Fluid Mechanics I / 2nd Year/ Dept. of Petroleum and Refining Engineering

وزارة التعليم العالي والبحث العلمي

جامعة الموصل/كلية هندسة النفط والتعدين

مفردات منهج قسم هندسة النفط والتكرير

المرحلة الدراسية: الثانية

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عدد الوحدات: 6 / الفصل الدراسي

عدد الساعات: 2 نظري + 2 عملي

لغة المقرر: انكليزي الجزء النظري

Week No.	Subject
1	Dimensions and units analysis-concept of fluid
2	Fluid and their properties-difference between solids, liquids and gases, Ideal and real fluids
3	Capillarity, surface tension
4	Cavitation issue and it's solution
5	Compressibility and bulk modulus, Newtonian and non-Newtonian fluids
6	Viscosity, newton law of viscosity, dynamic viscosity, units of viscosity
7	Effects of temperature and pressure on viscosity, velocity and shear stress through pipes
8	Fluid static, concept of pressure, pascal's law and its application, action of fluid pressure on a plane (horizontal. Vertical, and inclined)
9	submerged surface, resultant force and center of pressure, force on a curved surface
10	Buoyancy and flotation, stability of floating and submerged bodies, metacentric height
11	pressure distribution in a liquid subjected to constant horizontal/ vertical acceleration, rotation of liquid in a cylindrical container.
12	Fluid kinematics, Classification of fluid flows, velocity and acceleration of fluid particle, local and convective acceleration
13	normal and tangential acceleration, streamline, path line and streak line, flow rate and discharge mean velocity
14	continuity equation in Cartesian and cylindrical, polar coordinates. Rotational flows, rotation velocity and circulation, stream and velocity potential functions, flow net.
15	Fluid dynamic, Euler's equation, Bernoulli's equation and steady flow energy equation; representation of energy changes in fluid system,
16	impulse momentum equation, kinetic energy and momentum correction factors,

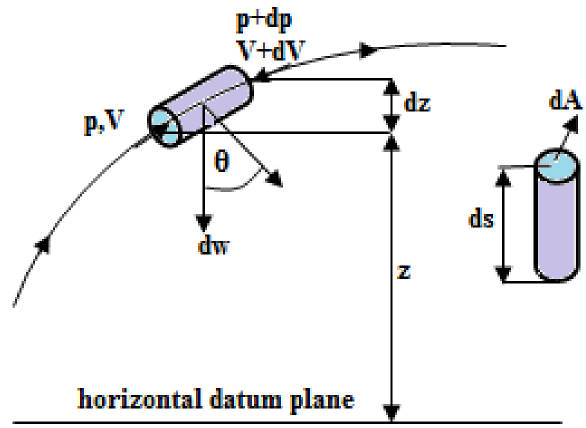
Conservation of Energy

The energy of the fluid flowing in any system remains constant, unless a certain amount of energy is added to or subtracted from the fluid. But the energy (or a part of it) can be changed from one form to another.

Flow of Steady Incompressible One-Dimension Ideal Flow

Euler's equation:

Consider a flow along the stream line 'S' and consider a cylindrical fluid element of length 'ds' and cross-sectional area 'dA'.



Appling Newton's 2nd law along the streamline.

$$\sum dF = dm.a$$

$$PdA - (P + dP)dA - dw\sin\theta = dm.a$$

$$-dPdA - dw\sin\theta = dm.a ----(1)$$

But $dm = d(\rho\forall) = \rho d\forall + \forall d\rho$

$\rho = const. d\rho = 0$ Incompressible fluid

$$dm = \rho d\forall = \rho dAds ---------------- (2a)$$

$$dw = gdm = g\rho d\forall = g\rho dAds \qquad \times \quad \sin\theta$$

$$dw\sin\theta = g\rho dAds\sin\theta = g\rho dAdz ---------- (2b)$$

$$a = \frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = V\frac{dV}{ds} ---------------- (2c)$$

Substituting equations 2a, 2b, and 2c, in equ. 1:

$$-dPdA - g\rho dAdz = \rho dAds\, V\frac{dV}{ds} \qquad \div \rho g dA$$

$$\frac{dP}{\gamma} + \frac{VdV}{g} + dz = 0 \qquad \text{By integreting}$$

$$\int\frac{dP}{\gamma} + \int\frac{VdV}{g} + \int dz = 0 \qquad \text{Eulers equation for steady flow along stream line}$$

For incompressible flow ρ = cons.

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = cons. \qquad \text{Bernoullis equation}$$

$$\frac{P_1}{\gamma} + \frac{{V_1}^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + z_2$$

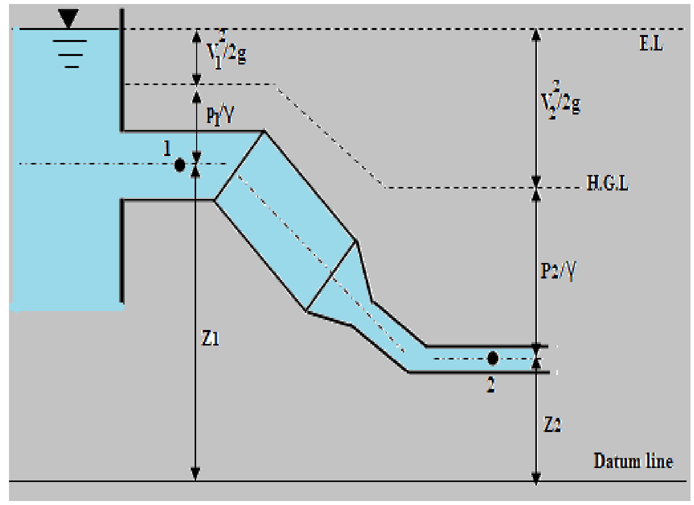
Bernoulli's equation:

It states as follow: in an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential or (datum) energy is constant along a stream line where

$\frac{P}{\gamma}$ – **pressure energy or pressure head (m)**

$\frac{V^2}{2g}$ – **kinetic energy or velocity head (m)**

Z – datum or elevation energy elevation head (m)



The elevation term, *z*, is related to the potential energy of the particle and is called the ***elevation head***. The pressure term, is called the ***pressure head*** and represents the height of a column of the fluid that is needed to produce the pressure *p*. The velocity term, is the ***velocity head*** and represents the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity *V* from rest. The Bernoulli equation states that the sum of the pressure head, the velocity head, and the elevation head is constant along a streamline.

BERNOULLI'S EQUATION

Bernoulli's equation states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

where,

$\frac{p}{w}$ = Pressure energy,

$\frac{V^2}{2g}$ = Kinetic energy, and

z = Datum (or elevation) energy.

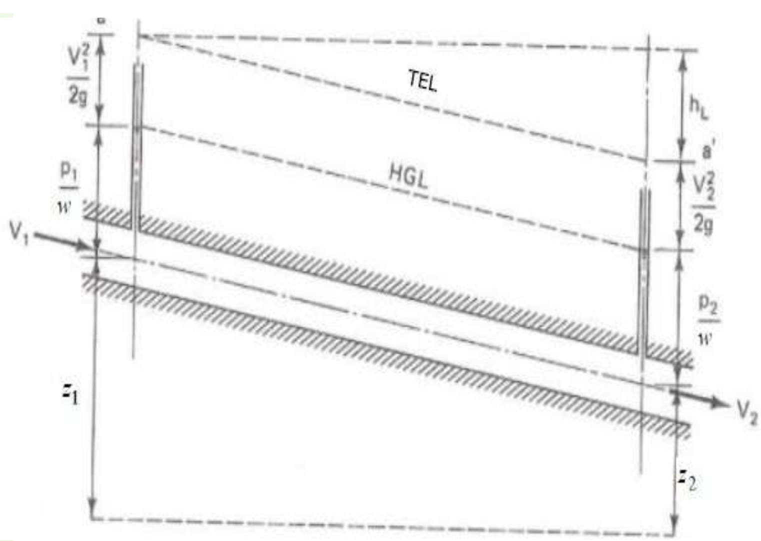
NOT ɣ OR w

Total energy line (T.E.L) – Line represents the sum of pressure head, potential head, and velocity head. $z + \frac{V^2}{2g} + \frac{p}{w}$

Hydraulic Grade Line H.G.L represents the sum of pressure head and potential head $\frac{p}{w} + z$

In ideal condition, the T.E.L is Horizontal (means that there is NO Losses)

Hydraulic Grade Line (H.G.L)
Total Energy Line (T.E.L) Or Energy Grade Line (E.G.L)



Hydraulic and Energy Grade Lines:

A useful visual interpretation of Bernoulli's equation is to sketch two grade lines of a flow. **The energy grade line (EGL)** shows the height of the total Bernoulli constant

$$\frac{P}{\gamma}+\frac{V^2}{2g}+z=h_o$$

In frictionless flow with no work or heat transfer, the EGL has constant height. **The hydraulic grade line (HGL)** shows the height corresponding to elevation and pressure head **z+p/γ** that is the EGL minus

the velocity head **$V^2/(2g)$.** The HGL is the height to which liquid would rise in a piezometer tube attached to the flow. In an open-channel flow the HGL is identical to the free surface of the water.

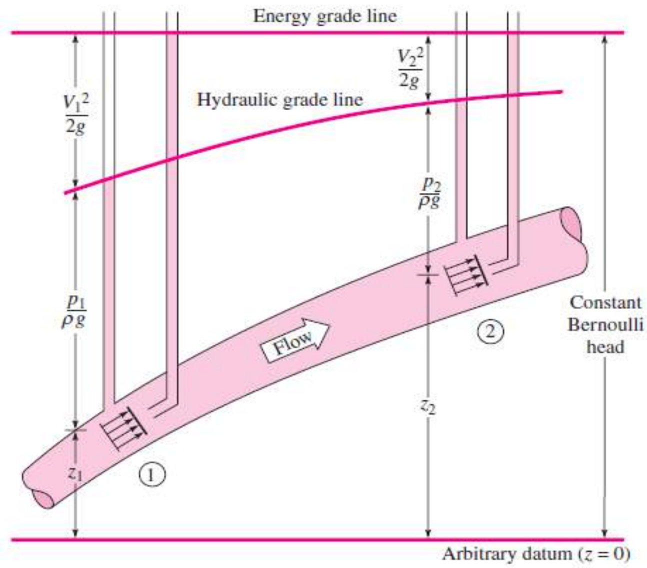


Figure illustrates the EGL and HGL for frictionless flow at sections 1 and 2 of a duct. The piezometer tubes measure the static pressure head $z+p/\gamma$ and thus outline the **HGL**. The pitot stagnation-velocity tubes measure the total head $\frac{P}{\gamma}+\frac{V^2}{2g}+z$, which corresponds to the EGL. In this particular case the EGL is constant, and the HGL rises due to a drop in velocity.

In more general flow conditions, the EGL will drop slowly due to friction losses and will drop sharply due to a substantial loss (a valve or obstruction) or due to work extraction (to a turbine). The EGL can rise only if there is work addition (as from a pump or propeller). The HGL generally follows the behavior of the EGL with respect to losses or work transfer, and it rises and/or falls if the velocity decreases and/or increases.

Basic assumption:

1. Steady incompressible fluid.
2. Inviscid flow.
3. No losses between any two points in the flow.
4. No energy added or removed in the flow.

Example 1. *In a pipe of 90 mm diameter water is flowing with a mean velocity of 2 m/s and at a gauge pressure of 350 kN/m². Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.*

Solution. Diameter of the pipe = 90 mm

Pressure, p = 350 kN/m²

Velocity of water, V = 2 m/s

Datum head, z = 8 m

Specific weight of water, w = 9.81 kN/m³

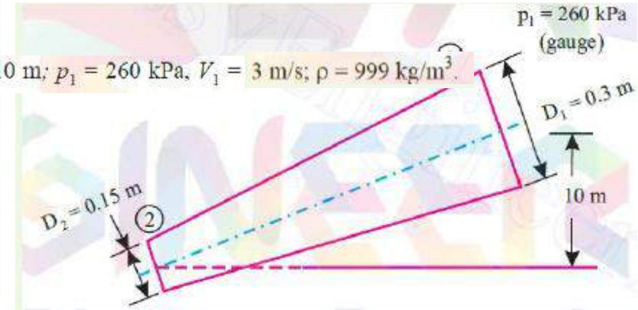
Total head of water, H:

$$H = z+\frac{V^2}{2g}+\frac{p}{w}$$

$$= 8+\frac{2^2}{2\times 9.81}+\frac{350}{9.81}=43.88\text{ m}$$

$$H = \mathbf{43.88\ m}$$

Example 2 *Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section. Frictional effects may be neglected. density of water to be 999 kg/m³.*



From continuity equation, $A_1 V_1 = A_2 V_2$,

Solution. Refer to Fig. 6.7. $D_1 = 0.3$ m; $D_2 = 0.15$ m; $z_1 = 0$; $z_2 = 10$ m; $p_1 = 260$ kPa, $V_1 = 3$ m/s; $\rho = 999$ kg/m³.

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left[\frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2}\right] \times V_1$$

$$= \left(\frac{D_1}{D_2}\right)^2 \times V_1 = \left(\frac{0.3}{0.15}\right)^2 \times 3 = 12 \text{ m/s}$$

Weight density of water, $w = \rho g = 999 \times 9.81 = 9800.19$ N/m³

From Bernoulli's equation between sections 1 and 2 (neglecting friction effects as given), we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{260 \times 1000}{9800.19} + \frac{(3)^2}{2 \times 9.81} + 10 = \frac{p_2}{9800.19} + \frac{(12)^2}{2 \times 9.81} + 0 \qquad 26.53 + 0.459 + 10 = \frac{p_2}{9800.19} + 7.34$$

H. W. *The water is flowing through a tapering pipe having diameters 300 mm and 150 mm at sections 1 and 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum and section 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m².*

Solution. At Section 1:

Diameter, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$

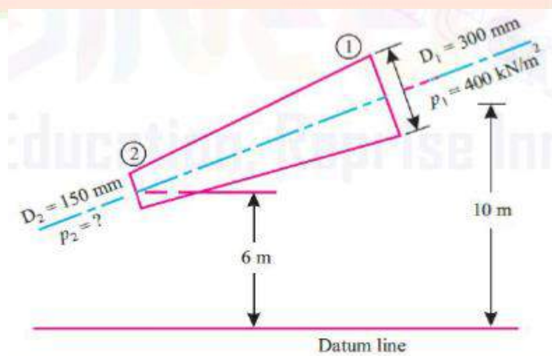
Pressure, $p_1 = 400$ kN/m²

Height of upper end above the datum, $z_1 = 10$ m

At Section 2:

Diameter, $D_2 = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$



Height of lower end above the datum, $z_2 = 6$ m

Rate of flow (*i.e.*, discharge), $Q = 40 \text{ litres/sec} = \frac{40 \times 10^3}{10^6} = 0.04 \text{ m}^3/\text{s}$

Example 3 *A pipe line carrying oil (sp. gr. 0.8) changes in diameter from 300 mm at position 1 to 600 mm diameter at position 2 which is 5 metres at a higher level. If the pressures at positions 1and 2 are 100 kN/m² and 60 kN/m² respectively and the discharge is 300 litres/sec., determine:*

(i) Loss of head, and *(ii) Direction of flow.*

Solution. Discharge, $Q = 300$ litres/sec $= \frac{300}{1000} = 0.3$ m³ $\therefore$

Sp. gr. of oil $= 0.8$

Weight of oil, $w = 0.8 \times 9.81 = 7.85$ kN/m³

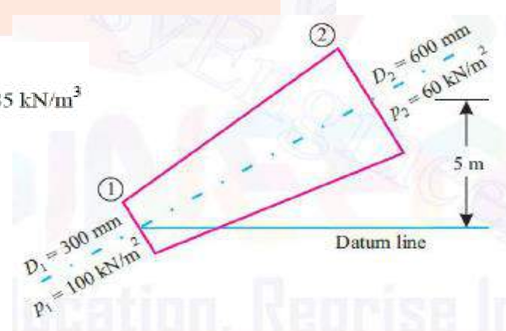
Diameter of pipe, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area of pipe, $A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$

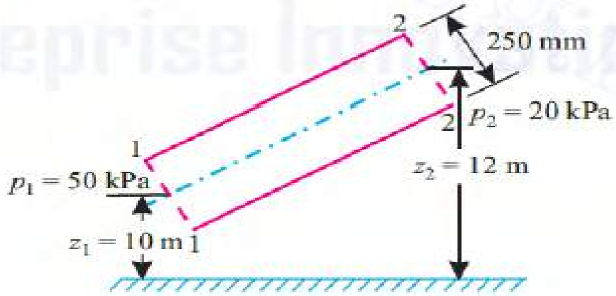
Pressure, $p_1 = 100$ kN/m²

If the datum line passes through section 1 (Fig. 6.16) then datum, $z_1 = 0$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.3}{0.0707} = 4.24$ m/s



Example 4 *In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1 which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20°C and 760 mm Hg is 998 kg/m³.* **(PTU)**



Solution. *Given:*

$D = 250 \text{ mm} = 0.25 \text{ m},$

$p_1 = 50 \text{ kPa} = 50 \times 10^3 \text{ N/m}^2;$

$z_1 = 10 \text{ m}; z_2 = 12 \text{ m};$

$p_2 = 20 \text{ kPa} = 20 \times 10^3 \text{ N/m}^2,$

$V_1 = V_2 = 1.25 \text{ m/s}, \rho = 998 \text{ kg/m}^3.$

Refer to Fig. 6.15.

