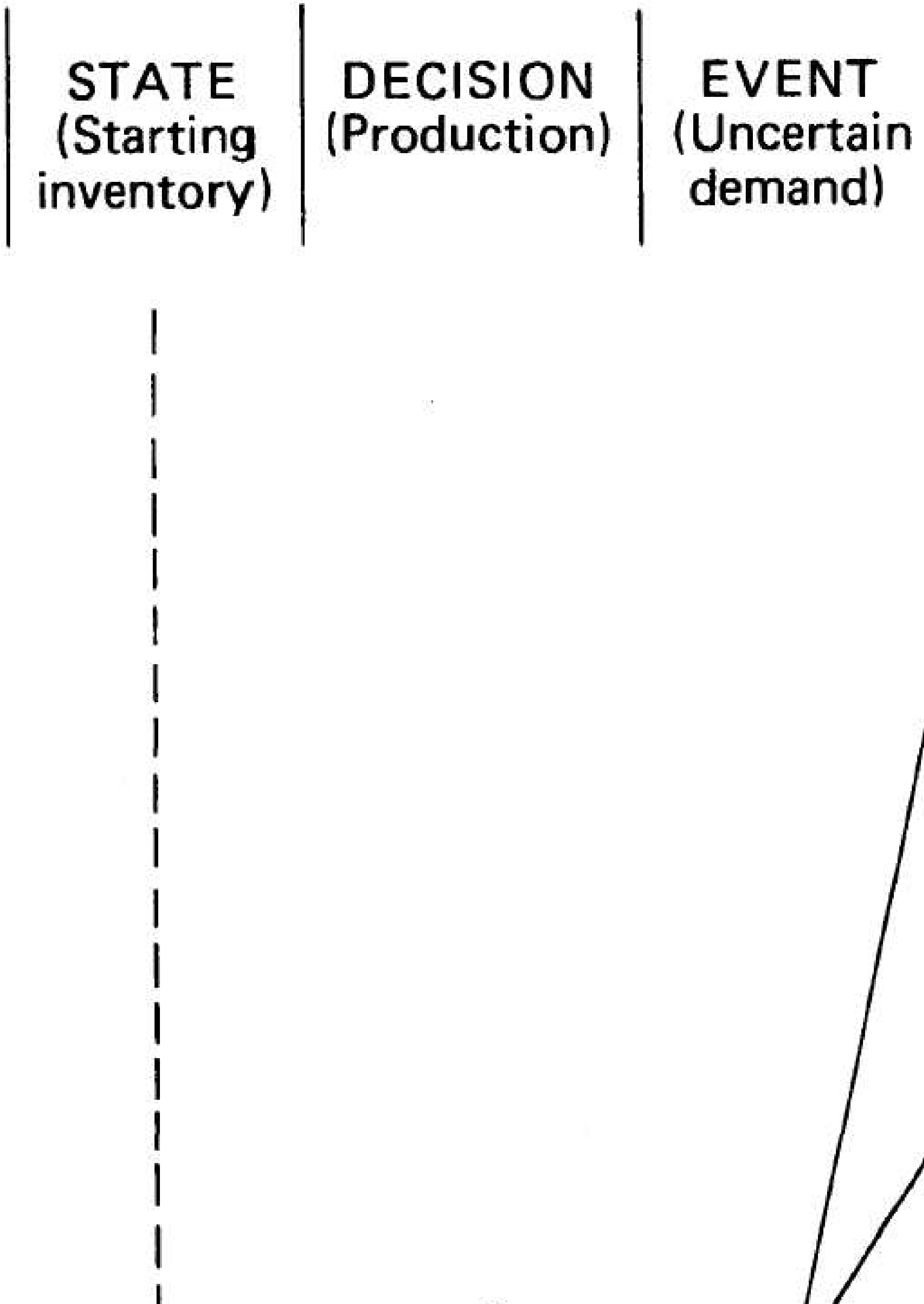
STATE
(Starting
inventory)
DECISION
(Production)
EVENT
(Uncertain
demand)

EXERCISES

Solutions to exercises marked with an asterisk (*) involve extensive computations. Formulate these problems as dynamic programs and provide representative computations to indicate the nature of the dynamic programming recursions; solve to completion only if a computer system is available.

