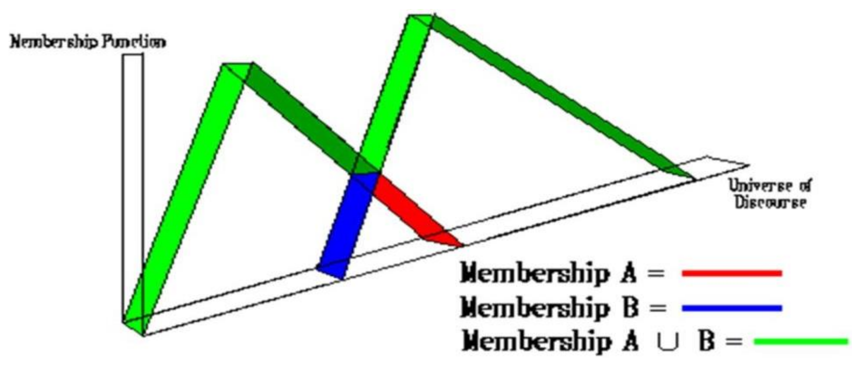


Fig: The Union operation in Fuzzy set theory is the equivalent of the **OR** operation inBoolean algebra.

Intersection

The membership function of the Intersection of two fuzzy sets A and B with membership functions $\boldsymbol{\mu_A}$ and $\boldsymbol{\mu_B}$ respectively is defined as the minimum of the two individual membership functions.

This is called the minimum criterion.

$$\mu_{A \cap B} = \min(\mu_A, \mu_B)$$

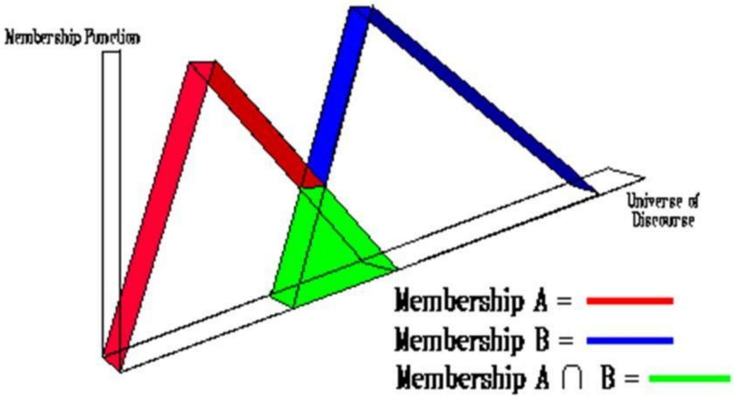
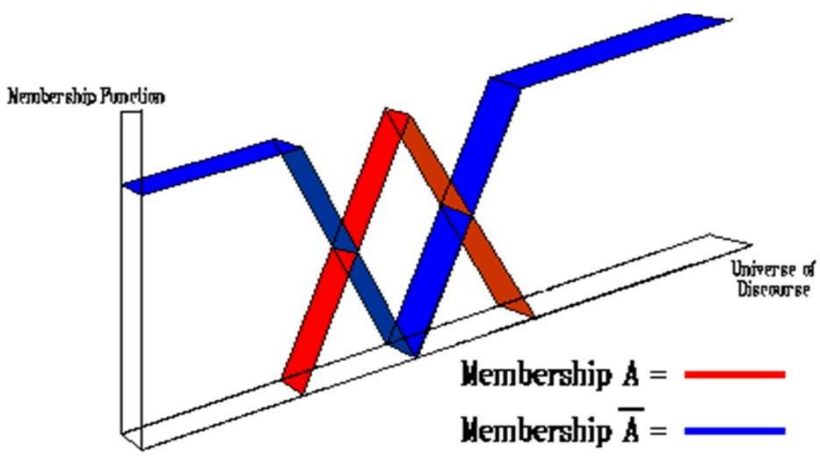


Fig: The Intersection operation in Fuzzy set theory is the equivalent of the **AND** operation in Boolean algebra.

Complement

The membership function of the Complement of a Fuzzy set A with membership $\boldsymbol{\mu_A}$ function is defined as the negation of the specified membership function. This is called the negation criterion.

$$\mu_{\overline{A}} = 1 - \mu_A$$



The Complement operation in Fuzzy set theory is the equivalent of the **NOT** operation inBoolean algebra.

The following rules which are common in classical set theory also apply to Fuzzy set theory.

