Inference in First-Order Logic

Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

FOL inference rules for quantifier:

As propositional logic we also have inference rules in first-order logic, so following are some basic inference rules in FOL:

1. Universal Generalization:

Universal generalization is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x). It can be represented as:

$$\frac{P(c)}{\forall x\, P(x)}.$$

- This rule can be used if we want to show that every element has a similar property. In this rule, x must not appear as a free variable.

Example: Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits**.", it will also be true.

2. Universal Instantiation:

Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences. The new KB is logically equivalent to the previous KB. As per UI, **we can infer any sentence obtained by substituting a ground term for the variable**.

The UI rule state that can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from **∀ x P(x) for any object in the universe of discourse**. It can be represented as:

$$\frac{\forall x\, P(x)}{P(c)}$$

Example: 1.

IF "Every person like ice-cream"=> ∀x P(x) so can infer that "John likes ice-cream" => P(c)

Example: 2.

Let's take a famous example,

"All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

∀x king(x) ∧ greedy (x) → Evil (x),

So from this information, we can infer any of the following statements using Universal Instantiation:

- **King(John) ∧ Greedy (John) → Evil (John),**
- **King(Richard) ∧ Greedy (Richard) → Evil (Richard),**
- **King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John)),**

3. Existential Instantiation:

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic. It can be applied only once to replace the existential sentence. The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable. This rule states that one can infer P(c) from the formula given in the form of ∃x P(x) for a new constant symbol c. The restriction with this rule is that c used in the rule must be a new term for which P(c) is true. It can be represented as:

$$\frac{\exists x\, P(x)}{P(c)}$$

Example:

From the given sentence: **∃x Crown(x) ∧ OnHead(x, John),**

So we can infer: **Crown(K) ∧ OnHead(K, John),** as long as K does not appear in the knowledge base.

4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P, then we can infer that there exists something in the universe which has the property P.
- It can be represented as: $\frac{P(c)}{\exists x P(x)}$

Example: Let's say that,

"Priyanka got good marks in English." "Therefore, someone got good marks in English."

What is Unification?

- Unification is a process of making two different logical atomic expressions identical by finding a substitution. Unification depends on the substitution process.
- It takes two literals as input and makes them identical using substitution.

- Let Ψ_1 and Ψ_2 be two atomic sentences and σ be a unifier such that, $\boldsymbol{\Psi_1\sigma = \Psi_2\sigma}$, then it can be expressed as **UNIFY (Ψ_1, Ψ_2)**.

  Example: Find the MGU for Unify{King(x), King(John)}

Let Ψ_1 = King(x), Ψ_2 = King(John),

Substitution θ = {John/x} is a unifier for these atoms and applying this substitution, and both expressions will be identical.

- The UNIFY algorithm is used for unification, which takes two atomic sentences and returns a unifier for those sentences (If any exist).
- Unification is a key component of all first-order inference algorithms.
- It returns fail if the expressions do not match with each other.
- The substitution variables are called Most General Unifier or MGU.

E.g. Let's say there are two different expressions, **P(x, y), and P(a, f(z))**.

In this example, we need to make both above statements identical to each other. For this, we will perform the substitution.

P(x, y)......... (i)

P(a, f(z))......... (ii)

- Substitute x with a, and y with f(z) in the first expression, and it will be represented as **a/x** and f(z)/y.
- With both the substitutions, the first expression will be identical to the second expression and the substitution set will be: **[a/x, f(z)/y]**.

Conditions for Unification:

Following are some basic conditions for unification:

- Predicate symbol must be same, atoms or expression with different predicate symbol can never be unified.
- Number of Arguments in both expressions must be identical.
- Unification will fail if there are two similar variables present in the same expression.