1. In solving the minimum-delay routing problem in Section 11.1, we assumed the same delay along each street (arc) in the network. Suppose, instead, that the delay when moving along any arc upward in the network is 2 units greater than the delay when moving along any arc downward. The delay at the intersections is still given by the data in Fig. 11.1. Solve for the minimum-delay route by both forward and backward induction.

2. Decatron Mills has contracted to deliver 20 tons of a special coarsely ground wheat flour at the end of the current month, and 140 tons at the end of the next month. The production cost, based on which the Sales Department has bargained with prospective customers, is $c_1(x_1) = 7500 + (x_1 - 50)^2$ per ton for the first month, and $c_2(x_2) = 7500 + (x_2 - 40)^2$ per ton for the second month; x_1 and x_2 are the number of tons of the flour produced in the first and second months, respectively. If the company chooses to produce more than 20 tons in the first month, any excess production can be carried to the second month at a storage cost of $3 per ton.

   Assuming that there is no initial inventory and that the contracted demands must be satisfied in each month (that is, no back-ordering is allowed), derive the production plan that minimizes total cost. Solve by both backward and forward induction. Consider x_1 and x_2 as continuous variables, since any fraction of a ton may be produced in either month.

3. A construction company has four projects in progress. According to the current allocation of manpower, equipment, and materials, the four projects can be completed in 15, 20, 18, and 25 weeks. Management wants to reduce the completion times and has decided to allocate an additional $35,000 to all four projects. The new completion times as functions of the additional funds allocated to each projects are given in Table E11.1.

   How should the $35,000 be allocated among the projects to achieve the largest total reduction in completion times? Assume that the additional funds can be allocated only in blocks of $5000.

Table E11.1 Completion times (in weeks)

Additional funds ($\times$1000 dollars)	*Project 1*	*Project 2*	*Project 3*	*Project 4*
0	15	20	18	25
5	12	16	15	21
10	10	13	12	18
15	8	11	10	16
20	7	9	9	14
25	6	8	8	12
30	5	7	7	11
35	4	7	6	10

4. The following table specifies the unit weights and values of five products held in storage. The quantity of each item is unlimited.

Product	*Weight* (W_i)	*Value* (V_i)
1	7	9
2	5	4
3	4	3
4	3	2
5	1	$\frac{1}{2}$

A plane with a capacity of 13 weight units is to be used to transport the products. How should the plane be loaded to maximize the value of goods shipped? (Formulate the problem as an integer program and solve by dynamic programming.)

5. Any linear-programming problem with n decision variables and m constraints can be converted into an n-stage dynamic-programming problem with m state parameters.

   Set up a dynamic-programming formulation for the following linear program:

$$\text{Minimize} \sum_{j=1}^{n} c_j x_j,$$

subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad (i = 1, 2, \ldots, m),$$

$$x_j \geq 0 \qquad (j = 1, 2, \ldots, n).$$

   Why is it generally true that the simplex method rather than dynamic programming is recommended for solving linear programs?

6. Rambling Roger, a veteran of the hitchhiking corps, has decided to leave the cold of a Boston winter and head for the sunshine of Miami. His vast experience has given him an indication of the expected time in hours it takes to hitchhike over certain segments of the highways. Knowing he will be breaking the law in several states and wishing to reach the warm weather quickly, Roger wants to know the least-time route to take. He summarized his expected travel times on the map in Fig. E11.1. Find his shortest time route.

7. J. J. Jefferson has decided to move from the West Coast, where he lives, to a mid-western town, where he intends to buy a small farm and lead a quiet life. Since J. J. is single and has accumulated little furniture, he decides to rent a small truck for $200 a week or fraction of a week (one-way, no mileage charge) and move his belongings by himself. Studying the map, he figures that his trip will require four stages, regardless of the particular routing. Each node shown in Fig. E11.2 corresponds to a town where J. J. has either friends or relatives and where he plans to spend one day resting and visiting if he travels through the town. The numbers in brackets in Fig. E11.2 specify the travel time in days between nodes. (The position of each node in the network is not necessarily related to its geographical position on the map.) As he will travel through different states, motel rates, tolls, and gas prices vary significantly; Fig. E11.2 also shows the cost in dollars for traveling (excluding truck rental charges) between every two nodes. Find J. J.'s cheapest route between towns 1 and 10, including the truck rental charges.