Total energy at *section 1–1*,

$$E_1 = \frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{50 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 10 = 15.187 \text{ m}$$

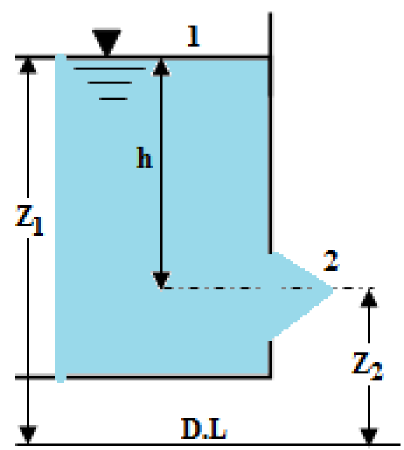
Total energy of *section 2–2*,

$$E_2 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 = \frac{20 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 12 = 14.122 \text{ m}$$

∴ Loss of head, $h_L = E_1 - E_2 = 15.187 - 14.122 = \mathbf{1.065 \text{ m}}$

Direction of flow:

Since $E_1 > E_2$ *direction of flow is from* **section 1-1 to section 2-2.**



Application of Bernoulli's equation:

1. Torricelli's theorem

A_1-surface area of liquid at '1' $A_1 >> A_2$

Points '1' and '2' are both exposed to atmospheric pressure

i.e. $p_1 = p_2 = 0$

applying Bernoulli's equation between points 1 and 2

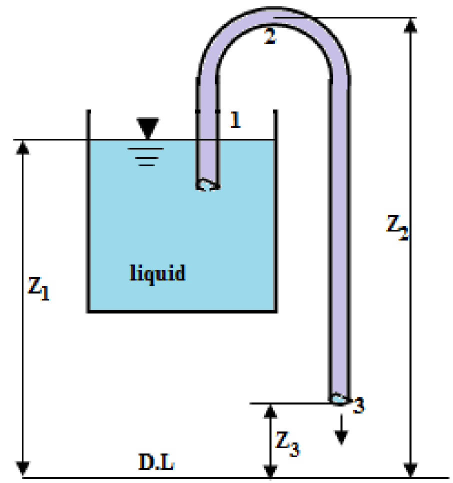
$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2$$

$V_1 = 0$ - large tank P_1 and $P_2 = 0$ atmospheric pressure

$$\frac{V_2^2}{2g} = z_1 - z_2 \qquad z_1 - z_2 = h$$

$V_2 = \sqrt{2gh}$ **Torricellis equation**

2. Siphon:



Conditions for siphon performance:

- $Z_1 > Z_3$
- Initially the fluid must be forced to flow.

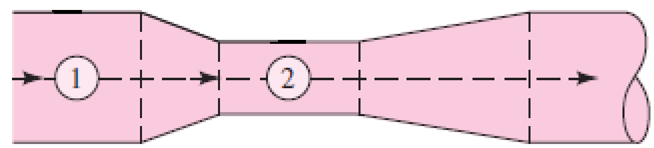
$$(Z_2 - Z_1)\gamma < P_{atm.}$$

3. closed duct or pipe:

In this case p_1 and p_2 not equal zero ; $Z_1 = Z_2$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Applying continuity equation ; $A_1V_1 = A_2V_2$



WORK – ENERGY EQUATIONS

Energy: 1. Added mechanically ⟶ (pump)

2. Removed: a. mechanically ⟶ (turbine)

b. frictional resistance (losses)

1) Valves 2) elbows 3) reducers

- Bernoulli's equation may be modified to account for energy added or energy removed between any two points in the flow.

Bernoulli's equation with pump:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + E_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Bernoulli's equation with turbine:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + E_T$$

For real flow with losses:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + H_L$$

Where, E_p – pump head (m)

E_T – turbine head (m)

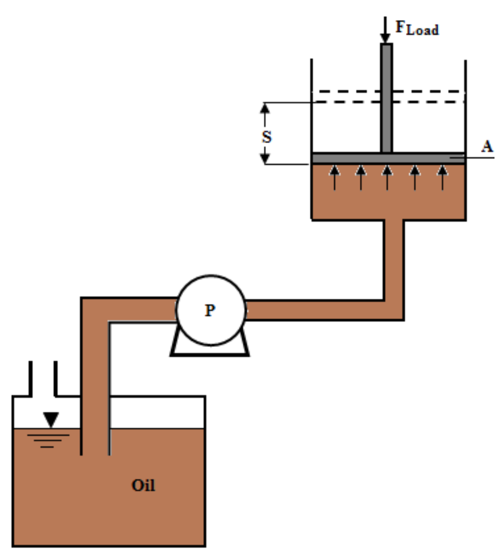
H_L – head losses (m)

Pump power:

$$P_{pump} = \gamma Q E_p$$

Turbine power:

$$P_{Turbine} = \gamma Q E_T$$



$F_{Load} \times S = \text{work}$ [J]

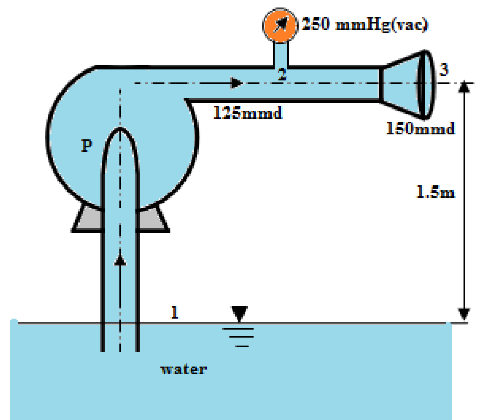
$\text{Power} = F \times \frac{s}{t} \left[\frac{J}{s}\right]$ or watt

$\text{Power} = PAV = PQ$ but $P = \gamma E_p$

$\therefore P_p = \gamma Q E_p$

Output power from the pump which is less than the electrical power input to the pump.

Problem (1) : Calculate the pump power, assuming that the divergent tube flow full.



$P_2 = \gamma_{Hg} h_{Hg}$

$P_2 = -13570 \times 9.81 \times 0.25 = -33280$ Pa

Applying Bernoulli's equation between 2&3

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$z_2 = z_3 \quad P_3 = 0$ atm.

$$\frac{-33280}{9810} = \frac{V_3^2 - V_2^2}{2g}$$

$$V_3^2 - V_2^2 = -66.56 ------------------(1)$$

$$V_2 A_2 = V_3 A_3 \rightarrow V_2 = V_3 \frac{A_3}{A_2} = \frac{150^2}{125^2} V_3 = 1.44 V_3 \quad --- (2)$$

From equation 1&2

$V_3^2 - 1.44^2 V_3^2 = -66.56 \qquad V_3 = 7.87 m/s$

$$Q = V_3 A_3 = 7.87 \times \frac{\pi}{4} \times 0.15^2 = 0.139 m^3/s$$

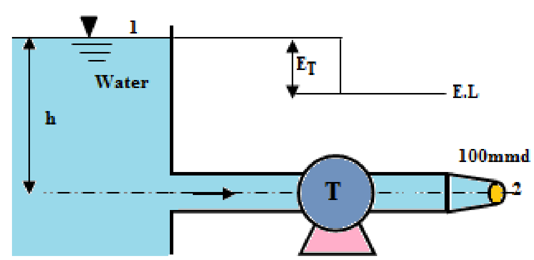
Applying Bernoulli's equation between 1&3

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + E_p = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \qquad V_1 = 0, \qquad P_1 = 0\,, \quad P_3 = 0$$

$$E_p = \frac{V_3^2}{2g} + z_3 - z_1 = \frac{7.87^2}{2 \times 9.81} + 1.5 = 4.66m$$

$$P_{pump} = \gamma Q E_p = 9810 \times 0.139 \times 4.66 = 6.35kW$$

Problem (2) : Calculate the height h that will produce a flowrate of 85 L/s and a turbine output power of 15 kW.



Solution:

$$P_{Turbine} = \gamma Q E_T \rightarrow 15000 = 0.085 E_T \rightarrow E_T = 18m$$

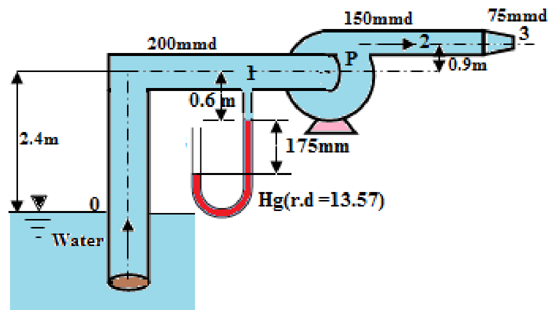
$$Q = V_2 A_2 \rightarrow V_2 = \frac{Q}{A_2} = \frac{0.085 \times 4}{\pi \times 0.1^2} = 10.8 m/s$$

Applying Bernoulli's equation between 1&2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + E_T \qquad V_1 = 0, \qquad P_1 = 0\,, \quad P_2 = 0$$

$$0 + 0 + h = 0 + \frac{10.8^2}{2 \times 9.81} + 0 + 18 \qquad \rightarrow \qquad h = 24m$$

Problem (3) : Calculate the pump power.



Solution: $P_1 = P_0 - \gamma_{Hg} h - \gamma_w \times 0.6$

$$\frac{P_1}{\gamma} = 0 - \frac{13570 \times 9.81}{9810} \times 0.175 - 0.6 = -2.975\text{m}$$

Applying Bernoulli's equation between 0&1

$$\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$

$$0 + 0 + 0 = -2.975 + \frac{V_1^2}{2 \times 9.81} + 2.4 \quad \rightarrow \quad V_1 = 3.36\ \text{m/s}$$

$$Q = V_1 A_1 = 3.36 \times \frac{\pi}{4} \times 0.2^2 = 0.1055 m^3/s$$

$$Q = V_2 A_2 \rightarrow V_2 = \frac{Q}{A_2} = \frac{0.1055 \times 4}{\pi \times 0.15^2} = 5.97 m/s$$

$$Q = V_3 A_3 \rightarrow V_3 = \frac{Q}{A_3} = \frac{0.1055 \times 4}{\pi \times 0.075^2} = 23.88 m/s$$

Applying Bernoulli's equation between 0&3

$$\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + z_0 + E_p = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$0 + 0 + 0 + E_p = 0 + \frac{23.88^2}{2 \times 9.81} + (2.4 + 0.9) \quad \rightarrow \quad E_p = 32.38\ \text{m}$$

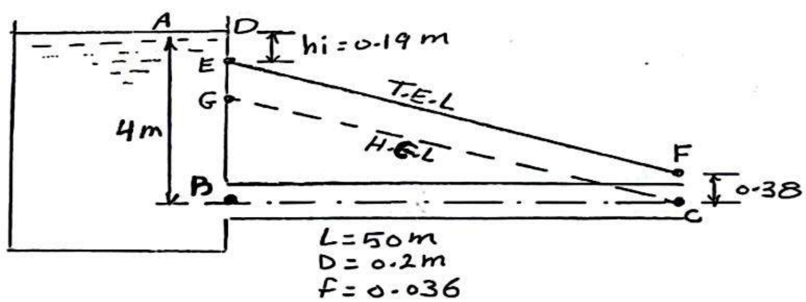
$$P_{pump} = \gamma Q E_p = 9810 \times 0.1055 \times 32.38 = 33.508\ \text{kW}$$

Hydraulic Gradient and Total Energy Line

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes.

- Hydraulic gradient line: the line which gives the sum of pressure head ($\frac{P}{\gamma}$) and datum head (Z) of a flowing fluid in a pipe with respect to some reference line. It is briefly written as H.G.L
- Total energy line: the line which gives the sum of pressure head ($\frac{P}{\gamma}$), kinetic head ($\frac{V^2}{2g}$) and datum head (Z) of a flowing fluid in a pipe with respect to some reference line.

Example: Draw the hydraulic gradient line (H.G.L) and Total energy line (T.E.L) for the system shown in the figure.



Sol:

Consider the velocity was calculated in previous example and is equal to (2.734) m/s

h_i = head lost at the entrance of the pipe

$$h_i = 0.5\frac{V^2}{2g} = \frac{0.5\times(2.734)^2}{2\times9.81} = 0.19 \text{ m}$$

$$h_f = \text{due to friction} = f\frac{L}{D}\cdot\frac{V^2}{2g}$$

$$= 0.036\times\frac{50\times(2.734)^2}{0.2\times2\times9.81} = 3.428$$

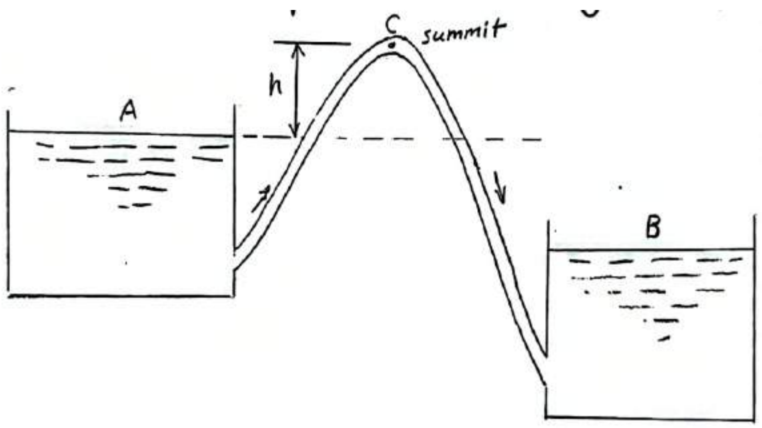
Total Energy Line (T.E.L)

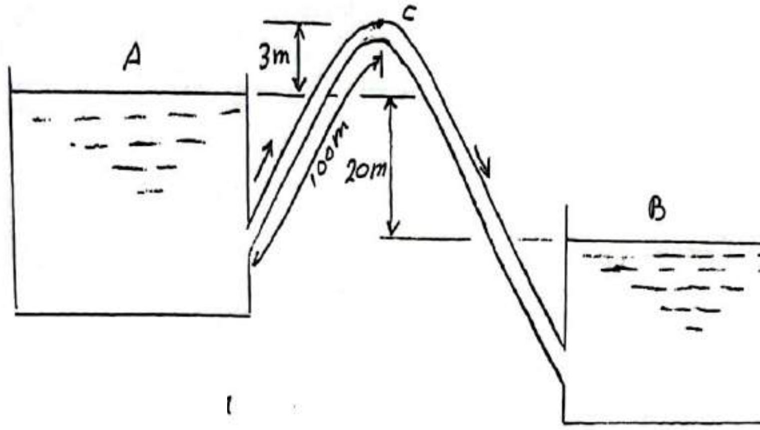
1- Total energy at point A $= \frac{P}{\gamma}+\frac{V^2}{2g}+Z = 0+0+4 = 4\text{ m}$

2- Total energy at point B = total energy at A $- h_i = 4-0.19 = 3.81\text{ m}$

3- Total energy at point C $= \frac{P_c}{\gamma}+\frac{V_c^2}{2g}+Z_c = 0+\frac{V^2}{2g}+0 = \frac{(2.734)^2}{2\times9.81}$

$= 0.38\text{ m}$





Example :

A syphon of diameter 0.2 m connects two reservoirs having difference in elevation of 20 m. The length of the syphon is 500 m and the summit is 3 m higher than the level of water at the upper reservoir. The length of pipe from the upper reservoir to the summit is 100 m. Determine the discharge through the syhon and the pressure at the summit. the friction factor is 0.02 Neglect all minor losses.

length of syphon, $L = 500\ m$ height of summit, $h = 3\ m$

Length of syphon up to summit $= 100\ m\ (L_1)$

friction factor, $f = 0.02$ $D = 0.2\ m$ $H = 20\ m$

Sol: Apply Bernouli's equation between points (A) and (B)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + \text{Losses (due to friction)}$$

$$0 + 0 + Z_A = 0 + 0 + Z_B + h_f \qquad P_A = P_B = 0 \quad V_A = V_B = 0$$

$$\therefore Z_A - Z_B = h_f = \frac{f.L.V^2}{D \cdot 2g}$$

$$Z_A - Z_B = 20\ m$$

$$\therefore 20 = h_f = \frac{f.L.V^2}{D.2g} = \frac{0.02 \times 500 \times V^2}{0.2 \times 2 \times 9.81} = 2.548\, V^2$$

$$\therefore V = \sqrt{\frac{20}{2.548}} = 2.8\ m/s$$

$$Q = VA = 2.8 \times \frac{\pi}{4}(0.2)^2 = 0.088\ m^3/s$$

To find the pressure at the summit (C) again apply Bernouli's eq. between points (A) and (C)

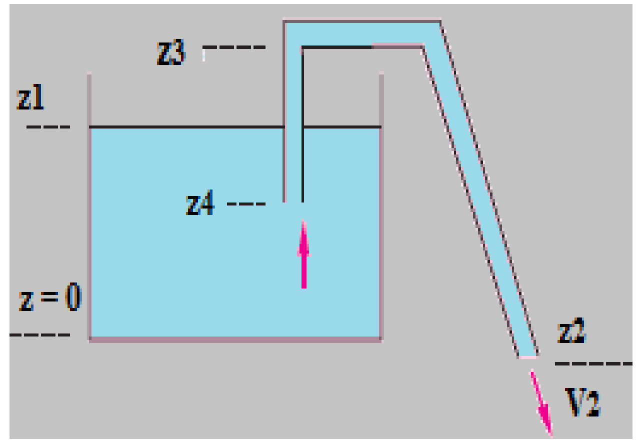
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + \text{losses} \quad \text{(friction losses between A & C)}$$

$$0 + 0 + 0 = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + 3 + h_f \quad \text{(datum through A)}$$

$$0 = \frac{P_C}{\gamma} + \frac{(2.8)^2}{2 \times 9.81} + 3 + \frac{0.02 \times 100 \times (2.8)^2}{2 \times 9.81} \qquad V_C = V = 2.8\ m/s$$

$$0 = \frac{P_C}{\gamma} + 0.3996 + 3 + 4$$

$$\therefore \frac{P_C}{\gamma} = -7.3996\ m$$



Problem (H.W.): **Consider the water siphon shown in figure. Assuming that Bernoulli's equation is valid, (*a*) find an expression for the velocity V_2 exiting the siphon tube. (*b*) If the tube is 1 cm in diameter and Z_1 = 60 cm, Z_2 = 25 cm, Z3 = 90 cm, and Z_4 = 35 cm, estimate the flow rate in cm^3/s.**

Solution: Note that the velocity is approximately zero at z_1, and a streamline goes from z_1 to z_2. Note further that p_1 and p_2 are both atmospheric, p = p_{atm}, and therefore cancel.