Unification Algorithm:

Algorithm: Unify(Ψ_1, Ψ_2)

```
Step. 1: If $\Psi_1$ or $\Psi_2$ is a variable or constant, then:
    a) If $\Psi_1$ or $\Psi_2$ are identical, then return NIL.
    b) Else if $\Psi_1$is a variable,
            a. then if $\Psi_1$ occurs in $\Psi_2$, then return FAILURE
            b. Else return { ($\Psi_2$/ $\Psi_1$)}.
        c) Else if $\Psi_2$ is a variable,
            a. If $\Psi_2$ occurs in $\Psi_1$ then return FAILURE,
            b. Else return {( $\Psi_1$/ $\Psi_2$)}.
        d) Else return FAILURE.
Step.2: If the initial Predicate symbol in $\Psi_1$ and $\Psi_2$ are not same, then return FAILURE.
Step. 3: IF $\Psi_1$ and $\Psi_2$ have a different number of arguments, then return FAILURE.
Step. 4: Set Substitution set (SUBST) to NIL.
Step. 5: For i=1 to the number of elements in $\Psi_1$.
        a) Call Unify function with the $i^{th}$ element of $\Psi_1$ and ith element of $\Psi_2$, and put the result into S.
        b) If S = failure then returns Failure
        c) If S $\neq$ NIL then do,
            a. Apply S to the remainder of both L1 and L2.
            b. SUBST= APPEND(S, SUBST).
Step.6: Return SUBST.
```

Implementation of the Algorithm

Step.1: Initialize the substitution set to be empty.

Step.2: Recursively unify atomic sentences:

a. Check for Identical expression match.

b. If one expression is a variable v_i, and the other is a term t_i which does not contain variable v_i, then:

   a. Substitute t_i / v_i in the existing substitutions

   b. Add t_i /v_i to the substitution setlist.

   c. If both the expressions are functions, then function name must be similar, and the number of arguments must be the same in both the expression.

For each pair of the following atomic sentences find the most general unifier MGU (If exist) which mean unifier of two terms.

1. Find the MGU of {p(f(a), g(Y)) and p(X, X)}

Sol: S_0 => Here, Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(X, X)

SUBST θ= {f(a) / X}

S1 => Ψ_1 = p(f(a), g(Y)), and Ψ_2 = p(f(a), f(a))

SUBST θ= {f(a) / g(y)}, **Unification failed**.

Unification is not possible for these expressions.

2. Find the MGU of {p(b, X, f(g(Z))) and p(Z, f(Y), f(Y))}

Here, Ψ_1 = p(b, X, f(g(Z))) , and Ψ_2 = p(Z, f(Y), f(Y))

S_0 => { p(b, X, f(g(Z))); p(Z, f(Y), f(Y))}

SUBST θ={b/Z}

S_1 => { p(b, X, f(g(b))); p(b, f(Y), f(Y))}

SUBST θ={f(Y) /X}

S_2 => { p(b, f(Y), f(g(b))); p(b, f(Y), f(Y))}

SUBST θ= {g(b) /Y}

S_2 => { p(b, f(g(b)), f(g(b)); p(b, f(g(b)), f(g(b))} **Unified Successfully.**

And Unifier = { b/Z, f(Y) /X , g(b) /Y}.

3. Find the MGU of {p (X, X), and p (Z, f(Z))}

Here, Ψ_1 = {p (X, X), and Ψ_2 = p (Z, f(Z))

S_0 => {p (X, X), p (Z, f(Z))}

SUBST θ= {X/Z}

S1 => {p (Z, Z), p (Z, f(Z))}

SUBST θ= {f(Z) / Z}, **Unification Failed**.

Hence, unification is not possible for these expressions.

4. Find the MGU of UNIFY(prime (11), prime(y))

Here, Ψ_1 = {prime(11) , and Ψ_2 = prime(y)}

S_0 => {prime(11) , prime(y)}

SUBST θ= {11/y}

S_1 => {prime(11) , prime(11)} , **Successfully unified.**

Unifier: {11/y}.

5. Find the MGU of Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)}

Here, Ψ_1 = Q(a, g(x, a), f(y)), and Ψ_2 = Q(a, g(f(b), a), x)

S_0 => {Q(a, g(x, a), f(y)); Q(a, g(f(b), a), x)}

SUBST θ= {f(b)/x}

S_1 => {Q(a, g(f(b), a), f(y)); Q(a, g(f(b), a), f(b))}

SUBST θ= {b/y}

S_1 => {Q(a, g(f(b), a), f(b)); Q(a, g(f(b), a), f(b))}, **Successfully Unified.**

Unifier: [a/a, f (b)/x, b/y].