8. At THE CASINO in Las Vegas, a customer can bet only in dollar increments. Betting a certain amount is called ''playing a round.'' Associated with each dollar bet on a round, the customer has a 40% chance to win another dollar and a 60% chance to lose his, or her, dollar. If the customer starts with $4 and wants to maximize the chances of finishing with at least $7 after two rounds, how much should be bet on each round? [*Hint.* Consider the number of dollars available at the beginning of each round as the state variable.]

*9. In a youth contest, Joe will shoot a total of ten shots at four different targets. The contest has been designed so that Joe will not know whether or not he hits any target until after he has made all ten shots. He obtains 6 points if any shot hits target 1, 4 points for hitting target 2, 10 points for hitting target 3, and 7 points for hitting target 4. At each shot there is an 80% chance that he will miss target 1, a 60% chance of missing target 2, a 90% chance of missing target 3, and a 50% chance of missing target 4, given that he aims at the appropriate target.

   If Joe wants to maximize his expected number of points, how many shots should he aim at each target?

10. A monitoring device is assembled from five different components. Proper functioning of the device depends upon its total weight q so that, among other tests, the device is weighted; it is accepted only if $r_1 \leq q \leq r_2$, where the two limits r_1 and r_2 have been prespecified.

   The weight $q_j (j = 1, 2, \ldots, 5)$ of each component varies somewhat from unit to unit in accordance with a normal distribution with mean μ_j and variance σ_j^2. As $q_1, q_2, \ldots, q_5$ are independent, the total weight q will also be a normal variable with mean $\mu = \sum_{j=1}^{5} \mu_j$ and variance $\sigma^2 = \sum_{j=1}^{5} \sigma_j^2$.

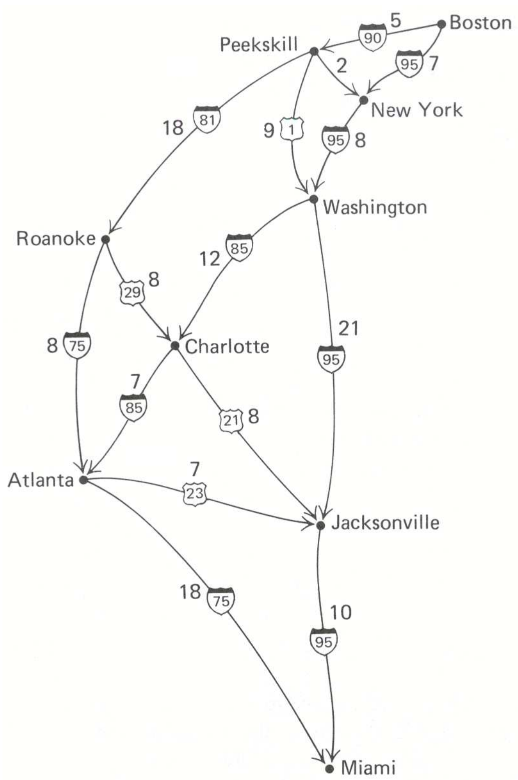

Figure E11.1 Travel times to highways.

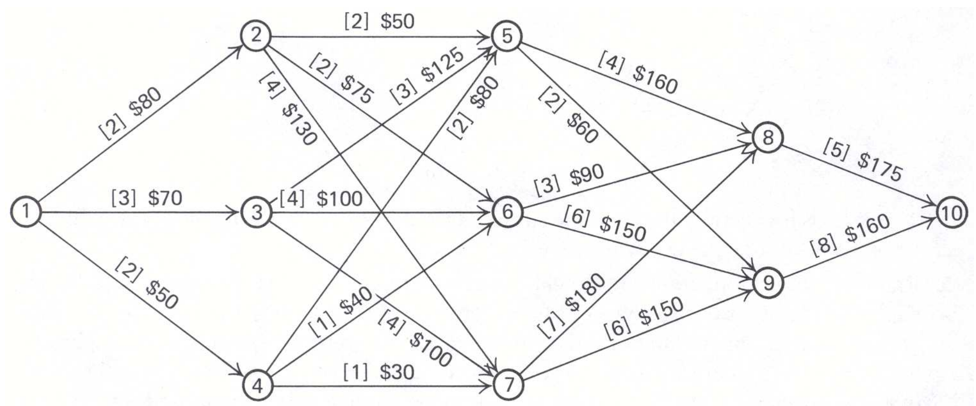

Figure E11.2 Routing times and costs.