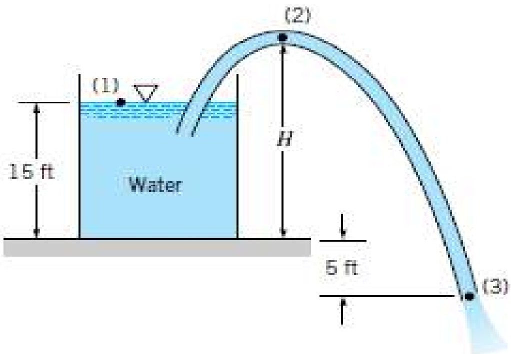
$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2$$

$$v_2=\sqrt{2g\,(z_1-z_2)}=\sqrt{2\times 9.81\times(0.6-(-0.25))}=4.08m/s$$

$$Q_2=V_2A_2=4.08\times\frac{\pi}{4}\times(0.01)^2=0.0003m^3/s$$

Flow siphoned:

Water at 60 °F is siphoned from a large tank through a constant diameter hose as shown in Fig. E3.10. Determine the maximum height of the hill, H, over which the water can be siphoned without cavitation occurring. The end of the siphon is 5 ft below the bottom of the tank. Atmospheric pressure is 14.7 psia.



■ FIGURE E3.10

SOLUTION

If the flow is steady, inviscid, and incompressible we can apply the Bernoulli equation along the streamline from (1) to (2) to (3) as follows:

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2 = p_3 + \tfrac{1}{2}\rho V_3^2 + \gamma z_3 \quad (1)$$

With the tank bottom as the datum, we have $z_1 = 15$ ft, $z_2 = H$, and $z_3 = -5$ ft. Also, $V_1 = 0$ (large tank), $p_1 = 0$ (open tank), $p_3 = 0$ (free jet), and from the continuity equation $A_2V_2 = A_3V_3$, or because the hose is constant diameter, $V_2 = V_3$. Thus, the speed of the fluid in the hose is determined from Eq. 1 to be

$$V_3 = \sqrt{2g(z_1 - z_3)} = \sqrt{2(32.2 \text{ ft/s}^2)[15 - (-5)] \text{ ft}}$$
$$= 35.9 \text{ ft/s} = V_2$$

Use of Eq. 1 between points (1) and (2) then gives the pressure p_2 at the top of the hill as

$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 - \tfrac{1}{2}\rho V_2^2 - \gamma z_2 = \gamma(z_1 - z_2) - \tfrac{1}{2}\rho V_2^2 \quad (2)$$

From Table B.1, the vapor pressure of water at 60 °F is 0.256 psia. Hence, for incipient cavitation the lowest pressure in the system will be $p = 0.256$ psia. Careful consideration of Eq. 2 and Fig. E3.10 will show that this lowest pressure will occur at the top of the hill. Since we have used gage pressure at point (1) ($p_1 = 0$), we must use gage pressure at point (2) also. Thus, $p_2 = 0.256 - 14.7 = -14.4$ psi and Eq. 2 gives

$$(-14.4 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2) = (62.4 \text{ lb/ft}^3)(15 - H)\text{ft} - \tfrac{1}{2}(1.94 \text{ slugs/ft}^3)(35.9 \text{ ft/s})^2$$

or

$$H = 28.2 \text{ ft} \qquad \textbf{(Ans)}$$

Week No.	Subjects
17	flow along a curved streamline, free and forced vortex motions.
18	Conservation of mass (mass balance)
19	Rayleigh's and Buckingham's Pi method for dimensional analysis.
20	Dimensionless numbers and their significance, geometric, kinematic and dynamic similarity, model studies
21	Flow regimes and Renlods number, flow classification
22	critical velocity and critical Reynolds number, laminar flow in circular cross section pipes
23	Turbulent flows and flow losses in pipes, Darcy equation
24	minor head losses in pipes and pipe fittings
25	hydraulic and energy gradient lines.
26	Water hammering and it's solution
27	Fluid measurements devices
28	Fluid measurements devices
29	Problems solutions
30	Review

Text Book:

Elementary Fluid Mechanics / Vennard and Street. 6thedition, 1982.

References:

Fluid Mechanics / 5th edition / Frank M. White. 1999.

CONCEPT OF FLUID MECHANICS

Definition :

Fluid mechanics is that branch of science, which deal with the behavior of the fluid (liquids or gases) at rest as well as at motion. Thus, this branch of science deal with the static, kinematic and dynamic aspect of fluid. The study of fluid at rest is called fluid statics. The study of fluid in motion where **pressure** forces are **not considered** is called fluid **kinematics**, and if the pressure forces are **considered** in fluid motion that branch of science is called fluid **dynamics**.

The fluid mechanics may be divided into three parts:

- ***Fluid Statics:*** *The study of* **incompressible** *fluids under static conditions is called* **hydrostatics**, *and that dealing with the* ***compressible*** *static gases is termed as* ***aerostatics***.
- ***Fluid Dynamics:*** *It deals with the relations between velocities, accelerations of fluid with the forces or energy causing them.*
- ***Fluid Kinematics:*** *It deals with the velocities, accelerations and the patterns of flow only.* ***Forces or energy*** *causing velocity and acceleration are* ***not dealt*** *under this heading.*

Fluid is defined as a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be. It is either gas or liquid.
Shear force is the force component tangent to surface.
Shear stress (force per unit area) is the shear force divided by the area of the surface.

Fluid:
- ✓ gasses
- ✓ liquids

Fluid statics:
- ✓ Fluid at rest.
- ✓ Fluid with constant linear acceleration.
- ✓ Fluid with constant angular acceleration.

Type of fluid:

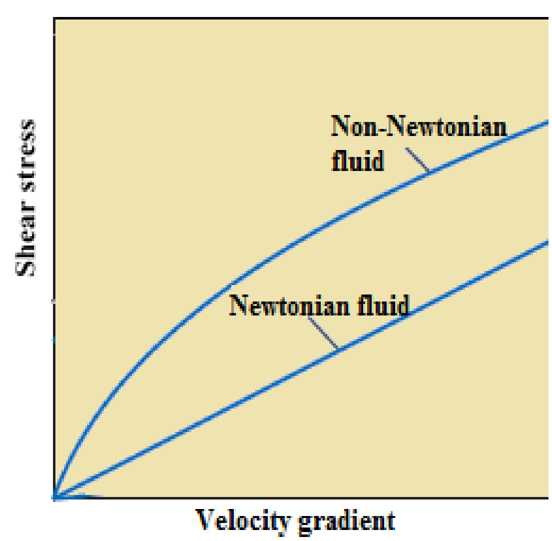
1. **Ideal fluid**: A fluid which is incompressible and is having **no viscosity**.
2. **Real fluid:** A fluid which is **having viscosity**.

An ***ideal fluid*** *is one which has* ***no viscosity*** *and* ***surface tension*** *and is* ***incompressible***. *In true sense no such fluid exists in nature. However fluids which have low viscosities such as* ***water*** *and* ***air*** *can be treated as ideal fluids under certain conditions. The assumption of ideal fluids helps in simplifying the mathematical analysis.*

3. **Newtonian fluid:** A real fluid in which the shear stress is ***directly*** proportional to the velocity gradient.

4. **Non- Newtonian fluid:** A real fluid in which the shear stress is ***not directly*** proportional to the velocity gradient.

Fluids for which the rate of deformation is linearly proportional to the shear stress are called ***Newtonian fluids***.



*Most common fluids such as **water, air, gasoline, and oils are Newtonian fluids**. **Blood and liquid plastics are examples of non-Newtonian fluid***

Fluid dynamics:

- ✓ Ideal flow.
- ✓ Real flow.

Flow:

- ✓ Internal flow (pipelines, ducts…).
- ✓ External flow (ships, wings, airplanes…).

Fundamental equations:

- ✓ Conservation of mass.
- ✓ Conservation of momentum.
- ✓ Conservation of energy.

Type of fluid flow:

- ✓ Viscous or non-viscous flow (ideal).
- ✓ Steady or non-steady flow.
- ✓ Compressible or incompressible flow.
- ✓ Uniform or non-uniform flow.

Solid and Fluid (liquid & gas)

In *solids*, the molecules are very closely spaced whereas in *liquids* (such as water, oil, and gasoline) the spacing between the different molecules is relatively large and in *gases* (such as CO2 and methane) the spacing between the molecules is still large.

- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid.**
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain, but in fluids stress is proportional* to *strain rate*.

Difference between liquid and gases

Liquid	Gases
Difficult to compress and often regarded as incompressible	Easily to compress – changes of volume is large, cannot normally be neglected and are related to temperature
Occupies a fixed volume and will take the shape of the container	No fixed volume, it changes volume to expand to fill the containing vessels
A free surface is formed if the volume of container is greater than the liquid.	Completely fill the vessel so that no free surface is formed.

Dimensions and Units:

A standard unit for length might be a (meter or foot), for time might be (hour or second), and for mass a (slug or kilogram). Such standards are called ***units***, and several systems of units are in common use as described in the following section. The qualitative description is conveniently given in terms of certain ***primary*** quantities, such as length, (**L**), time, (**T**), mass, (**M**), and temperature, (**θ**). These primary quantities can then be used to provide a qualitative description of any other ***secondary*** quantity: for example, Area=L2, Velocity=LT-1, Density=ML-3 and so on, where the symbol is used to indicate the dimensions of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, V, we would write V =LT-1 and say that "the dimensions of a velocity equal length divided by time." ***The primary quantities are also referred to as basic dimensions***.

For a wide variety of problems involving fluid mechanics, only the three basic dimensions, (L, T, and M) are required. Alternatively, (L, T, and F) could be used, where F is the basic dimensions of force. Since Newton's law states that force is equal to mass times acceleration, it follows that F=MLT-2

For the SI system there are four basic dimensions through which fluid properties are expressed.
• Basic Dimensions are:

- Mass (M)
- Length (L)
- Time (T)
- Force (F)

There are two systems of dimensions:

1. **M - L - T** systems
2. **F - L - T** systems

quantity **dimension** **units**

length	**(L)**	**Meter (m)**
mass	**(M)**	**Kilogram (kg)**
Time	**(t)**	**Second (s)**
Temperature	**(T)**	**Kelvin (ºk)**

Derived units:

Force = mass × acceleration = $\mathbf{F} = \mathbf{M} \times \mathbf{a} =$ kg × $\frac{m}{s^2}$ ≡ N=**M L T**$^{-2}$ (Newton's second law)

Velocity = distance/time = m/s = V = Length /Time = L / T OR V = L T^{-1}

Quantity	Dimension	SI Units	English Units
Area A	L^2	m^2	ft^2
Volume* $\forall$	L^3	m^3 or L (liter)	ft^3
Velocity V	L/T	m/s	ft/sec
Acceleration a	L/T^2	m/s^2	ft/sec^2
Angular velocity Ω	T^{-1}	s^{-1}	sec^{-1}
Force F	ML/T^2	kg · m/s^2 or N	slug · ft/sec^2 or lb
Density ρ	M/L^3	kg/m^3	slug/ft^3
Specific weight γ	M/L^2T^2	N/m^3	lb/ft^3
Frequency f	T^{-1}	s^{-1}	sec^{-1}
Pressure p	M/LT^2	N/m^2 or Pa	lb/ft^2
Stress τ	M/LT^2	N/m^2 or Pa	lb/ft^2
Surface tension σ	M/T^2	N/m	lb/ft
Work W	ML^2/T^2	N · m or J	ft · lb
Energy E	ML^2/T^2	N · m or J	ft · lb
Heat rate $\dot{Q}$	ML^2/T^3	J/s	Btu/sec
Torque T	ML^2/T^2	N · m	ft · lb
Power $\dot{W}$	ML^2/T^3	J/s or W	ft · lb/sec
Mass flux $\dot{m}$	M/T	kg/s	slug/sec
Flow rate Q	L^3/T	m^3/s	ft^3/sec
Specific heat c	$L^2/T^2\Theta$	J/kg · K	Btu/slug · °R
Viscosity μ	M/LT	N · s/m^2	lb · sec/ft^2
Kinematic viscosity ν	L^2/T	m^2/s	ft^2/sec

*We use the special symbol $\forall$ to denote volume and V to denote velocity.

Fluid properties:

Density (ρ) is defined as the ratio of mass of fluid to its volume. (*mass per unit volume at a standard temperature and pressure)*. The value of density of water is 1000kg/m^3

$$\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}} \quad \left[\frac{kg}{m^3}\right]$$

- **Specific weight** (weight density) the ratio of fluid weight to its volume. *weight per unit volume, at a standard tempe. and pressure. It is denoted by (γ) Mathematically is (ρg)*

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}} = \frac{mg}{\vartheta} = \rho g \quad \left[\frac{N}{m^3}\right]$$

- **Specific gravity** (relative density):The ratio of the density of fluid to the density of water.(ratio of the specific weight of the liquid to the specific weight of a standard fluid). t is usually denoted by s.g or sp.gr or $\mathbf{r.d}$) , It is dimensionless and has no units.

$$\text{Specific gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}} = \frac{w_{liquid}}{w_{water}}$$

$$(\mathbf{r.d})_{liquid} = \frac{\textbf{density of liquid}}{\textbf{density of water}} \qquad [\textbf{dimensionless}]$$

$$(\mathbf{r.d})_{gas} = \frac{\textbf{density of gas}}{\textbf{density of air}} \qquad [\textbf{dimensionless}]$$

- **Specific volume:** is defined as the volume of fluid per unit mass, volume per unit mass , It is usually denoted by $\boldsymbol{\vartheta}$, mathematically is (1/ρ)

$$\boldsymbol{\vartheta} = \frac{\textbf{volume of fluid}}{\textbf{mass of fluid}} = \frac{\mathbf{m^3}}{\mathbf{kg}} = \frac{\mathbf{1}}{\frac{\mathbf{kg}}{\mathbf{m^3}}} = \frac{\mathbf{1}}{\boldsymbol{\rho}}$$

Table (1.1)

Quantity	FLT system	MLT system
Acceleration	LT^{-2}	LT^{-2}
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^{2}	L^{2}
Density	$FL^{-4}T^{2}$	ML^{-3}
Energy	FL	$ML^{2}T^{-2}$
Force	F	MLT^{-2}
Heat	FL	$ML^{2}T^{-2}$
Length	L	L
Mass	$FL^{-1}T^{2}$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of force	FL	$ML^{2}T^{-2}$
Moment of inertia	L^{4}	L^{4}
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	$ML^{2}T^{-3}$
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	1	1
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}

Temperature	**Ɵ**	**Ɵ**
Torque	**FL**	**ML^2T^{-2}**
Velocity	**LT^{-1}**	**LT^{-1}**
Dynamic viscosity	**$FL^{-2}T$**	**$ML^{-1}T^{-2}$**
Kinematic viscosity	**L^2T^{-1}**	**L^2T^{-1}**
Work	**FL**	**ML^2T^{-2}**

<u>Problem 1:</u> calculate the specific weight, density and relative density of 1 liter of liquid which weights 7 N.