6. UNIFY (knows (Richard, x), knows (Richard, John))

Here, Ψ_1 = knows(Richard, x), and Ψ_2 = knows(Richard, John)

S_0 => { knows(Richard, x); knows(Richard, John)}

SUBST θ= {John/x}

S_1 => { knows(Richard, John); knows(Richard, John)}, **Successfully Unified.**

Unifier: {John/x}.

Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions.

Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements. Unification is a key concept in proofs by resolutions. Resolution is a single inference rule which can efficiently operate on the **conjunctive normal form or clausal form**.

Clause: Disjunction of literals (an atomic sentence) is called a **clause**. It is also known as a unit clause.

Conjunctive Normal Form: A sentence represented as a conjunction of clauses is said to be **conjunctive normal form** or **CNF**.

Example:

We can resolve two clauses which are given below:

[Animal (g(x) V Loves (f(x), x)] and [¬ Loves(a, b) V ¬Kills(a, b)]

Where two complimentary literals are: **Loves (f(x), x) and ¬ Loves (a, b)**

These literals can be unified with unifier **θ= [a/f(x), and b/x]** , and it will generate a resolvent clause:

[Animal (g(x) V ¬ Kills (f(x), x)].

Steps for Resolution:

1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

To better understand all the above steps, we will take an example in which we will apply resolution.

Example:

a. **John likes all kind of food.**

b. **Apple and vegetable are food**

c. **Anything anyone eats and not killed is food.**

d. **Anil eats peanuts and still alive**

e. **Harry eats everything that Anil eats.**

Prove by resolution that:

f. **John likes peanuts.**

Step-1: Conversion of Facts into FOL

In the first step we will convert all the given statements into its first order logic.

a. ∀x: food(x) → likes(John, x)

b. food(Apple) Λ food(vegetables)

c. ∀x ∀y: eats(x, y) Λ ¬ killed(x) → food(y)

d. eats (Anil, Peanuts) Λ alive(Anil).

e. ∀x : eats(Anil, x) → eats(Harry, x)

f. ∀x: ¬ killed(x) → alive(x)

g. ∀x: alive(x) →¬ killed(x)

} **added predicates.**

h. likes(John, Peanuts)

Step-2: Conversion of FOL into CNF

In First order logic resolution, it is required to convert the FOL into CNF as CNF form makes easier for resolution proofs.

- **Eliminate all implication (→) and rewrite**
  a. ∀x ¬ food(x) V likes(John, x)
  b. food(Apple) Λ food(vegetables)
  c. ∀x ∀y ¬ [eats(x, y) Λ ¬ killed(x)] V food(y)
  d. eats (Anil, Peanuts) Λ alive(Anil)
  e. ∀x ¬ eats(Anil, x) V eats(Harry, x)
  f. ∀x¬ [¬ killed(x)] V alive(x)
  g. ∀x ¬ alive(x) V ¬ killed(x)
  h. likes(John, Peanuts).
- **Move negation (¬)inwards and rewrite**
  a. ∀x ¬ food(x) V likes(John, x)
  b. food(Apple) Λ food(vegetables).
  c. ∀x ∀y ¬ eats(x, y) V killed(x) V food(y).
  d. eats (Anil, Peanuts) Λ alive(Anil).
  e. ∀x ¬ eats(Anil, x) V eats(Harry, x).
  f. ∀x ¬killed(x) V alive(x).

g. ∀x ¬ alive(x) V ¬ killed(x).
h. likes(John, Peanuts).

- **Rename variables or standardize variables**
  a. ∀x ¬ food(x) V likes(John, x).
  b. food(Apple) Λ food(vegetables).
  c. ∀y ∀z ¬ eats(y, z) V killed(y) V food(z)
  d. eats (Anil, Peanuts) Λ alive(Anil)
  e. ∀w¬ eats(Anil, w) V eats(Harry, w)
  f. ∀g ¬killed(g)] V alive(g)
  g. ∀k ¬ alive(k) V ¬ killed(k)
  h. likes(John, Peanuts).

c. **Eliminate existential instantiation quantifier by elimination.**

In this step, we will eliminate existential quantifier ∃, and this process is known as **Skolemization**. But in this example problem since there is no existential quantifier so all the statements will remain same in this step.

d. **Drop Universal quantifiers.**

In this step we will drop all universal quantifier since all the statements are not implicitly quantified so we don't need it.

a. ¬ food(x) V likes(John, x).
b. food(Apple).
c. food(vegetables).
d. ¬ eats(y, z) V killed(y) V food(z).
e. eats (Anil, Peanuts).
f. alive(Anil).
g. ¬ eats(Anil, w) V eats(Harry, w).
h. killed(g) V alive(g).
i. ¬ alive(k) V ¬ killed(k).
j. Likes (John, Peanuts).