De Morgans law

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}) \quad , \overline{(A \cup B)} = \bar{A} \cap \bar{B})$$

Associativity:

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Commutativity

$$A \cap B = B \cap A, A \cup B = B \cup A$$

Distributivism

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

All other operations on classical sets also hold for fuzzy sets, **except** for the **excluded middleaxiom**

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X$$
$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \emptyset$$

RELATIONS

Relations represent mappings between sets and connectives in logic. A classical binary relation represents the presence or absence of a connection or interaction or association between the elements of two sets. Fuzzy binary relations are a generalization of crisp binary relations, andthey allow various degrees of relationship (association) between elements.

Fuzzy Relations

A **crisp relation** represents the presence or absence of association, interaction, or interconnectedness between the elements of two or more sets. It alow for various degrees or strengths of relation or interaction between elements. Degrees of association can be represented by membership grades in a **fuzzy relation** in the same way as degrees of set membership are represented in the fuzzy set.

Cartesian product

The *Cartesian product* of two crisp sets *X* and *Y*, denoted by $X \times Y$ is the crisp set of all ordered pairs such that the first element in each pair is a member of *X* and the second element isa member of *Y*. Formally,

$$X \times Y = \{(x, y) | x \in X, y \in Y\}$$

Note, that if $X \neq Y$, then $X \times Y \neq Y \times X$.

The Cartesian product can be generalized for a family of crisp sets$\{X | i \in N^n\}$ and denoted either by $(X_1 \times X_2 \times \ldots \ldots X_r$or by $\times_{\in k'}\ X_r)$. Elements of the Cartesian product of *n* crisp sets are *n*-tuples $(x_1, x_2, \ldots, x_n)\ such\ that\ x_i \in N^x$

Thus,

$$\times_{i \in \mathbb{N}^n} X_i = \{(x_1, x_2, \ldots, x_n) | x_i \in X_i \text{ for all } i \in \mathbb{N}^n\}$$

It is possible for all sets X to be equal, that is, to be a single set *X*. In this case, the Cartesian product of a set *X* with itself *n* times is usually denoted by X^n.

Relation among sets

A relation among crisp sets $X_1 \times X_2 \times \ldots\ldots X_n$ is a subset of the Cartesian product $X_1 \times X_2 \times \ldots\ldots X_n$. it is denoted either by $R(X_1 \times X_2 \times \ldots\ldots X_n)$ or by the abbreviated form$R(X_i|i \in N^x)$. Thus,

$$R(X_1, X_2, \ldots, X_n) \subset X_1 \times X_2 \times \ldots \times X_n,$$

For relations among sets, the Cartesian product $X_1 \times X_2 \times \ldots\ldots X_n$ represents the universal set. Because a relation is itself a set$X_1 \times X_2 \times \ldots\ldots X_n$, the basic set concepts such as containment or subset, union, intersection, and complement can be applied without modification to relations.

Each crisp relation *R* can be defined by a characteristic function that assigns a value 1 to every tuple of the universal set belonging to the relation and a 0 to every tuple that does not belong. Thus,

$$\mu_R(x_1, x_2, \ldots, x_n) = \begin{cases} 1 & \text{if and only if } (x_1, x_2, \ldots, x_n) \in R, \\ 0 & \text{otherwise.} \end{cases}$$

The membership of a tuple in a relation signifies that the elements of the tuple are related or associated with one another.

A relation can be written as a set of ordered tuples. Another convenient way of representing a relation $R(X_1 \times X_2 \times \ldots\ldots X_n)$ involves an *n*-dimensional membership array:

$$M_R = [\mu_{i_1, i_2, \ldots\ldots i_n}]$$

Each element of the first dimension i_1 of this array corresponds to exactly one member of X_1, each

$$\mu_{i_1 i_2 \ldots i_n} = \begin{cases} 1 & \text{if and only if } (x_1, x_2, \ldots, x_n) \in R, \\ 0 & \text{otherwise.} \end{cases}$$

element of the first dimension i_2 to exactly one member of X_2, and so on. If the *n*-tuple, then

Just as the characteristic function of a crisp set can be generalized to allow for degrees of set membership, the characteristic function of a crisp relation can be generalized to allow tuples to have degrees of membership within the relation.

Thus, a fuzzy relation is a fuzzy set defined on the Cartesian product of crisp sets $X_1 \times X_2 \times \ldots \ldots X_n$ where tuples $(x_1, x_2 \ldots, x_n)$ may have varying degrees ofmembership within the relation. The membership grade is usually represented by a real numberin the closed interval [0.1] and indicates the strength of the relation present between the elements of the tuple.