<u>Solution</u>:

- $$1\text{ liter} = \frac{1}{1000}\text{m}^3, \quad \text{weight} = 7\text{N}$$

$$\text{specific weight } \gamma = \frac{\text{weight}}{\text{volume}} = \frac{7}{1/1000} = 7000\frac{\text{N}}{\text{m}^3} \qquad \textbf{ANS.}$$

$$\text{Density} = \frac{\text{specific weight } \gamma}{\text{g}} = \frac{7000}{9.81} = 713.5\frac{\text{kg}}{\text{m}^3} \qquad \textbf{ANS.}$$

$$\text{relative density(r.d)} = \frac{\text{density of liquid}}{\text{density of water}} = \frac{713.5}{1000} = 0.713 \qquad \textbf{ANS.}$$

<u>Problem2:</u> Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7.

<u>Solution</u>: $$1\text{ liter} = \frac{1}{1000}\text{m}^3$$

$$(\text{r.d}) = \frac{\text{density of liquid}}{\text{density of water}} \rightarrow \rho = \text{r.d} \times \rho_{\text{water}} = 0.7 \times 1000 = 700\frac{\text{kg}}{\text{m}^3} \qquad \textbf{ANS.}$$

$$\text{specific weight } \gamma = \rho g = 700 \times 9.81 = 6867\frac{\text{N}}{\text{m}^3} \qquad \textbf{ANS.}$$

$$\text{specific weight } \gamma = \frac{\text{weight}}{\text{volume}} \rightarrow \text{weight} = \gamma \times \forall = 6867 \times \frac{1}{1000} = 6.867\text{N} \quad \textbf{ANS.}$$

1.1 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

(a) volume $\doteq \underline{\underline{L^3}}$

(b) acceleration = time rate of change of velocity

$$\doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$$

(c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^2}}$

(d) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(e) work = force × distance

$$\doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

1.2 Determine the dimensions, in both the *FLT* system and *MLT* system, for **(a)** the product of force times volume, **(b)** the product of pressure times mass divided by area, and **(c)** moment of a force divided by velocity.

(a) force × volume $\doteq (F)(L^3) \doteq \underline{\underline{FL^3}}$

Since $F \doteq MLT^{-2}$

force × volume $\doteq (MLT^{-2})(L^3) \doteq \underline{\underline{ML^4T^{-2}}}$

(b)
$$\frac{\text{pressure} \times \text{mass}}{\text{area}} \doteq \frac{(FL^{-2})(M)}{L^2} \doteq \frac{(FL^{-2})(FL^{-1}T^2)}{L^2}$$

$$\doteq \underline{\underline{F^2L^{-5}T^2}}$$

$$\doteq \frac{(MLT^{-2})(L^{-2})(M)}{L^2}$$

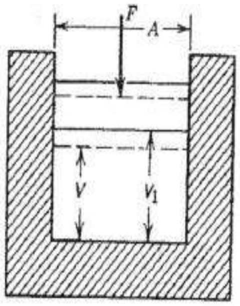
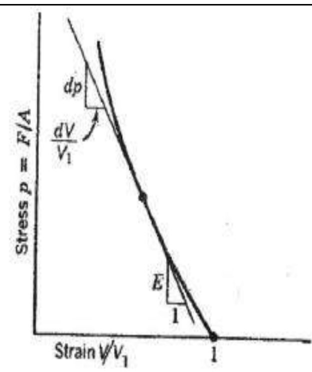
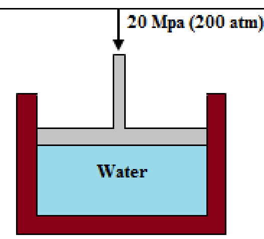
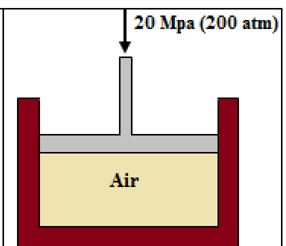
$$\doteq \underline{\underline{M^2L^{-3}T^{-2}}}$$

(c)
$$\frac{\text{moment of a force}}{\text{velocity}} \doteq \frac{FL}{LT^{-1}} \doteq \underline{\underline{FT}}$$

$$\doteq (MLT^{-2})(T) \doteq \underline{\underline{MLT^{-1}}}$$

➢ <u>Compressibility</u>

All fluids may be compressed by application of pressure. Elastic energy is stored in the compressed fluids and the fluids return to their original volumes when the pressure is released. This show us that the fluid is 'elastic'. In engineering, this is summarized by **'bulk modulus of elasticity E'.**

$E = \dfrac{dp}{\frac{-d\forall}{\forall}}$ $= \dfrac{N/m^2}{m^3/m^3}$			 $d\forall \approx 0.1\% \rightarrow d\rho \sim 0$ **Fluid is incompressible**.	 $d\rho \neq 0$ **Fluid is compressible**

where **dp** is the differential pressure change, **d∀** is the differential volume change, and ∀ is the volume of fluid. Because is negative for a positive dp, a negative sign is used in the definition to yield a positive E. The elasticity is often called the compressibility of the fluid. The fractional change in volume can be related to the change in material density **using** $\mathbf{m = \rho\forall}$ **; Since the mass is constant**

$$\mathbf{dm = \rho d\forall + \forall d\rho = 0 \rightarrow \rho d\forall = -\forall d\rho \rightarrow \frac{d\rho}{\rho} = -\frac{d\forall}{\forall}} \therefore E = \frac{dp}{\frac{d\rho}{\rho}} = \frac{\text{Change of pressure}}{\text{Fractional change of density}} \rightarrow \frac{dp}{d\rho} = \frac{E}{\rho}$$

OR $$m = \rho\forall \Rightarrow d\rho = d\left(\frac{m}{\forall}\right) = -m\frac{d\forall}{\forall^2} = -\rho\frac{d\forall}{\forall} \Rightarrow \frac{d\rho}{\rho} = -\frac{d\forall}{\forall} \Rightarrow \therefore E = \rho\frac{dp}{d\rho}$$

Elasticity is a measure of liquid incompressibility. **The bulk modulus of elasticity** of water is approximately 2.2GN/m^2 which corresponds to a 0.05% change in volume for a change of 1MN/m^2 in pressure. Obviously, the term **incompressible** is justifiably applied to water because it has such a **small** change in volume for a very **large** change in pressure.

Problem 1: Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of the liquid decreases by 0.15 per cent.

Solution: Initial pressure = 70 N/cm^2 ; Final pressure = 130 N/cm^2

$\therefore dp = \text{increase of pressure} = 130 - 70 = 60\text{N/cm}^2$ ***(Decrease in volume = 0.15%)***

$$\therefore -\frac{d\forall}{\forall} = +\frac{15}{100} \rightarrow \quad \therefore E = \frac{dp}{\frac{-d\forall}{\forall}} = \frac{60}{\frac{0.15}{100}} = \frac{6000}{0.15} = 4\times10^4\frac{\text{N}}{\text{cm}^2}$$ **ANS.**

Problem 2: *What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m^3 at 80 N/cm^2 pressure to a volume of 0.0124 m^3 at 150 N/cm^2 pressure?*

Solution: Initial volume = *0.0125 m^3* ; Final pressure = *0.0124 m^3*

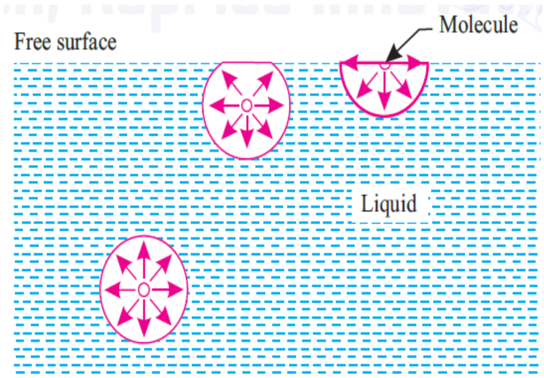
$$\therefore d\forall = \text{decrease in volume} = 0.0125 - 0.0124 = 0.0001\text{m}^3 \rightarrow\rightarrow \therefore -\frac{d\forall}{\forall} = +\frac{\mathbf{0.0001}}{\mathbf{0.0125}}$$

Initial pressure=80 N/cm^2 ; Final pressure=150 N/cm^2 $\therefore dp = \text{increase of pressure} = 150 - 80 = 70\text{N/cm}^2$

$$E = \frac{dp}{\frac{-d\forall}{\forall}} = \frac{70}{\frac{0.0001}{0.0125}} = 70\times125 = 8.75\times10^3\frac{\text{N}}{\text{cm}^2}$$ **ANS.**

- **Vapor Pressure:** It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere. Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere. If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules. When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the vapor pressure. Ex., water of 20C has vapor pressure of 2.451Kpa absolute.
- **Cohesion:** Cohesion means intermolecular attraction between molecules of the same liquid. Cohesion is a tendency of the liquid to remain as one assemblage of particles.
- **Adhesion:** Adhesion means attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick to another body.

- **Surface tension:** At the interface between a liquid and a gas, or between two different liquids, forces develop in the liquid surface which causes the surface to behave as a "skin" stretched over the fluid mass. **it** is caused by the force of cohesion at the free surface. At liquid–air interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion).



Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet

Case I. Water droplet:

Let, p = Pressure inside the droplet above outside pressure (*i.e.*, $\Delta p = p - 0 = p$ above atmospheric pressure)

d = Diameter of the droplet and

σ = Surface tension of the liquid. $\frac{F}{L}$

From free body diagram (Fig. 1.19 *d*), we have:

(*i*) Pressure force $= p \times \frac{\pi}{4} d^2$, $= F_p$, and

(*ii*) Surface tension force acting around the circumference $= \sigma \times \pi d$.

Under equilibrium conditions these two forces will be equal and opposite,

i.e., $$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\therefore \quad p = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}$$

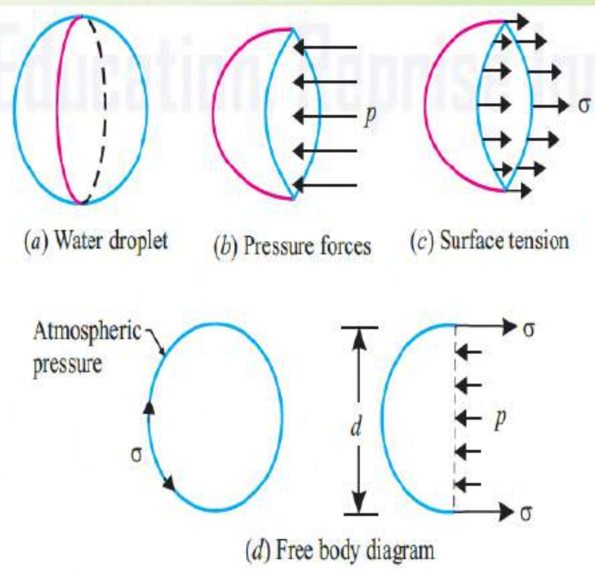


Fig. 1.19. Pressure inside a water droplet.

The equation above shows that, $P \propto \frac{1}{d}$

Case II. Soap (or hollow) bubble:

Soap bubbles have two surfaces on which surface tension σ acts.

From the free body diagram (Fig. 1.20), we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$\therefore \quad p = \frac{2\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad \text{...(1.18)}$$

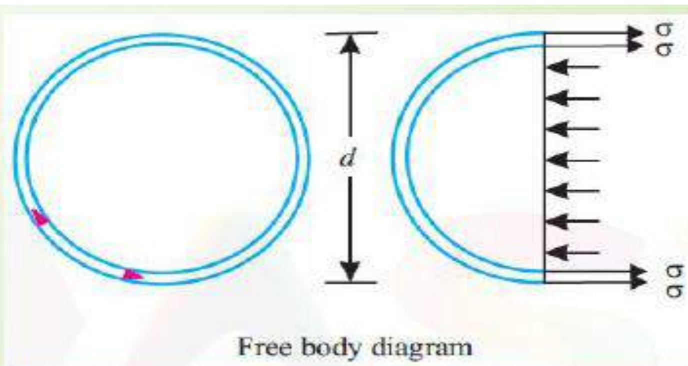
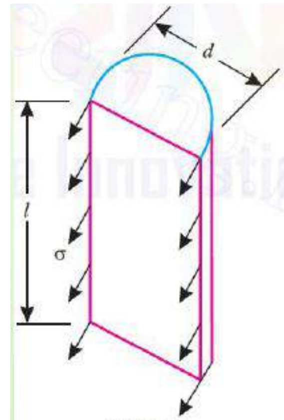


Fig. 1.20. Pressure inside a soap bubble.

Case III. A Liquid jet:



Semi-jet

Let us consider a cylindrical liquid jet of diameter d and length l.

Fig. shows a semi-jet.

Pressure force $= p \times l \times d$

Surface tension force $= \sigma \times 2l$

Ec $p \times l \times d = \sigma \times 2l$ ⇨ $p = \dfrac{\sigma \times 2l}{l \times d} = \dfrac{2\sigma}{d}$

OR

$P \cdot \dfrac{\pi}{4} d^2 = \sigma \pi d$ ⇨ $P = 4 \dfrac{\sigma}{d}$ قوة الضغط = مقاومة الشد السطحي

$P.\,d.\,L = \sigma(2)(L + d)$ ⇨ $P.\,d.\,L = 2\sigma L$ ⇨ $p = 2 \dfrac{\sigma}{d}$ ⇦ $d <<< L$ وفي حالة نفاث السائل

$P = \dfrac{8\sigma}{d}$ ⇨ وللفقاعة مثلا هناك سطحان متلامسان مع الهواء . ولذلك يكون الضغط داخل الفقاعة .

Example *A soap bubble 62.5 mm diameter has an internal pressure in excess of the outside pressure of 20 N/m². What is tension in the soap film?*

Solution. *Given:* Diameter of the bubble, $d = 62.5\ \text{mm} = 62.5 \times 10^{-3}$ m;

Internal pressure in excess of the outside pressure, $p = 20\ \text{N/m}^2$.

Surface tension, σ:

Using the relation, $p = \dfrac{8\sigma}{d}$ ⇨ $20 = \dfrac{8\sigma}{62.5 \times 10^{-3}}$ ⇨ $\therefore\ \sigma = 20 \times \dfrac{62.5 \times 10^{-3}}{8} = \mathbf{0.156\ N/m}$

- *Capillarity is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of Cohesion and Adhesion of liquid particles.*

Figure shows the phenomenon of rising water in the tube of *smaller* diameters.

Let, d = Diameter of the capillary tube,

θ = Angle of contact of the water surface,

σ = Surface tension force for unit length, and

w = Weight density (ρg).

Now, upward surface tension force (lifting force) = weight of the water column in the tube (*gravity force*)

$$\pi d.\sigma \cos\theta = \frac{\pi}{4} d^2 \times h \times w$$

$$\therefore\ h = \frac{4\sigma \cos\theta}{wd}$$

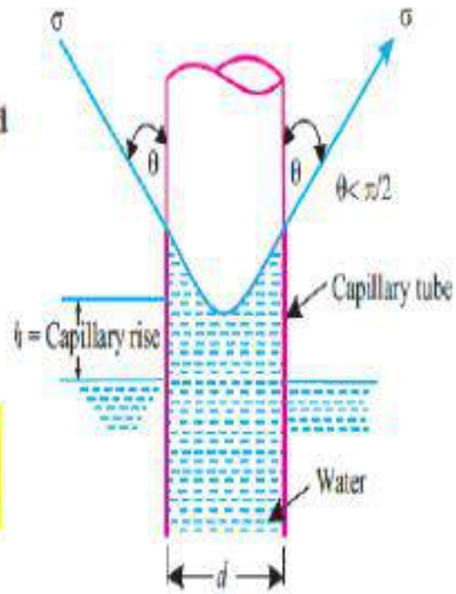
$w = \gamma$

usually use the symbol γ to refer to the weight density

For water and glass: $\theta = 0$.