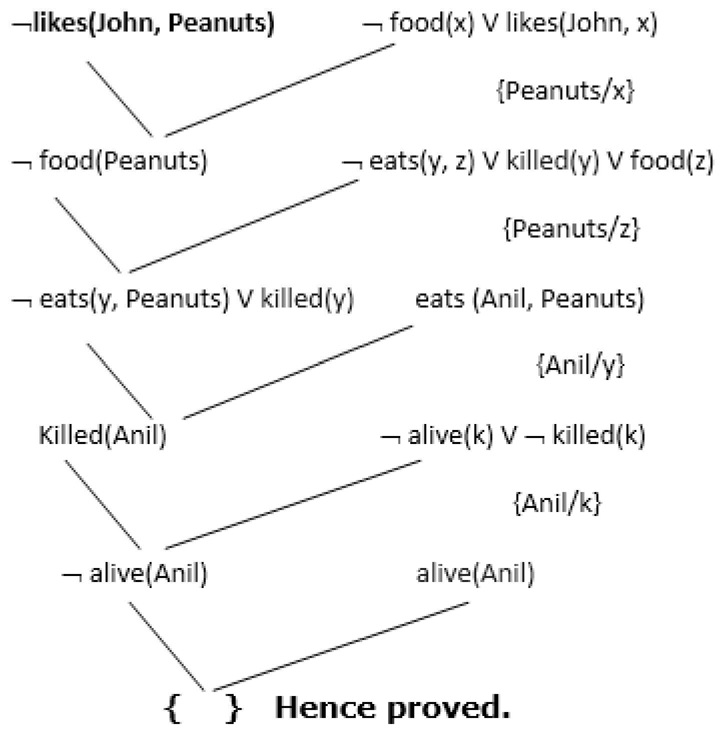
- **Distribute conjunction ∧ over disjunction ¬.**
  - This step will not make any change in this problem.

Step-3: Negate the statement to be proved

In this statement, we will apply negation to the conclusion statements, which will be written as ¬likes(John, Peanuts)

Step-4: Draw Resolution graph:

Now in this step, we will solve the problem by resolution tree using substitution. For the above problem, it will be given as follows:



Hence the negation of the conclusion has been proved as a complete contradiction with the given set of statements.

Explanation of Resolution graph:

- In the first step of resolution graph, **¬likes(John, Peanuts)** , and **likes(John, x)** get resolved(canceled) by substitution of **{Peanuts/x}**, and we are left with **¬ food(Peanuts)**
- In the second step of the resolution graph, **¬ food(Peanuts)** , and **food(z)** get resolved (canceled) by substitution of **{ Peanuts/z}**, and we are left with **¬ eats(y, Peanuts) V killed(y)** .
- In the third step of the resolution graph, **¬ eats(y, Peanuts)** and **eats (Anil, Peanuts)** get resolved by substitution **{Anil/y}**, and we are left with **Killed(Anil)** .
- In the fourth step of the resolution graph, **Killed(Anil)** and **¬ killed(k)** get resolve by substitution **{Anil/k}**, and we are left with **¬ alive(Anil)** .
- In the last step of the resolution graph **¬ alive(Anil)** and **alive(Anil)** get resolved.

Forward Chaining and backward chaining in AI

In artificial intelligence, forward and backward chaining is one of the important topics, but before understanding forward and backward chaining lets first understand that from where these two terms came.

Inference engine:

The inference engine is the component of the intelligent system in artificial intelligence, which applies logical rules to the knowledge base to infer new information from known facts. The first inference engine was part of the expert system. Inference engine commonly proceeds in two modes, which are:

a. **Forward chaining**

b. **Backward chaining**

Horn Clause and Definite clause:

Horn clause and definite clause are the forms of sentences, which enables knowledge base to use a more restricted and efficient inference algorithm. Logical inference algorithms use forward and backward chaining approaches, which require KB in the form of the **first-order definite clause**.