Clearly, even if μ can be adjusted to fall within the interval $[r_1, r_2]$, the rejection rate will depend upon σ^2; in this case, the rejection rate can be made as small as desired by making the variance σ^2 sufficiently small. The design department has decided that $\sigma^2 = 5$ is the largest variance that would make the rejection rate of the monitoring device acceptable. The cost of manufacturing component j is $c_j = 1/\sigma_j^2$.

Determine values for the design parameters σ_j^2 for $j = 1, 2, \ldots, 5$ that would minimize the manufacturing cost of the components while ensuring an acceptable rejection rate. [*Hint*. Each component is a stage; the state variable is that portion of the total variance σ^2 not yet distributed. Consider σ_j^2's as continuous variables.]

*11. A scientific expedition to Death Valley is being organized. In addition to the scientific equipment, the expedition also has to carry a stock of spare parts, which are likely to fail under the extreme heat conditions prevailing in that area. The estimated number of times that the six critical parts, those sensitive to the heat conditions, will fail during the expedition are shown below in the form of probability distributions.

Part 1

# of Failures	*Probability*
0	0.5
1	0.3
2	0.2

Part 2

# of Failures	*Probability*
0	0.4
1	0.3
2	0.2
3	0.1

Part 3

# of Failures	*Probability*
0	0.7
1	0.2
2	0.1

Part 4

# of Failures	*Probability*
0	0.9
1	0.1

Part 5

# of Failures	*Probability*
0	0.8
1	0.1
2	0.1

Part 6

# of Failures	*Probability*
0	0.8
1	0.2

The spare-park kit should not weight more than 30 pounds. If one part is needed and it is not available in the spare-park kit, it may be ordered by radio and shipped by helicopter at unit costs as specified in Table E11.2, which also gives the weight of each part.

Table E11.2 Spare-Part Data

Part	*Weight* (pounds/unit)	*Shipping cost* ($/unit)
1	4	100
2	3	70
3	2	90
4	5	80
5	3	60
6	2	50

Determine the composition of the spare-park kit to minimize total expected ordering costs.

*12. After a hard day at work I frequently wish to return home as quickly as possible. I must choose from several alternate routes (see Fig. E11.3); the travel time on any road is uncertain and depends upon the congestion at the nearest major

Table E11.3 Travel time on the road

		Travel-time distribution	
Road i–j	*Congestion at initial intersection* (i)	*Travel time* (minutes)	*Probability*
5–4	Heavy	4	$\frac{1}{4}$
		6	$\frac{1}{2}$
		10	$\frac{1}{4}$
	Light	2	$\frac{1}{3}$
		3	$\frac{1}{3}$
		5	$\frac{1}{3}$
5–3	Heavy	5	$\frac{1}{2}$
		12	$\frac{1}{2}$
	Light	3	$\frac{1}{2}$
		6	$\frac{1}{2}$
4–2	Heavy	7	$\frac{1}{3}$
		14	$\frac{2}{3}$
	Light	4	$\frac{1}{2}$
		6	$\frac{1}{2}$
3–2	Heavy	5	$\frac{1}{4}$
		11	$\frac{3}{4}$
	Light	3	$\frac{1}{3}$
		5	$\frac{1}{3}$
		7	$\frac{1}{3}$
3–1	Heavy	3	$\frac{1}{2}$
		5	$\frac{1}{2}$
	Light	2	$\frac{1}{2}$
		3	$\frac{1}{2}$
2–1	Heavy	2	$\frac{1}{2}$
		4	$\frac{1}{2}$
	Light	1	$\frac{1}{2}$
		2	$\frac{1}{2}$

intersection preceding that route. Using the data in Table E11.3, determine my best route, given that the congestion at my starting point is heavy.

Assume that if I am at intersection i with heavy congestion and I take road i–j, then

$$\text{Prob (intersection } j \text{ is heavy)} = 0.8.$$

If the congestion is light at intersection i and I take road i–j, then

$$\text{Prob (intersection } j \text{ is heavy)} = 0.3.$$