Hence the capillary rise of water in the glass tube,

$$h = \frac{4\sigma}{wd}$$



Adhesion > Cohesion
(Miniscus concave)

$$\gamma\pi \frac{d^2}{4} h = \sigma \cos\theta\, \pi\, d$$

$$\gamma \frac{d}{4} h = \sigma \cos\theta$$

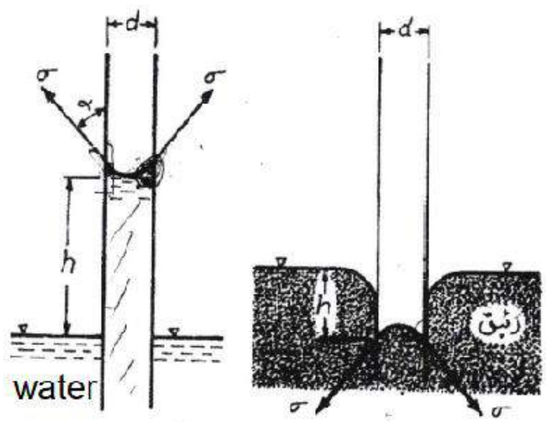
$$h = \frac{4\sigma \cos\theta}{\gamma d}$$

حيث أن :

σ : الشد السطحي (N / m)

γ : الوزن النوعي للماء (N / m^3)

θ : زاوية التماس بين الماء والوعاء



For water ⇨ θ very small ⇨ $\cos\theta = 1$ ⇨ $h = \frac{4\sigma}{\gamma d}$

For Mercury ⇨ $\theta = 129°$

Example *A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m³. What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.*

Solution. *Given:* $d = 2.5$ mm ; $\sigma = 4$ N/m, $\theta = 135°$; $\rho = 13600$ kg/m^3

Level of the liquid in the tube, *h*:

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall).

$$h = \frac{4\sigma \cos\theta}{wd}$$

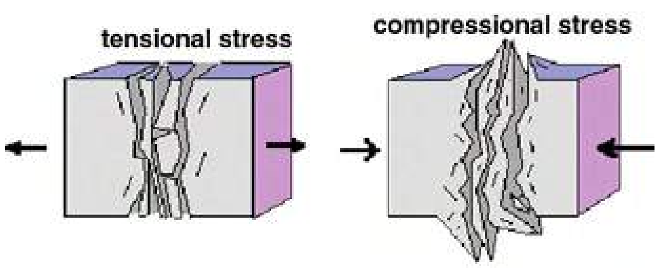
$$= \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} \qquad (\because w = \rho g)$$

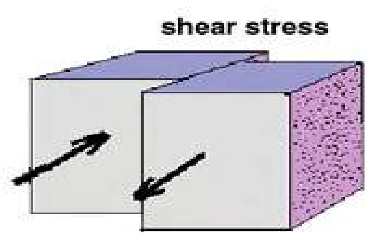
$$= -3.39 \times 10^{-3} \text{m or } -3.39 \text{ mm}$$

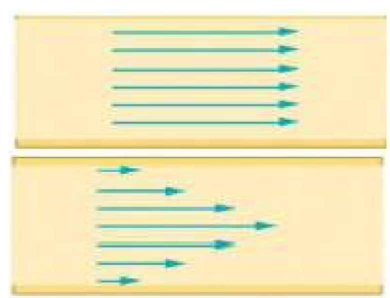
Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm.**

➢ VISCOSITY (μ)

- *Viscosity* may be defined as the *property of a fluid which determines its resistance to shearing stresses*.
- It is a measure of the internal fluid friction which causes resistance to flow (shearing of fluid)
- Viscosity of fluids is due to cohesion and interaction between particles.





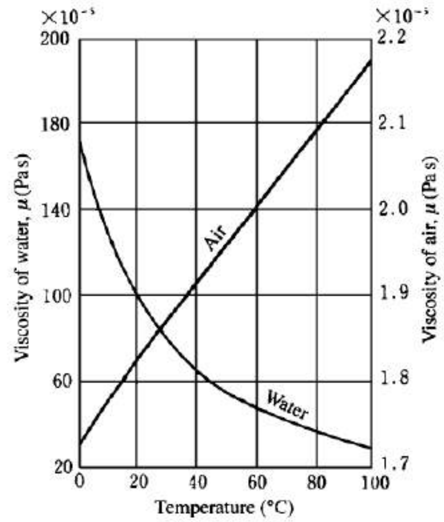


- **An ideal fluid has no viscosity.**

Ideal Fluid Non-Viscous Fluid , $\mu = 0$

Real Fluid Viscous Fluid , $\mu \neq 0$

Factors Effecting Viscosity (μ)



✓ ***Viscosity and temperature:***

$T\uparrow \Rightarrow \mu \downarrow$ *for liquid*

$T\uparrow \Rightarrow \mu \uparrow$ *for gases*

The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature.

The liquid molecules are closely spaced, with strong cohesive forces between molecules, and the resistance to relative motion between adjacent layers of fluid is related to these intermolecular forces.

As the temperature increases, these cohesive forces are reduced with a corresponding reduction in resistance to motion. Since viscosity is an index of this resistance, it follows that the viscosity is reduced by an increase in temperature.

In gases, however, the molecules are widely spaced and intermolecular forces negligible. In this case, resistance to relative motion arises due to the exchange of momentum of gas molecules between adjacent layers.

✓ Pressure

• The viscosity under ordinary conditions is not noticeably affected by the changes in pressure. however, the viscosity of some oils has been found to increase with increase in pressure.

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance h, as shown in Figure. Now a constant parallel force *F* is applied to the upper plate while the lower plate is held fixed.

Dynamic Viscosity or (Absolute viscosity): The viscosity can be defined as the fluid resistance to move (flow) under any magnitude of shear stress. When a fluid is flowing, it begins to move at a strain rate proportional to shear stress, and the constant of proportionality is called *coefficient of viscosity* μ. Consider a fluid element sheared in one plane by a single shear stress (τ), as shown in Fig. (1.1).The velocity δu will continuously grow along the normal distance between the fluid layers δy as long as the stress τ is maintained constant. The upper surface is moving at speed δu larger than the lower surface. Such common fluids as water, oil, and air show a linear relation between applied shear stress and resulting strain rate, $\tau \alpha \frac{\delta u}{\delta y}$. Where, $\frac{\delta u}{\delta y}$ is called velocity gradient or strain rate. Then the constant of proportionality is called viscosity as shown as

Equation ($\tau = \mu \frac{\delta u}{\delta y}$) is dimensionally consistent; therefore μ has dimensions of (shear stress ×time) which means $\{F.T/L^2\}$ or $\{M/(L.T)\}$. The BG unit is (slugs /foot×second), and the SI unit is (kilograms /meter×second). The linear fluids which called Newtonian fluids, after Sir Isaac Newton, who first postulated this resistance law in 1687.

The second form of viscosity that the ratio of dynamic viscosity to mass density which it has the name of *(kinematic viscosity), ν:* The ratio between the dynamic viscosity μ and the density.

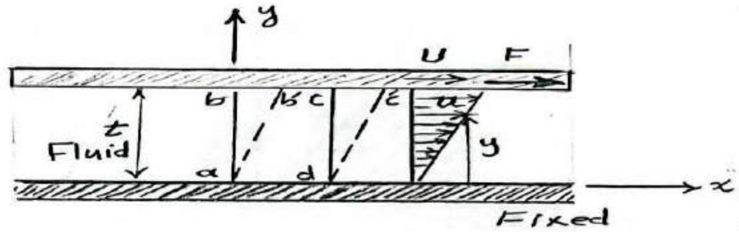
Kinematic viscosity unit: - $\vartheta = \frac{\mu}{\rho} = \frac{Ns/m^2}{kg/m^3} = \frac{Ns}{m^2} \times \frac{m^3}{Ns^2} \times m = \frac{m^2}{s}$

A common unit for kinematic viscosity is Stoke = 10^{-4} m^2/s

It is called kinematic because the mass units cancel, having the units of {m^2/s} in SI unit and {ft^2/s} in BG unit. It has another units such as (poises, and stokes). Each 1 poise = $(N/m^2)*10^{-1}$. Each 1 stoke $(cm^2/s)=10^{-4}m^2/s$.

$$\mu = \frac{\tau}{dv/dy} = \frac{N/m^2}{\frac{m/s}{m}} = \frac{N}{m^2} \cdot s = \text{Pa. s} = 10 \text{ Poises}$$

$$\mu = \frac{N}{m^2} \cdot s = \frac{Kg.\,m}{m^2.\,s^2}\, s = \frac{Kg}{m.\,s} = \frac{M}{LT} = ML^{-1}T^{-1}$$



The fluid in the area abcd flows to the new position ab'c'd.
Velocity u varying from zero at the stationary plate to U at the upper plate

$F = \mu \frac{AU}{t}$ — A is the area of upper plate

$\frac{F}{A} = \mu \frac{U}{t}$ — $\frac{U}{t}$ is the angular velocity or rate of angular deformation

$\tau = \frac{F}{A}$ = shear stress

$\therefore \tau = \mu \frac{U}{t}$ — μ = viscosity of fluid. $(\frac{N.s}{m^2})$

in general

$\tau = \mu \frac{du}{dy}$ — This equation is Newton law of viscosity

and the surface is assumed to be linear. What force is required if the plate and surface are horizontal?

velocity gradient : $\frac{du}{dy} = \frac{5-0}{0.002} = 2500$ m/s.m

$\tau = \mu \frac{du}{dy}$ $\quad \tau = \frac{F}{A}$ $\quad$ μ of oil = 0.1 $\frac{N.s}{m^2}$

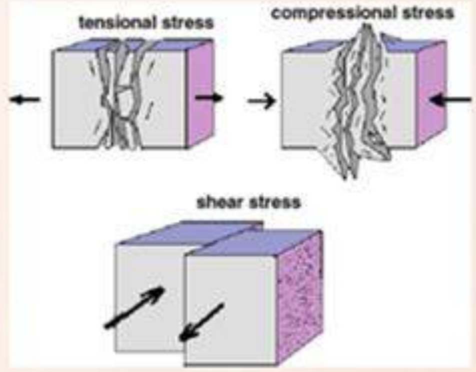
$\frac{F}{A} = \mu \frac{du}{dy}$

$\therefore F = \mu . A \frac{du}{dy} = 0.1 \times 0.5 \times 2 \times 2500$

$F = 250$ N

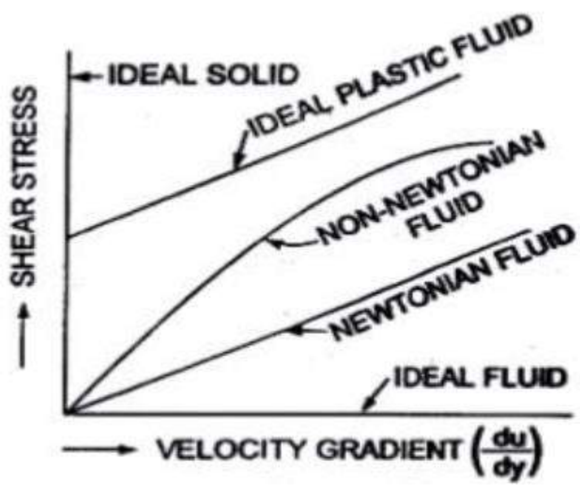
VISCOSITY (μ)

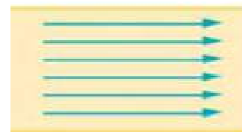
- *Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses.*
- It is a measure of the internal fluid friction which causes resistance to flow (shearing stresses between the moving layers of fluid)
- Viscosity of fluids is due to cohesion and interaction between particles.



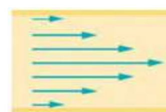
Ideal Plastic Fluid.

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.





Ideal Fluid
Non-Viscous Fluid
$\mu = 0$



Real Fluid
Viscous Fluid
$\mu \neq 0$

Factors Effecting Viscosity (μ)

Temperature

- The viscosity of *liquids* ($\mu_{liquids}$) *decreases* with *increase in temperature* (**T**). But, the viscosity of *gases* (μ_{gases}) increases with *increase in temperature* (**T**).

This is due to the reason that in *liquids* the shear stress is due to the inter-molecular cohesion which *decreases* with increase of temperature. As **T** ↑ **Cohesive force** ↓ then $\mu_{liquids}$ ↓

- In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules. The molecular activity increases with rise in temperature and so does the viscosity of gas.
- As **T** ↑ **Cohesive force** (Negligible), **Exchange of momentum of the molecules** ↑ then μ_{gases} ↑

Pressure

- The viscosity under ordinary conditions is not noticeably affected by the changes in pressure. however, the viscosity of some oils has been found to increase with increase in pressure.

$$F \propto V$$

$$F \propto \frac{1}{y}$$

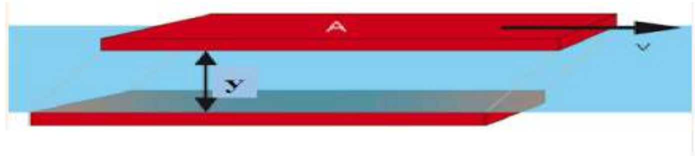
$$F \propto A$$

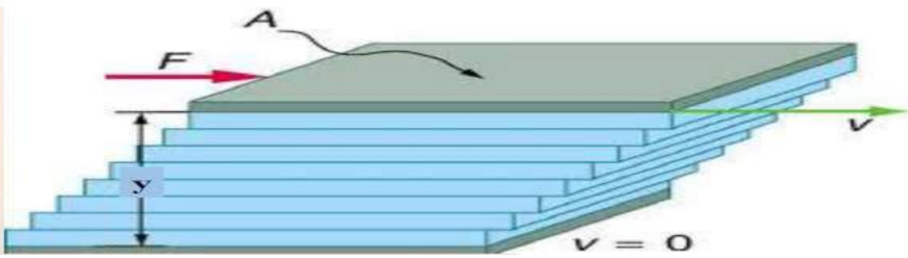
$$F \propto \frac{V}{y} A$$

$$F = \mu \frac{V}{y} A$$

$$\frac{F}{A} = \mu \frac{V}{y}$$

$$\tau = \mu \frac{V}{y}$$





Unit of Viscosity.

The unit of viscosity is obtained by putting the dimension of the quantities

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}} = \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$$\text{SI unit of viscosity} = \frac{\text{Newton second}}{m^2} = \frac{Ns}{m^2}$$

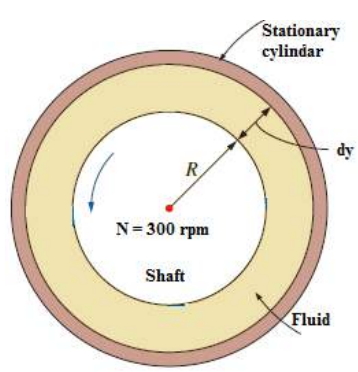
Kinematic Viscosity.

- It is defined as the ratio between the dynamic viscosity and density of fluid.It is denoted by the Greek symbol (ν) called 'nu' . Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

- The SI unit of kinematic viscosity is m^2/s.

<u>Example :</u> The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders as shown. The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N.m. Determine the viscosity of the fluid.



<u>Solution :</u>

L = 40 cm, R = 6 cm, dy = 0.15 cm,

N = 300 r.p.m, T = 1.8 N.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.4s^{-1} \rightarrow \; v = \omega R = 31.4 \times 0.06 = 1.88m/s$$

$$A = 2\pi RL = 2\pi \times 0.06 \times 0.40 = 0.15m^2$$

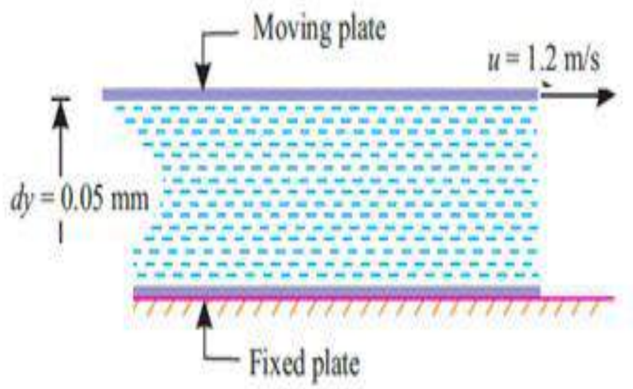
$$\tau = \mu\frac{dv}{dy} = \frac{F}{A} = \frac{T}{RA} \rightarrow \mu = \frac{T \cdot dy}{R \cdot A \cdot dv} = \frac{1.8 \times 0.0015}{0.06 \times 0.15 \times 1.88} = 0.159Pa.s$$

EXAMPLE *A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/m² to maintain this speed. Find the viscosity of the fluid between the plates.*

Solution: Velocity of the moving plate, $u = 1.2$ m/s

Distance between the plates, $dy = 0.05$ mm $= 0.05 \times 10^{-3}$ m

Force on the moving plate, $F = 2.2$ N/m²



Viscosity of the fluid, μ:

We know, $\tau = \mu.\frac{du}{dy}$ where τ = shear stress or force per unit area = 2.2 N/m²,

du = change of velocity

$= u - 0 = 1.2$ m/s and

dy = change of distance

$= 0.05 \times 10^{-3}$m.

$\therefore \quad 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}} \qquad \mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5}$ N.s/m²

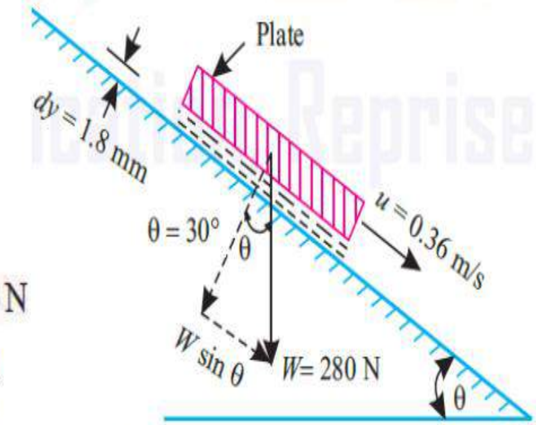
Example *A plate having an area of 0.6 m² is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.*

Solution: Area of plate, $A = 0.6\ \text{m}^2$

Weight of plate, $W = 280$ N

Velocity of plate, $u = 0.36$ m/s

Thickness of film, $t = dy = 1.8\ \text{mm} = 1.8 \times 10^{-3}$ m



Viscosity of the fluid, μ:

Component of W along the plate $= W \sin\theta = 280 \sin 30° = 140$ N

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33\ \text{N/m}^2 \qquad \text{We know,} \quad \tau = \mu \cdot \frac{du}{dy}$$

Where, du = change of velocity $= u - 0 = 0.36$ m/s • $dy = t = 1.8 \times 10^{-3}$ m

$$233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}} \Rightarrow \mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166\ \text{N.s/m}^2$$

Vapor pressure Pv

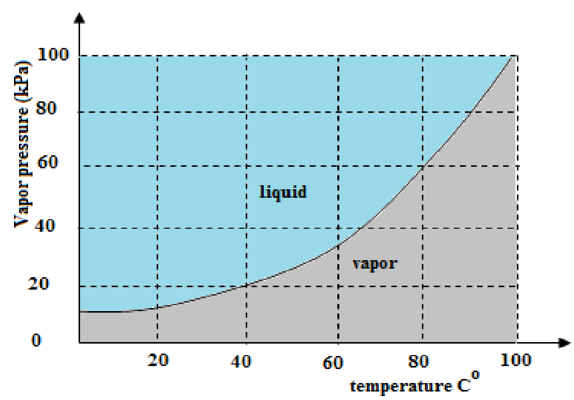
Is that pressure at which the liquid starts to boil (vaporize).