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Definite clause: A clause which is a disjunction of literals with **exactly one positive literal** is known as a definite clause or strict horn clause.

Horn clause: A clause which is a disjunction of literals with **at most one positive literal** is known as horn clause. Hence all the definite clauses are horn clauses.

Example: (¬ p V ¬ q V k). It has only one positive literal k.

It is equivalent to $p \wedge q \rightarrow k$.

A. Forward Chaining

Forward chaining is also known as a forward deduction or forward reasoning method when using an inference engine. Forward chaining is a form of reasoning which start with atomic sentences in the knowledge base and applies inference rules (Modus Ponens) in the forward direction to extract more data until a goal is reached.

The Forward-chaining algorithm starts from known facts, triggers all rules whose premises are satisfied, and add their conclusion to the known facts. This process repeats until the problem is solved.

Properties of Forward-Chaining:

- It is a down-up approach, as it moves from bottom to top.

- It is a process of making a conclusion based on known facts or data, by starting from the initial state and reaches the goal state.
- Forward-chaining approach is also called as data-driven as we reach to the goal using available data.
- Forward -chaining approach is commonly used in the expert system, such as CLIPS, business, and production rule systems.

Consider the following famous example which we will use in both approaches:

Example:

"As per the law, it is a crime for an American to sell weapons to hostile nations. Country A, an enemy of America, has some missiles, and all the missiles were sold to it by Robert, who is an American citizen."

Prove that **"Robert is criminal."**

To solve the above problem, first, we will convert all the above facts into first-order definite clauses, and then we will use a forward-chaining algorithm to reach the goal.

Facts Conversion into FOL:

- It is a crime for an American to sell weapons to hostile nations. (Let's say p, q, and r are variables)

  American (p) ∧ weapon(q) ∧ sells (p, q, r) ∧ hostile(r) → Criminal(p) ...(1)
- Country A has some missiles. **?p Owns(A, p) ∧ Missile(p)**. It can be written in two definite clauses by using Existential Instantiation, introducing new Constant T1.

  Owns(A, T1) ......(2)

  Missile(T1) ....(3)
- All of the missiles were sold to country A by Robert.

  ?p Missiles(p) ∧ Owns (A, p) → Sells (Robert, p, A) ......(4)
- Missiles are weapons.

  Missile(p) → Weapons (p) ....(5)
- Enemy of America is known as hostile.

  Enemy(p, America) →Hostile(p) ....(6)
- Country A is an enemy of America.

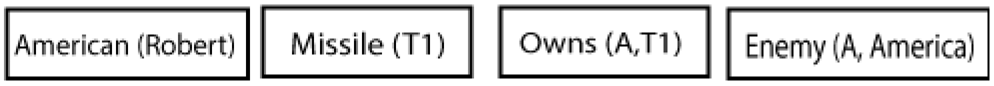
  Enemy (A, America) ....(7)
- Robert is American

  American(Robert). ....(8)

Forward chaining proof:

Step-1:

In the first step we will start with the known facts and will choose the sentences which do not have implications, such as: **American(Robert), Enemy(A, America), Owns(A, T1), and Missile(T1)**. All these facts will be represented as below.



Step-2:

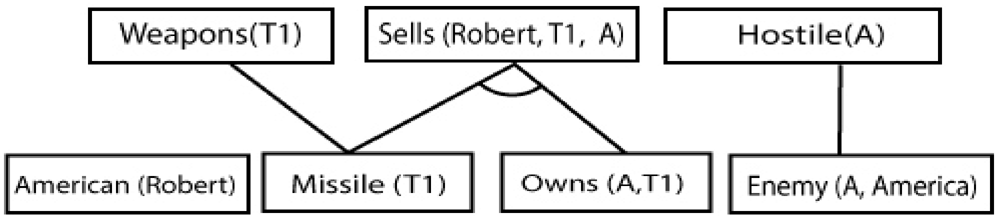
At the second step, we will see those facts which infer from available facts and with satisfied premises.

Rule-(1) does not satisfy premises, so it will not be added in the first iteration.