A fuzzy relation can also conveniently be represented by an *n*-dimensional membership array whose entries correspond to *n*-tuples in the universal set. These entries take values representing the membership grades of the corresponding *n*-tuples.

Examples

- Let *R* be a crisp relation among the two sets *X*={dollar, pound, franc, mark} and *Y*={United States, France, Canada, Britain, Germany}, which associates a country with a currency asfollows:

R(*X*,*Y*)= {(dollar,United States), (franc,France), (dollar,Canada), (pound,Britain), (mark,Germany) }

This relation can also be represented by the following two dimensional membership array:

	U.S.	France	Canada	Britain	Germany
dollar	1	0	1	0	0
pound	0	0	0	1	0
franc	0	1	0	0	0
mark	0	0	0	0	1

- Let *R* be a fuzzy relation among the two sets the distance to the target *X*={far, close, very close} and the speed of the car *Y*={very slow, slow, normal, quick, very quick}, which represents the relational concept "the break must be pressed very strong".

This relation can be written in list notation as :

R(*X*,*Y*) = {0/(far, very slow) + .3/(close, very slow) + .8/(very close, very slow) + 0/(far, slow) +.4/(close, slow) + .9/(very close, slow) + 0/(far, normal) + .5/(close, normal) + 1/(ver close, normal) + .1/(far, quick) + .6/(close, quick) + 1/(very close, quick) + .2/(far, very quick)+ .7/(close, very quick)+ 1/(very close, very quick)}. This relation can also be represented by the following two dimensional membership arrays:

	very slow	slow	normal	quick	very quick
far	0	0	0	.1	.2
close	.3	.4	.5	.6	.7
very close	.8	.9	1	1	1

Fuzzy Logic Controller:

Fuzzification

Establishes the fact base of the fuzzy system. It identifies the input and output of the system, defines appropriate IF THEN rules, and uses raw data to derive a membership function.

- Consider an air conditioning system that determine the best circulation level by samplingtemperature and moisture levels. The inputs are the current temperature and moisture level. The fuzzy system outputs the best air circulation level: “none”, “low”, or “high”. The following fuzzy rules are used:

1. If the room is hot, circulate the air a lot.
2. If the room is cool, do not circulate the air.
3. If the room is cool and moist, circulate the air slightly.

A knowledge engineer determines membership functions that map temperatures to fuzzy values and map moisture measurements to fuzzy values.

Inference

Evaluates all rules and determines their truth values. If an input does not precisely correspond to an IF THEN rule, partial matching of the input data is used to interpolatean answer.

Continuing the example, suppose that the system has measured temperature and moisture levels and mapped them to the fuzzy values of .7 and .1 respectively. The system now infers the truth of each fuzzy rule. To do this a simple method called MAX-MIN is used.This method sets the fuzzy value of the THEN clause to the fuzzy value of the IF clause. Thus, the method infers fuzzy values of 0.7, 0.1, and 0.1 for rules 1, 2, and 3 respectively.

Composition

Combines all fuzzy conclusions obtained by inference into a single conclusion. Since different fuzzy rules might have different conclusions, consider all rules.

Continuing the example, each inference suggests a different action

- rule 1 suggests a "high" circulation level
- rule 2 suggests turning off air circulation
- rule 3 suggests a "low" circulation level.

A simple MAX-MIN method of selection is used where the maximum fuzzy value of the inferences is used as the final conclusion. So, composition selects a fuzzy value of 0.7 since this was the highest fuzzy value associated with the inference conclusions.

Defuzzification

Convert the fuzzy value obtained from composition into a “crisp” value. This process is often complex since the fuzzy set might not translate directly into a crisp value.

Continuing the example, composition outputs a fuzzy value of 0.7. This imprecise value is not directly useful since the air circulation levels are “none”, “low”, and “high”. The defuzzification process converts the fuzzy output of 0.7 into one of the air circulation levels. In this case it is clear that a fuzzy output of 0.7 indicates that the circulation should be set to “high”.

There are many defuzzification methods. Two of the more common techniques are the centroid and maximum methods.

In the centroid method, the crisp value of the output variable is computed by finding the variable value of the center of gravity of the membership function for the fuzzy value.

In the maximum method, one of the variable values at which the fuzzy subset has its maximum truth value is chosen as the crisp value for the output variable.