Note: {boiling can initialed at a giving pressure acting on the liquid by rising the temperature or at given fluid temperature by lowering the pressure}.

For water:

$P_{saturation}$ =101.325 kPa at 100 C°

$T_{saturation}$ = 100 C°, at101.325 kPa

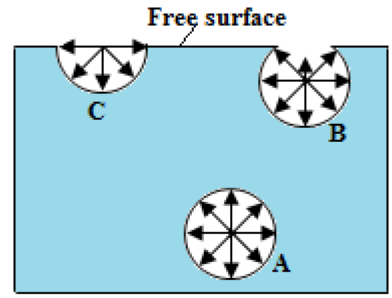


Dependence of vapor pressure with temperature for water

Cavitation: is the phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapor pressure and sudden collapsing of these vapor bubbles in a region of higher pressure. When the vapor bubbles collapse, a very high pressure is created. The metallic surfaces, above which the liquid is flowing, is subjected to these high pressures, which causing damage to pipes or parts of machinery. This phenomenon is a common cause for drop in performance and even the erosion of impeller blades.

Surface tension Surface tension σ (sigma) is the measure of energy stored in the free face (or an interface). Surface tension is defined as the tensile force acting on the surface of a liquid in contact with gas. It has unit of energy per unit area. $\sigma = \frac{\text{J}}{\text{m}^2} = \frac{\text{N.M}}{\text{m}^2} = \frac{\text{N}}{\text{m}}$

The phenomenon of surface tension is explained by Figure. Consider three molecules **A, B, C** of a liquid in a mass of liquid. The molecule **A** is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule A is zero. But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced.



Thus a net resultant force on molecule **B** is acting in the downward direction. The molecule **C**, situated on the free surface of liquid does experience a resultant downward force. All die molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrane under tension.

Cohesion: it means intermolecular attraction between molecules of the same liquid.

Adhesion: it means attraction between the molecules of the fluid and the molecules of a solid boundary surface in contact with liquid.

Surface tension caused by the force of cohesion at the free surface (rain drop…).

Capillarity, action is due to both cohesion and adhesion forces.

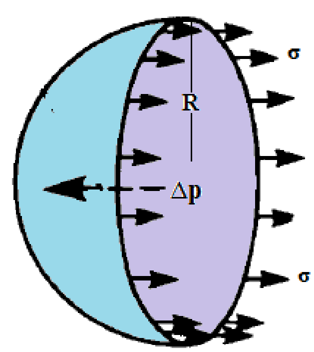
1. Surface Tension on Liquid Droplet. Consider a small spherical droplet of a liquid of radius R. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid. ; R= radius of droplet.

P = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

Let the droplet is cut into two halves.

The forces acting on one half (say left half) will be **tensile force** due to surface tension acting around the circumference of the cut portion as shown in Figure and this is equal to:



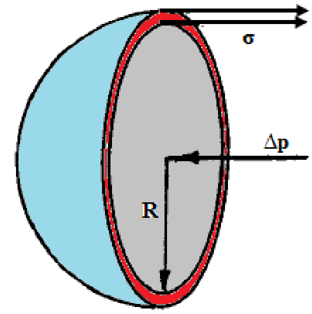
$$= \boldsymbol{\sigma \times \text{circumference} = \sigma 2\pi R}$$

$$\textbf{Pressure force on the area} = \mathbf{P} \times \boldsymbol{R^2 \pi}$$

These two forces will be equal and opposite under equilibrium conditions:

$$\sigma 2\pi R = P \times R^2 \pi \qquad \rightarrow \qquad P = \frac{2\sigma}{R}$$

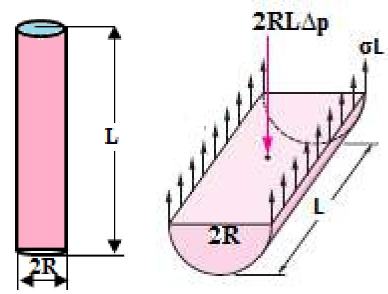
2. Surface tension on a bubble: A bubble has two surfaces in contact with air, one inside and the other outside. These two surfaces subjected to surface tension.



$$(2\pi R\sigma) \times 2 = R^2 \pi P$$

$$P = \frac{4\sigma}{R}$$

3. Surface tension on a liquid jet: consider a liquid jet of diameter 2R and length L as shown in figure.



$$2RLP = 2L\sigma$$

$$P = \frac{\sigma}{R}$$

Problem 1: The surface tension of water in contact with air is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.

σ = 0.0725 N/m, P = 0.02 N/ cm^2 = 0.02x10^4 N/m^2

$$P = \frac{2\sigma}{R} \rightarrow R = \frac{2 \times 0.0725}{0.02 \times 10^4} = 0.000725\text{m} \quad ; \quad D = 2R = 0.00145\text{m} = 1.45\text{mm}$$

Problem 2: Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

$$R = \frac{40}{2} = 20\text{mm} = 0.02\text{m}$$

$$P = \frac{4\sigma}{R} \rightarrow \sigma = \frac{PR}{4} = \frac{2.5 \times 0.02}{4} = 0.0125\text{N/m}$$

Problem 3: The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0:0725 N/m of water.

$$R = \frac{D}{2} = \frac{0.04}{2} = 0.02\text{mm} = 0.02 \times 10^{-3}\text{m} \quad ; \quad P = \frac{2\sigma}{R} = \frac{2 \times 0.0725}{0.02 \times 10^3} = \frac{7250N}{m^2} = 0.725N/cm^2$$

Pressure inside the droplet =p + Pressure outside the droplet **=0.725 + 10.32 = 11.045 N/cm^2**

Capillarity: is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. It is value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Fig. shows the phenomenon of rising water in the tube of *smaller* diameters.

Let, d = Diameter of the capillary tube,

θ = Angle of contact of the water surface,

σ = Surface tension force for unit length, and

w = Weight density (ρg). $= \gamma$

Now, upward surface tension force (lifting force) = weight of the water column in the tube (*gravity force*)

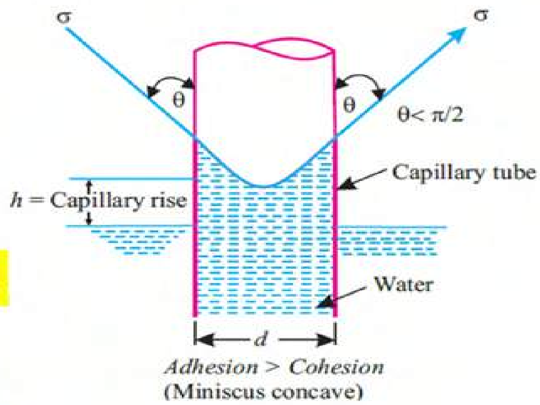
$$\pi d.\sigma \cos\theta = \frac{\pi}{4} d^2 \times h \times w \leftarrow w = \gamma$$

usually use the symbol γ to refer to the weight density

$$\therefore \quad h = \frac{4\sigma \cos\theta}{wd}$$

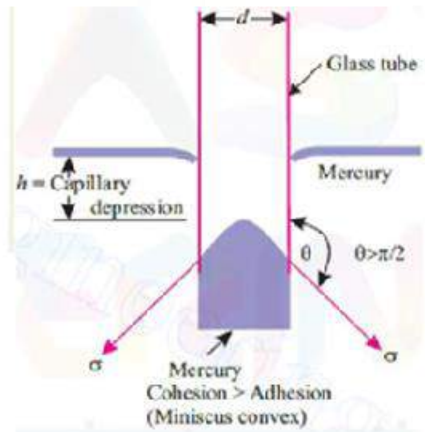
For water and glass: $\theta \simeq 0$.

Hence the capillary rise of water in the glass tube, $h = \frac{4\sigma}{wd}$



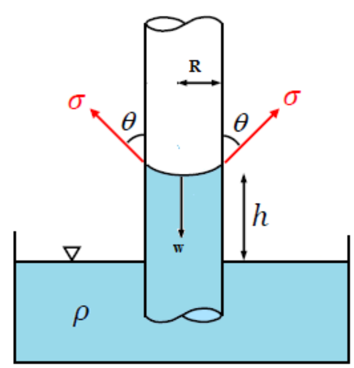
In case of mercury there is a capillary depression as shown in Figure , and the angle of depression is $\theta \simeq 140°$. (It may be noted that here $\cos\theta = \cos 140° = \cos(180 - 40°) = -\cos 40°$, therefore, h is *negative* indicating capillary depression).

$$\cos(180° - x) = -\cos(x)$$



1. Expression for Capillary Rise: Consider a glass tube of small diameter R opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.



$$W = \sigma \times 2\pi R\cos\theta$$

$$mg = \sigma \times 2\pi R\cos\theta \rightarrow \rho g\forall = \sigma \times 2\pi R\cos\theta$$

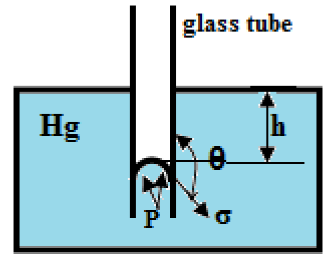
$$\gamma R^2\pi h = \sigma \times 2\pi R\cos\theta$$

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

For circular tube R < 2.5 mm

Value of θ for water and glass tube is 0°.

2. Expression for Capillary Fall: If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Figure.



Let *h* = Height of depression in tube. Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to:

$\sigma \times 2\pi R\cos\theta$

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth h x Area.

$$\pi R^2 P = \pi R^2 \rho g h \qquad P = \rho g h$$

$$2\pi R\sigma\cos\theta = \pi R^2 \rho g h$$

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

Value of θ for mercury and glass tube is 129°.

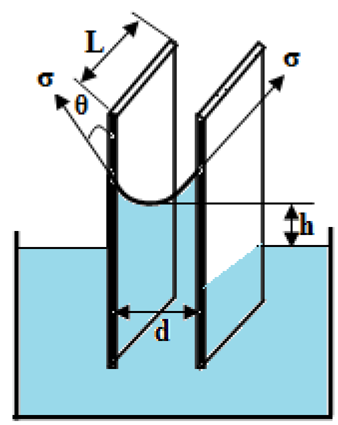
3. Capillary rise between two vertical parallel plats at a distance d apart:

Surface tension force = weight of column of water

$$\sigma\cos\theta \times 2L = \gamma\forall$$

$$\sigma\cos\theta \times 2L = \gamma L d h$$

$$h = \frac{2\sigma\cos\theta}{\gamma d}$$



Problem1: Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact 130°.

a- Capillary rise of water θ = 0°:

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{2 \times 0.0725 \times 1}{9810 \times 1.25 \times 10^{-3}} = 0.0118\text{m} = 1.18\text{cm}$$

b- Capillary fall of mercury θ = 130°:

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{2 \times 0.52 \times \cos 130}{13.6 \times 1000 \times 9.81 \times 1.25 \times 10^{-3}} = -0.0040\text{m} = -0.4\text{cm}$$

The negative sign indicates the capillary depression.

<u>Problem 2:</u> Calculate the capillary effect in millimeters in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20ºC and the value of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 130°. Take density of water at 20°C as equal to 998 kg/m^3.

a- Capillary rise of water θ = 0º:

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{2 \times 0.073575 \times 1}{998 \times 9.81 \times 2 \times 10^{-3}} = 0.00751\text{m} = 7.51\text{mm}$$

b- Capillary fall of mercury θ = 130º:

$$h = \frac{2\sigma\cos\theta}{\gamma R} = \frac{2 \times 0.51 \times \cos 130}{13.6 \times 1000 \times 9.81 \times 2 \times 10^{-3}} = -0.00245\text{m} = -2.45\text{mm}$$

<u>Problem 3:</u> The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.

$$h = \frac{2\sigma\cos\theta}{\gamma R} \rightarrow R = \frac{2\sigma\cos\theta}{\gamma h} = \frac{2 \times 0.0725 \times 1}{9810 \times 0.2 \times 10^{-3}} = 0.074\text{m} = 7.4\text{cm}$$

$$\therefore \quad D = 2R = 2 \times 7.4 = 14.8\text{cm}$$

<u>Problem 4:</u> Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

$$h = \frac{2\sigma\cos\theta}{\gamma R} \rightarrow R = \frac{2\sigma\cos\theta}{\gamma h} = \frac{2 \times 0.073575 \times 1}{9810 \times 2 \times 10^{-3}} = 0.0075m = 0.75cm$$

$$\therefore \quad D = 2R = 2 \times 0.75 = 1.5\text{cm}$$

<u>Problem 5:</u> A soup bubble 50 mm in diameter contain a pressure (in excess of atmospheric) of 20 Pa. Calculate the tension in in the soap film.

$$(2\pi R\sigma) \times 2 = R^2\pi P$$

$$P = \frac{4\sigma}{R} \rightarrow \sigma = \frac{PR}{4} = \frac{20 \times 25 \times 10^{-3}}{4} = 0.125N/m$$

Example *A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m³. What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.*

Solution. *Given:* $d = 2.5$ mm ; $\sigma = 4$ N/m, $\theta = 135°$; $\rho = 13600$ kg/m^3

Level of the liquid in the tube, *h*:

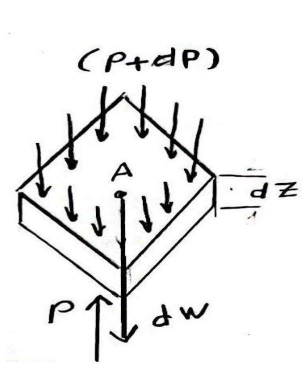
The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

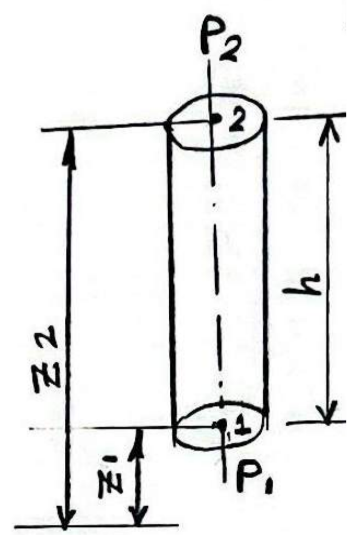
$$h = \frac{4\sigma\cos\theta}{wd} = \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} = -3.39 \times 10^{-3}\text{m or} -3.39\text{ mm} \qquad (\because w = \rho g)$$

Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm.**

<u>**Fluid statics:**</u> is the study of fluid problems in which there is no relative motion between fluid elements.

Pressure Variation in Static Fluids: $P = \frac{F}{A}$





$+\uparrow \sum \mathbf{F = 0} \;\rightarrow\rightarrow \quad \mathbf{-(P + Pd)A - dw + PA = 0}$

$\mathbf{-AdP = dw} \quad \rightarrow\rightarrow \; * \; \mathbf{dw = \gamma v = \gamma Adz} \;\rightarrow\rightarrow$

$\therefore \mathbf{-AdP = \gamma Adz} \;\rightarrow\rightarrow \quad \therefore \mathbf{-dP = \gamma dz} \quad * \boldsymbol{\gamma} = \mathbf{cons.}$

$$\int_1^2 - dp = \gamma \int_1^2 dz \quad \textbf{or} \quad (p_2 - p_1) = -\gamma (z_2 - z_1)$$

$h = z_2 - z_1$ since h is positive downwards (pressure head)

$\therefore (p_2 - p_1) = -\gamma h$ **in final form:** $\therefore p_1 = p_2 + \gamma h$ **or** $p_2 = p_1 - \gamma h$

If P_2 considered atmospheric pressure and taken as zero

$\therefore p_1 = \gamma h$ **(gauge pressure)**

The equation can be written as the ordinary differential equation $\frac{dP}{dz} = -\gamma$, it is one important principle of the hydrostatic, or shear-free, these equations show that the pressure does not depend on x or y (which means pressure don't varied horizontally). Since p depends only on z. ***The pressure is varied with vertical depth.***

<u>***Incompressible Fluid:***</u> Since the specific weight is equal to the product of fluid density and acceleration of gravity ($\gamma = \rho . g$) changes in are caused either by a change in ρ or *g*. For most engineering applications the variation in *g* is negligible, so our main concern is with the possible variation in the fluid density (<u>which it called compressible</u>). For liquids the variation in density is usually negligible (<u>which it called incompressible</u>), so that the assumption of constant specific weight when dealing with liquids. For this instance, Eq. ($\frac{dP}{dz} = -\gamma$) can be directly integrated:

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz \quad \text{or} \quad (p_2 - p_1) = -\gamma (z_2 - z_1) \quad \text{OR in final form:} \quad (p_1 - p_2) = \gamma (z_2 - z_1)$$

The reference pressure p_o would correspond to the pressure acting on the free surface (which would frequently be <u>*atmospheric pressure*</u>), and thus if we let $p_2 = p_o$ in above Equation it follows that the pressure *p* at any depth *h* below the free surface is given by the equation: $p = \gamma h + p_o$

where p_1 and p_2 are pressures at the vertical elevations as is illustrated in Fig. (2). Equation can be written in the compact form: $(p_1 - p_2) = \gamma h$,

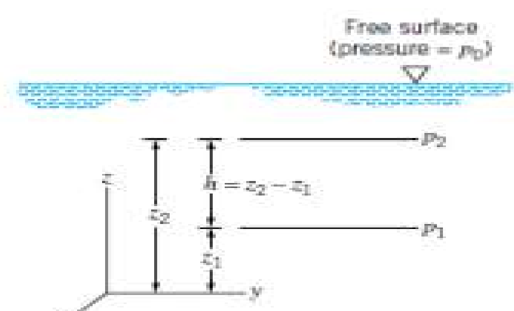


Fig. (2).