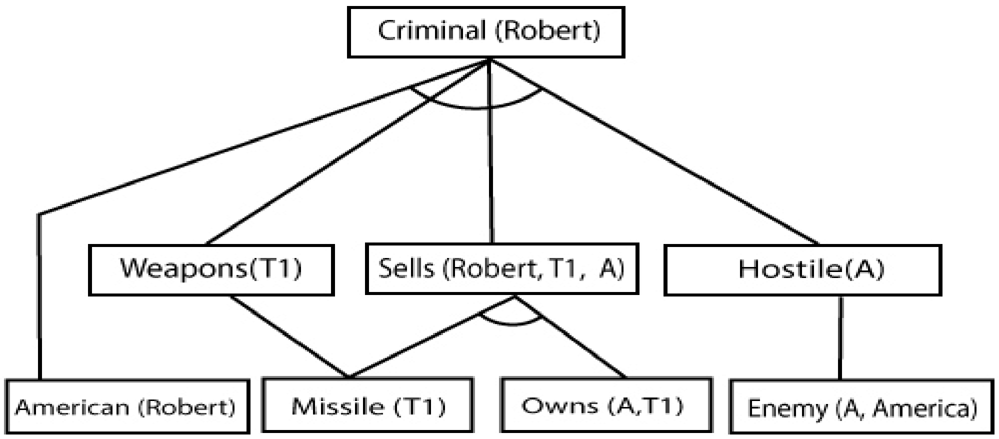
Rule-(2) and (3) are already added.

Rule-(4) satisfy with the substitution {p/T1}, **so Sells (Robert, T1, A)** is added, which infers from the conjunction of Rule (2) and (3).

Rule-(6) is satisfied with the substitution(p/A), so Hostile(A) is added and which infers from Rule-(7).



Step-3: At step-3, as we can check Rule-(1) is satisfied with the substitution **{p/Robert, q/T1, r/A}, so we can add Criminal (Robert)** which infers all the available facts. And hence we reached our goal statement.



Hence it is proved that Robert is Criminal using forward chaining approach.

B. Backward Chaining:

Backward-chaining is also known as a backward deduction or backward reasoning method when using an inference engine. A backward chaining algorithm is a form of reasoning, which starts with the goal and works backward, chaining through rules to find known facts that support the goal.

Properties of backward chaining:

- The process uses an up-down approach (top to bottom).
- Backward-chaining is based on modus ponens inference rule.
- In backward chaining, the goal is broken into sub-goal or sub-goals to prove the facts true.
- It is called a goal-driven approach, as a list of goals decides which rules are selected and used.
- Backward -chaining algorithm is used in game theory, automated theorem proving tools, inference engines, proof assistants, and various AI applications.
- The backward-chaining method mostly used a **depth-first search** strategy for proof.

Example:

In backward-chaining, we will use the same above example, and will rewrite all the rules.

- **American (p) ∧ weapon(q) ∧ sells (p, q, r) ∧ hostile(r) → Criminal(p) ...(1) Owns(A, T1) ....(2)**
- **Missile(T1)**
- **?p Missiles(p) ∧ Owns (A, p) → Sells (Robert, p, A) ......(4)**
- **Missile(p) → Weapons (p) ....(5)**
- **Enemy(p, America) →Hostile(p) ....(6)**
- **Enemy (A, America) ....(7)**
- **American(Robert). ....(8)**

Backward-Chaining proof:

In Backward chaining, we will start with our goal predicate, which is **Criminal(Robert)**, and then infer further rules.

Step-1:

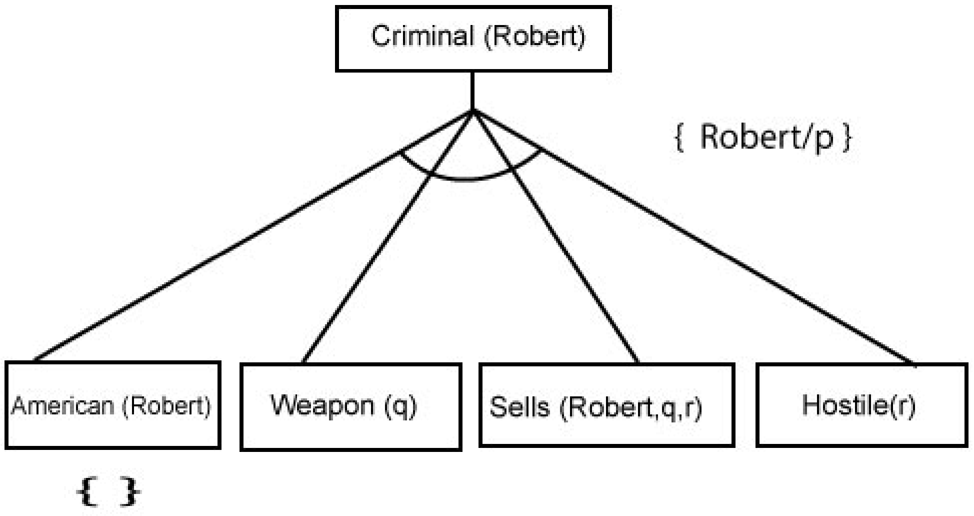
At the first step, we will take the goal fact. And from the goal fact, we will infer other facts, and at last, we will prove those facts true. So our goal fact is "Robert is Criminal," so following is the predicate of it.

Criminal (Robert)

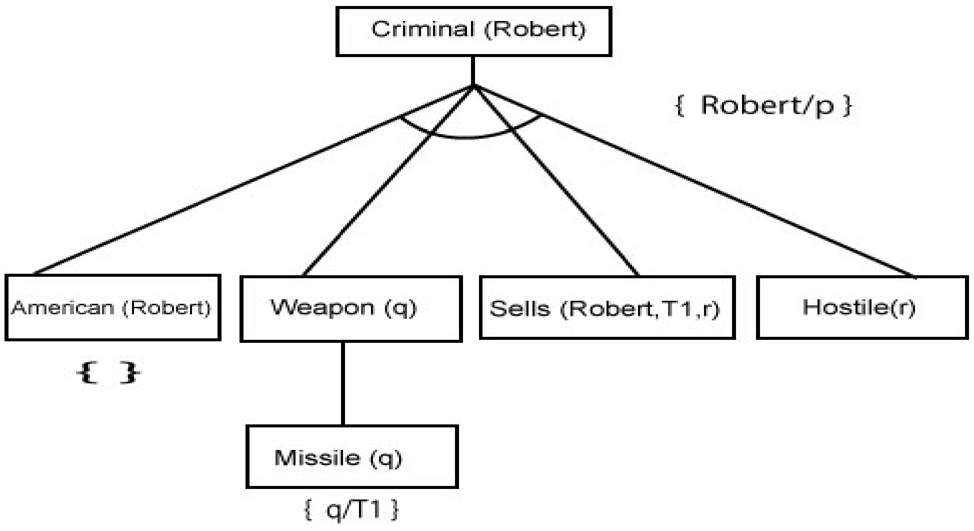
Step-2:

At the second step, we will infer other facts form goal fact which satisfies the rules. So as we can see in Rule-1, the goal predicate Criminal (Robert) is present with substitution {Robert/P}. So we will add all the conjunctive facts below the first level and will replace p with Robert.

Here we can see American (Robert) is a fact, so it is proved here.