Equation shows that in an incompressible fluid at rest the pressure varies linearly with depth and (*h is called pressure head*) which has units of length (m) or (ft). When one works with liquids there is often a free surface, as is illustrated in Fig. (2), and it is convenient to use this surface as a reference plane

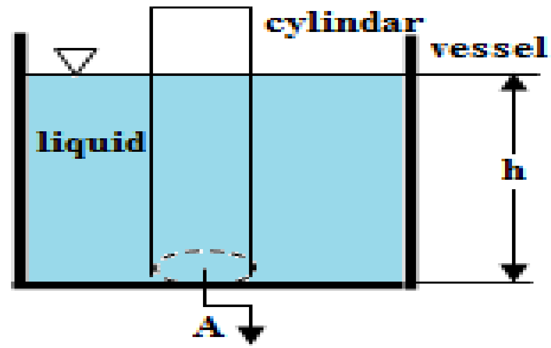
Pressure head of a liquid:

when fluid is contained in a vessel it exerts force at all points on side, bottom and top.

h- height of liquid in cylinder ; A- area of cylinder
γ - specific weight ; P- pressure of liquid; F – force
Now,

total pressure force on the base of the cylinder
= weight of liquid in the cylinder

$$PA = mg = \rho \forall g = \gamma A h \quad \rightarrow\rightarrow P = \gamma h$$



Problem 1: find the pressure at a depth of 15m below the free surface of water in a reservoir.

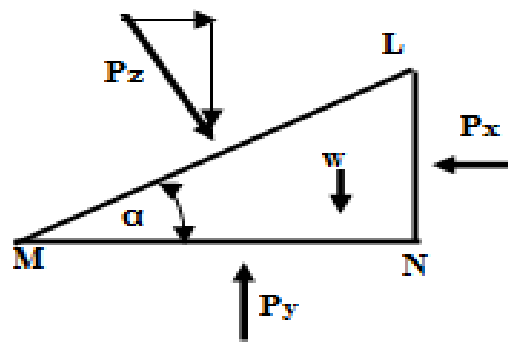
$$P = \rho g h = 1000 \times 9.81 \times 15 = 147.1 kPa$$

Pascal's law: the pressure at any point in the liquid at vessel is the same in all direction.

Proof: let us consider a very small wedge shaped element LMN of a liquid.
P_x – horizontal pressure ; P_y – vertical pressure
P_z – pressure on LM ; α – angle of element fluid
F_x, F_y, F_z - pressure forces on LN, NM, ML respectively.
As the element of fluid at rest, therefore:

$$\sum F_x = 0 \rightarrow F_x = F_z \sin\alpha$$

$$P_x LN = P_z LM \sin\alpha, \quad \text{but } LM \sin\alpha = LN$$



$$P_x LN = P_z LN \quad \rightarrow \quad P_x = P_z ----(1)$$

$$\sum F_z = 0 \rightarrow F_y = F_z \cos\alpha + w, \quad w = 0 \text{ very small element}$$

$$P_y MN = P_z LM \cos\alpha \quad \text{but } LM \cos\alpha = MN$$

$$P_y MN = P_z MN \quad \rightarrow \quad P_y = P_z --- (2)$$

From equations 1&2 $\quad P_x = P_y = P_z$

OR

Pressure at Point:

Figure 1 shows a small wedge of fluid at rest of size Δx by Δz by Δs and depth b into the paper. There is no shear by definition when fluid at rest), but we suppose that the pressures px, pz, and pn may be different on each face. The weight of the element also may be important. Summation of forces must equal zero (no acceleration) in both the x and z directions.

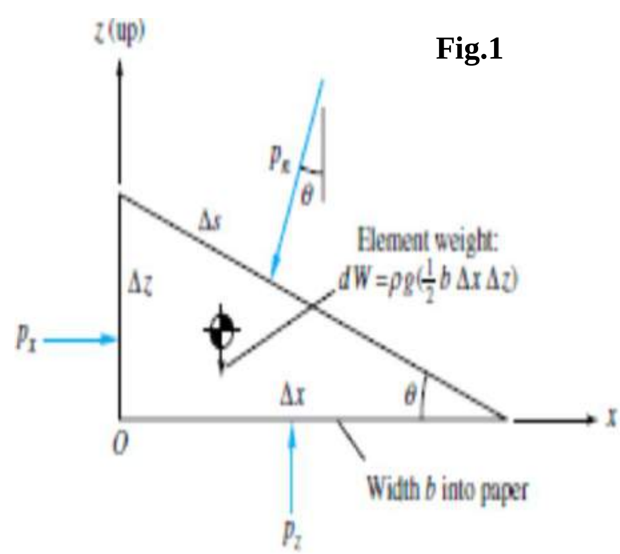


Fig.1

$$\sum F_x = 0 = p_x b \Delta z - p_n b \Delta s \sin\theta$$ **,also,**

$$\sum F_z = 0 = p_z b \Delta x - p_n b \Delta s \cos\theta - (1/2)\gamma b \Delta x \Delta z \quad .. (1)$$

But we know that: $\Delta s \sin\theta = \Delta z$ **, and** $\Delta s \cos\theta = \Delta x$
Then by substituting in Eq. (1), and re-arrangement:

$p_x = p_n$ **, and …………………………. (2a)**

$p_z = p_n + (1/2)\gamma\Delta z$ **…………...……… (2b)**

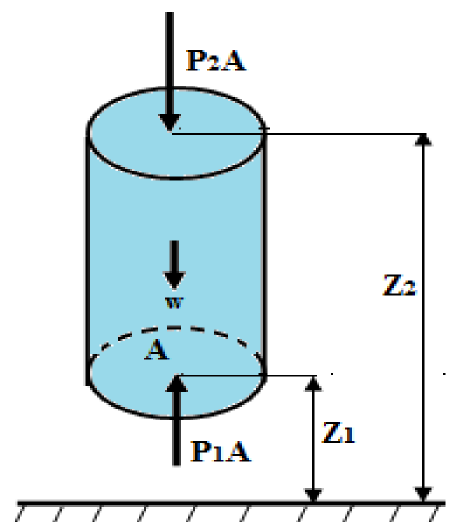
In the limit as the fluid wedge shrinks to a "point,'' $\Delta z \rightarrow 0$ and Eqs. (2) becomes:

$p_x = p_z = p_n = p$ **……………………………….(3)**

These relations illustrate one important principle of the hydrostatic, or shear-free, condition:
There is no pressure change in the horizontal direction. We conclude that the pressure p at a point in a static fluid is independent of direction as long as there are no shearing stresses present, This important result is known as Pascal's law named in honor of Blasé Pascal 11623–16622, a French mathematician who made important contributions in the field of hydrostatics or (the pressure at point inside static fluid is equal from all sides).

Variation of pressure vertically in fluid under gravity

As shown in figure, an element of fluid which is a vertical column of constant cross-sectional area A surrounding by the same fluid of mass density ρ. The pressure at the bottom of cylinder is P_1 at level Z_1, and at the top P_2 at level Z_2. The fluid at rest and in equilibrium, so all the forces in the vertical direction sum is zero.



$$\sum \mathbf{F = 0} \rightarrow \mathbf{P_1A - P_2A - w = 0}$$

$$\mathbf{P_1A - P_2A - \gamma A(z_2 - z_1) = 0 \qquad \div A\gamma}$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - z_2 + z_1 = 0$$

$$\frac{\mathbf{P_1}}{\gamma} + \mathbf{z_1} = \frac{\mathbf{P_2}}{\gamma} + \mathbf{z_2} = \mathbf{cons.}$$

Absolute, gage, vacuum and atmospheric pressure

$P_{abs.} = P_{atm.} + P_{gage}$

$P_{abs.} = P_{atm.} - P_{vac.}$

Example :

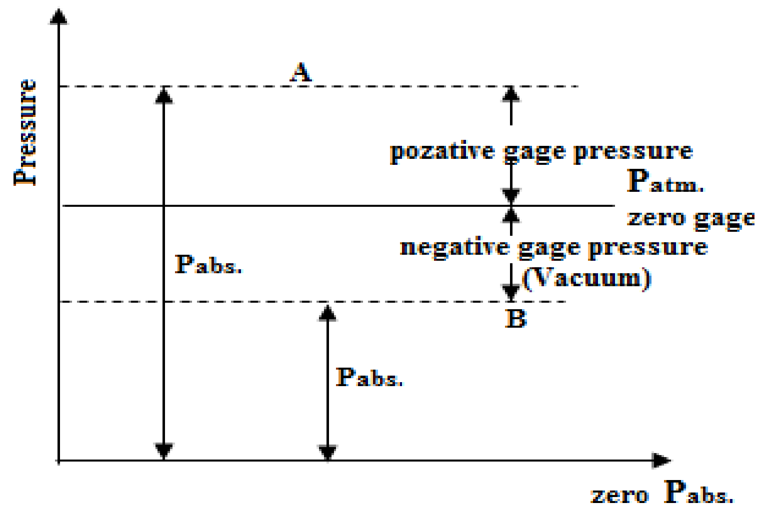
$P_A = 75kPa(gage)$

$P_B = 40\ kPa(Vacuum)$

$P_{atm.} = 100kPa,\ P_{Aabs.},\ P_{Babs.}$?

$P_{Aabs.} = 75 + 100 = 175kPa$

$P_{Babs.} = 100 - 40 = 60kPa$



Absolute pressure is measured relative to a perfect vacuum (absolute zero pressure), whereas gage pressure is measured relative to the local atmospheric pressure. Thus, a gage pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressures can be either positive or negative depending on whether the pressure is above atmospheric pressure (a positive value) or below atmospheric pressure (a negative value). A negative gage pressure is also referred to as a *suction* or *vacuum* pressure.

Problem 1: Calculate the pressure due to a column of 0.3m of (a) water (b) an oil of r.d = 0.8 and (c) mercury of r.d = 13.6. Take density of water 1000 kg/m^3.

Solution:

For water $P = \rho gh = 1000 \times 9.81 \times 0.3 = 2943\ N/m^2$

For oil $P = \rho gh = 1000 \times 0.8 \times 9.81 \times 0.3 = 2354.4\ N/m^2$

For mercury $P = \rho gh = 1000 \times 13.6 \times 9.81 \times 0.3 = 40025\ N/m^2$

Problem 2: The pressure intensity at a point in a fluid is given 3.924 N/cm^2. Find the corresponding height of fluid when the fluid is: (a) water, and (b) oil of r.d = 0.9.

Solution: **For water:** $P = \rho gh \rightarrow h = \frac{P}{\rho g} = \frac{3.924\times10^4}{9810} = 4m$

For oil: $P = \rho gh \rightarrow h = \frac{P}{\rho g} = \frac{3.924\times10^4}{0.8\times1000\times9.81} = 4.44m$

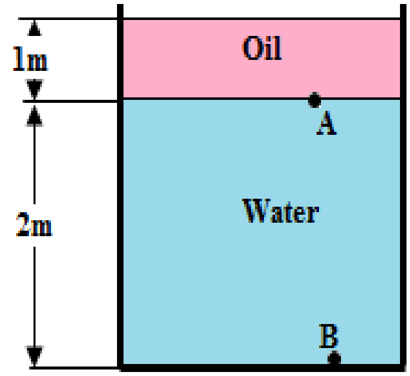
Problem 3: An oil of r.d = 0.9 is contained in a vessel. At a point the height of oil is 40 m, Find the corresponding height of water at the point.

Solution:

For oil: $\mathbf{P = \rho g h = 0.9 \times 1000 \times 9.81 \times 40 = 353160\ Pa}$

For water: $\mathbf{P = \rho g h \rightarrow h = \frac{P}{\rho g} = \frac{353160}{9810} = 36m}$

Problem 4: An open tank contains water up to a depth of 2 m and above it an oil of r.d = 0.9 for a depth of l m. Find the pressure intensity (i) at the interface of the two liquids A, and (ii) at the bottom of the tank B.



Solution: **At the interface point A:**

$$\mathbf{P = \rho g h = 0.9 \times 1000 \times 9.81 \times 1 = 8829\ N/m^2}$$

At the bottom of the tank B:

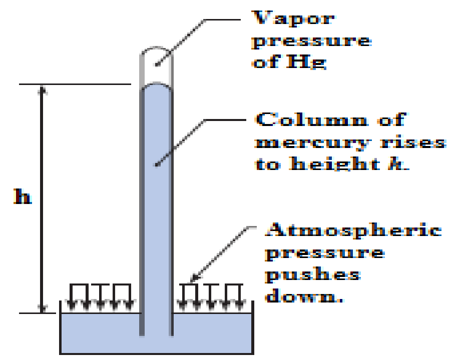
$$\mathbf{P = (\rho g h)_{oil} + (\rho g h)_{water}}$$
$$\mathbf{= 0.9 \times 1000 \times 9.81 \times 1 + 9810 \times 2 \quad 28449 N/m^2}$$

Pressure Measurements

Generally, pressure is measured by: **1. Barometers** ; **2. Manometers**

3. Mechanical and electronics measuring device, e.g. a. Burdon pressure gage ; b. Pressure transducer

1.Barometer: An instrument that is used to measure atmospheric pressure is called a *barometer*. The most common types are the mercury barometer and the aneroid barometer. A mercury barometer is made by inverting a mercury-filled tube in a container of mercury as shown in Figure. The pressure at the top of the mercury barometer will be the vapor pressure of mercury, which is very small:



$$\mathbf{P_{vap.} = 2.4 * 10^{-6}\ atm.\ at\ 20^{o}C.}$$

$$P_{atm.} = \gamma_{Hg} h + P_{vap.} \cong \gamma_{Hg} h$$

2.Manometers

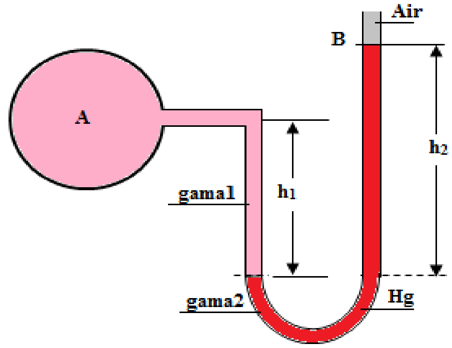
Manometers. Manometers are defined as the devices used for measuring the pressure at i- point in a fluid by balancing the column of fluid by the same or another column of the fluid.

They classified as: **(a) Simple Manometers,** **(b) Differential Manometers**.

2.1 Manometer equation

1. Start from any point given.
2. Move from that point:
   a- If you move downwards you add (+) the pressure reading.
   b- If you move upwards you subtract (-) the pressure reading.
3. Continue till the other end reached.
4. Equate all pressure reading to the pressure at the other end.

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 = P_B = P_{atm.} = 0$$



2.2 Simple Type of Manometers

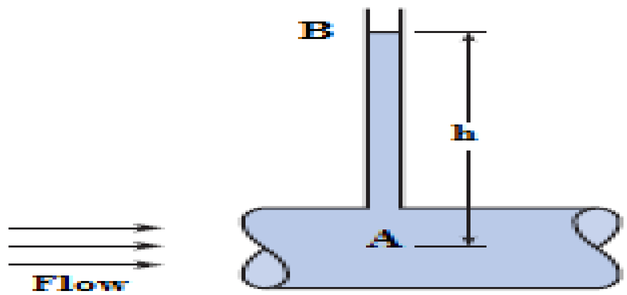
A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and another end remains open to atmosphere. Common types of simple manometers are:

1. **Piezometer,**
2. **Inclined piezometer**
3. **U-tube Manometer, and.**
4. **Differential Manometer**.

1. Piezometer

It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Figure. The rise of liquid gives the pressure head at that point.