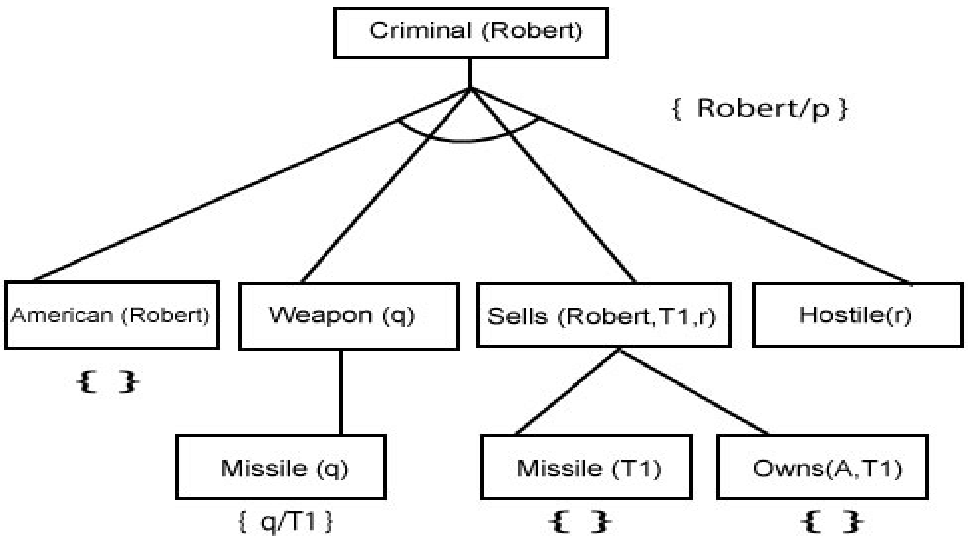


Step-3:t At step-3, we will extract further fact Missile(q) which infer from Weapon(q), as it satisfies Rule-(5). Weapon (q) is also true with the substitution of a constant T1 at q.



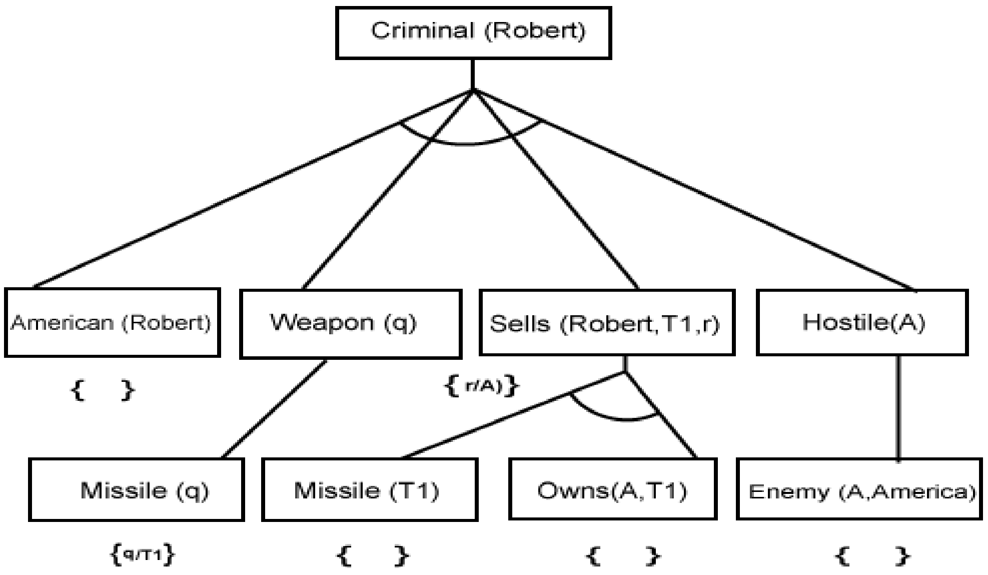
Step-4:

At step-4, we can infer facts Missile(T1) and Owns(A, T1) form Sells(Robert, T1, r) which satisfies the **Rule- 4**, with the substitution of A in place of r. So these two statements are proved here.



Step-5:

At step-5, we can infer the fact **Enemy(A, America)** from **Hostile(A)** which satisfies Rule- 6. And hence all the statements are proved true using backward chaining.



Difference between backward chaining and forward chaining

Following is the difference between the forward chaining and backward chaining:

- Forward chaining as the name suggests, start from the known facts and move forward by applying inference rules to extract more data, and it continues until it reaches to the goal, whereas backward chaining starts from the goal, move backward by using inference rules to determine the facts that satisfy the goal.
- Forward chaining is called a **data-driven** inference technique, whereas backward chaining is called a **goal-driven** inference technique.
- Forward chaining is known as the **down-up** approach, whereas backward chaining is known as a **top-down** approach.
- Forward chaining uses **breadth-first search** strategy, whereas backward chaining uses **depth-first search** strategy.
- Forward and backward chaining both applies **Modus ponens** inference rule.
- Forward chaining can be used for tasks such as **planning, design process monitoring, diagnosis, and classification**, whereas backward chaining can be used for **classification and diagnosis tasks**.
- Forward chaining can be like an exhaustive search, whereas backward chaining tries to avoid the unnecessary path of reasoning.
- In forward-chaining there can be various ASK questions from the knowledge base, whereas in backward chaining there can be fewer ASK questions.
- Forward chaining is slow as it checks for all the rules, whereas backward chaining is fast as it checks few required rules only.