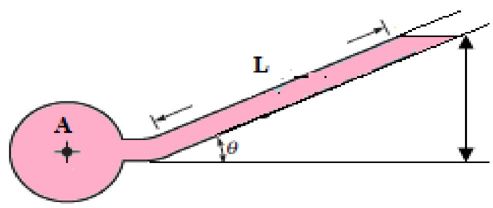
$$P_B = P_A - \gamma h = P_{atm.} = 0$$



2. Inclined-Tube Manometer

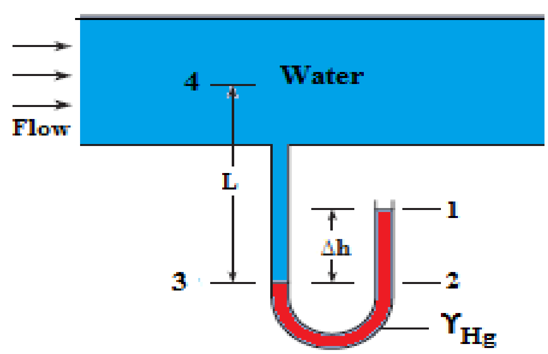
Usually used when more accurate reading is required.

$$h = L\sin\theta$$



3. U Tube Manometer

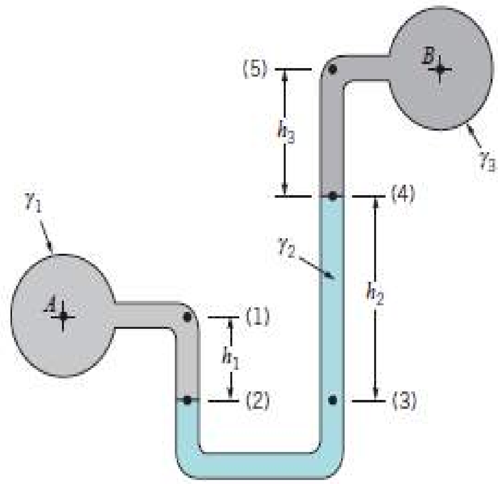
It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and another end remains open to the atmosphere as shown in Fig. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



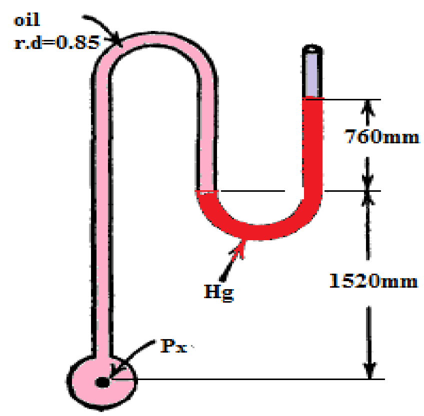
4. Differential manometer

The U-tube manometer is also widely used to measure the *difference* in pressure between two containers or two points in a given system. Consider a manometer connected between containers *A* and *B* as is shown in Figure. The difference in pressure between *A* and *B* can be found by again starting at one end of the system and working around to the other end.

$$\mathbf{P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B}$$
$$\mathbf{P_A - P_B = -\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3}$$



Problem1: With the manometer reading as shown, calculate P_x.



Solution:

$$\gamma_{oil} = \rho g = r.d \times \rho_w \times g$$

$$\gamma_{oil} = 0.85 \times 1000 \times 9.81 = 8338.5\ N/m^3$$

$$\gamma_{Hg} = 13.57 \times 1000 \times 9.81 = 133121.7 N/m^3$$

$$\gamma_{Hg} \times 0.760 + \gamma_{oil} \times 1.52 = P_x$$

$$P_x = 13.85\ kPa$$

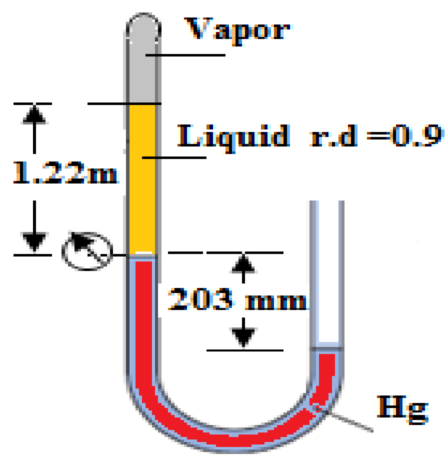
Problem2: Barometric (absolute) pressure is 91 kPa. Calculate the vapor pressure of the liquid and the gage reading.

Solution:

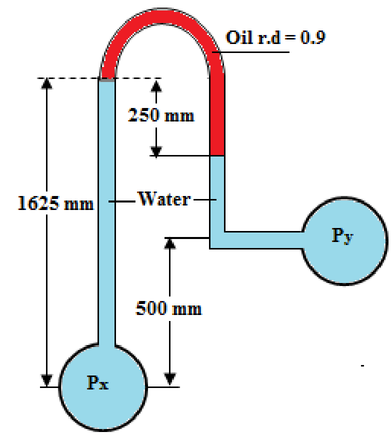
$$P_{vap.} + \gamma_{liq.} \times 1.22 + \gamma_{Hg} \times 0.203 - P_{atm.} = P_{abs.}$$

$$P_{vap.} = 91000 - 900 \times 9.81 \times 1.22 - 13570 \times 9.81 \times 0.203 = 53.2 kPa$$

Gage reading = 203mmHg vacuum



<u>Problem3:</u> Calculate $P_x - P_y$ for this inverted U-tube manometer.

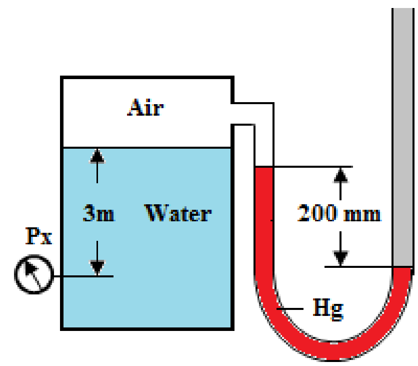


<u>Solution:</u>

$$P_x = P_y - \gamma_{wat.}(1.625 - 0.5 - 0.25) - \gamma_{oil} \times 0.25 + \gamma_{wat.} \times 1.625$$

$$P_x - P_y = -9810 \times 0.875 - 8829 \times 0.25 + 9810 \times 1.625 = 5.15\ kPa$$

<u>Problem 4:</u> Calculate the gage reading P_x.

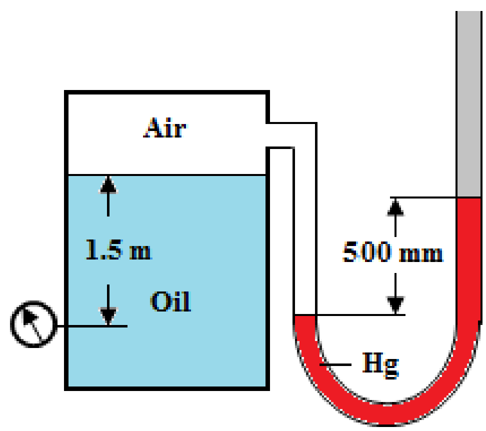


<u>Solution:</u>

$$P_x = -\gamma_{Hg} \times 0.2 + \gamma_{wat.} \times 3$$

$$P_x = -133121 \times 0.2 + 9810 \times 3 = 2.8kPa$$

<u>Problem 5:</u> Calculate the gage reading. Relative density of the oil is 0.85 and barometric pressure is 755 mm of mercury.



$$P_{abs.} = \gamma_{Hg} \times 0.5 + \gamma_{oil.} \times 1.5$$

$$P_{abs.} = 133121 \times 0.5 + 8338.5 \times 1.5 = 79kPa$$

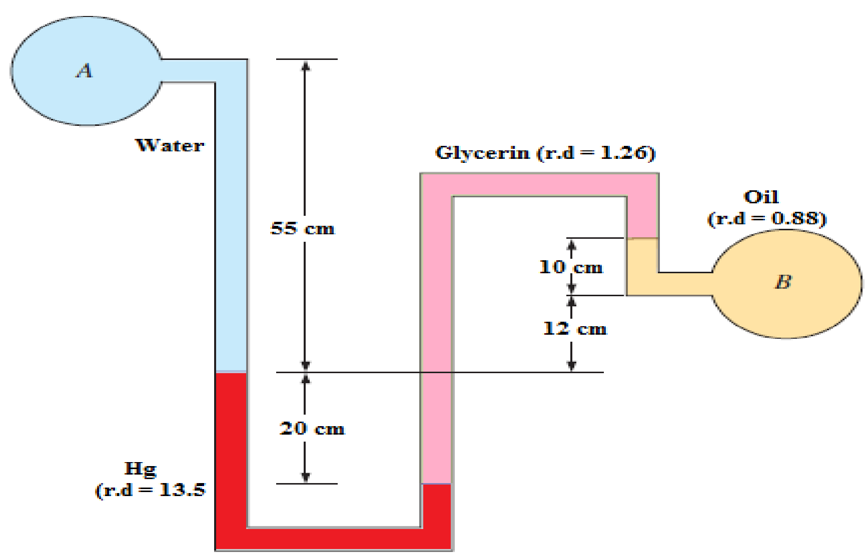
$$P_{atm.} = 0.755 \times 133121 = 100.5kPa$$

$$P_{gag.} = P_{abs.} - P_{atm.} = 79 - 100.5 = -21.5\ kPa$$

$$\text{Gage reading} = \frac{21500}{133121} = 161.5mmHg$$

<u>H.W.</u> : The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Figure. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$.

$$P_B - P_A = \gamma_w \times 0.55 + \gamma_{Hg} \times 0.2 - \gamma_G(0.2 + 0.12 + 0.1) + \gamma_{oil} \times 0.1$$



$$= 9810 \times 0.55 + 13500 \times 9.81 \times 0.2 - 1260 \times 9.81 \times 0.42 + 880 \times 9.81 \times 0.1$$

$$P_B - P_A = 27.6kPa$$

Fluid static, concept of pressure, pascal's law and its application, action of fluid pressure on a plane (horizontal. Vertical, and inclined)

Hydrostatic Forces on Surfaces

Total Pressure and Centre of Pressure

• **Total pressure.** It is defined as the ***force*** *exerted by static fluid on a surface* (*either plane or curved*) *when the fluid comes in contact with the surface. This force is always at right angle* (*or normal*) *to the surface.*

• **Centre of pressure.** It is defined as the point of application of the total pressure on the surface.

The immersed surfaces may be:

1. Horizontal plane surface;
2. Vertical plane surface;
3. Inclined plane surface;
4. Curved surface.

Horizontally immersed surface

Total Pressure (P):

Refer to Fig. Consider a plane horizontal surface immersed in a liquid.

Let, A = Area of the immersed surface,

$\bar{x}$ = Depth of horizontal surface from the liquid, and

w = Specific weight of the liquid.

The total pressure on the surface, Total Force

P = Weight of the liquid above the immersed surface

= Specific weight of liquid × volume of liquid

= Specific weight of liquid × area of surface × depth of liquid

$= wA\bar{x}$

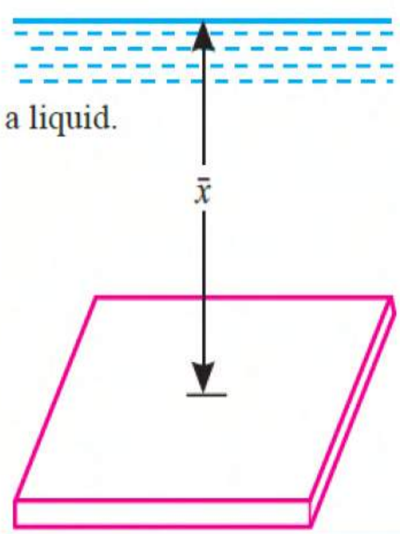


Fig. Horizontally immersed surface.

VERTICALLY IMMERSED SURFACE

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig.

Let, A = Total area of the surface,

G = Centre of the area of the surface,

$\bar{x}$ = Depth of centre of area,

OO = Free surface of liquid, and

$\bar{h}$ = Distance of centre of pressure from free surface of liquid.

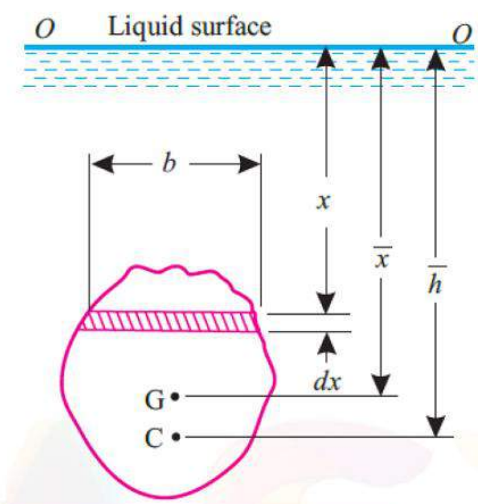


Fig. Vertically immersed surface.

(a) Total pressure (P):

Consider a thin horizontal strip of the surface of thickness dx and breadth b. Let the depth of the strip be x. Let the intensity of pressure on strip be p; this may be taken as uniform as the strip is extremely small. Then,

$$p = wx \quad \text{Pressure}$$

where, w = specific weight of the liquid.

Total pressure on the strip $= p.b.dx$. Force

$= wx \,.\, bdx$

Total pressure on the whole area, $P = \int wx \,.\, bdx = w\int bdx \,.\, x$ Force

Note that
The Intensity of Pressure = Pressure, (KN/m²)
Total Pressure = Total Force, (KN)

But, $\int bdx \,.\, x$ = Moment of the surface area about the liquid level $= A\bar{x}$

$\therefore \quad P = wA\bar{x}$ ...[same as in Art. 3.3] Force

VERTICALLY IMMERSED SURFACE

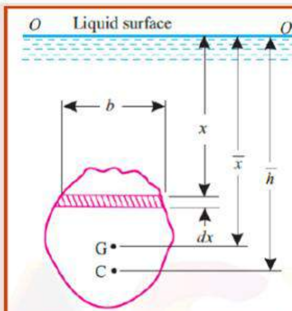


Fig. 3.2. Vertically immersed surface.

But, $\int x^2 .b.dx = I_0$ = Moment of inertia of the surface about the free surface OO (or second moment of area)

$$M = wI_0 \quad ...(i)$$

The sum of the moments of the pressure is also equal to $P \times \bar{h}$ ...(ii)

Now equating eqns. (i) and (ii), we get:

$$P \times \bar{h} = wI_0. \qquad (\because P = wA\bar{x})$$

$$wA\bar{x} \times \bar{h} = wI_0$$

$$\bar{h} = \frac{I_0}{A\bar{x}} \quad ...(iii)$$

Also, $I = I_G + Ah^2$ (Theorem of parallel axis)

where, I_G = Moment of inertia of the figure about horizontal axis through its centre of gravity, and

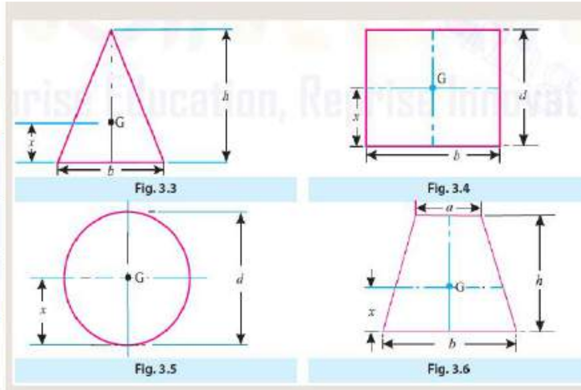
h = Distance between the free liquid surface and the centre of gravity of the figure ($\bar{x}$ in this case)

Thus rearranging equation (iii), we have $\bar{h} = \dfrac{I_G + A\bar{x}^2}{A\bar{x}} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

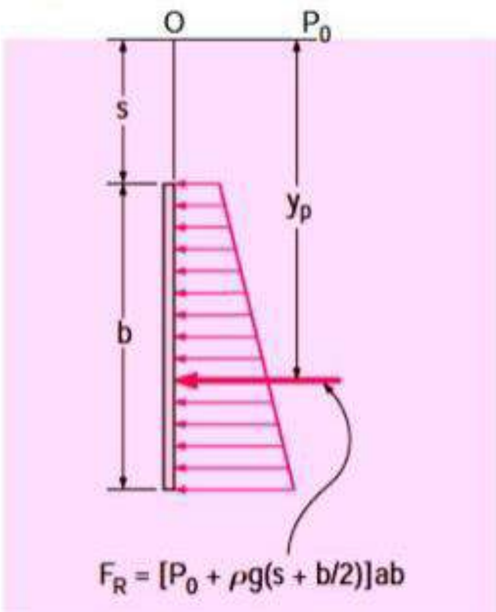
Hence, *centre of pressure*, $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

Table 3.1. The Centre of Gravity (G) and Moment of Inertia (I) of Some Important Geometrical Figures:

S.No.	Name of figure	C.G. from the base	Area	I about an axis passing through C.G. and parallel to the base	I about base
1.	Triangle Fig. 3.3	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle Fig. 3.4	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
3.	Circle Fig. 3.5	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4.	Trapezium Fig. 3.6	$x = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^2$	—

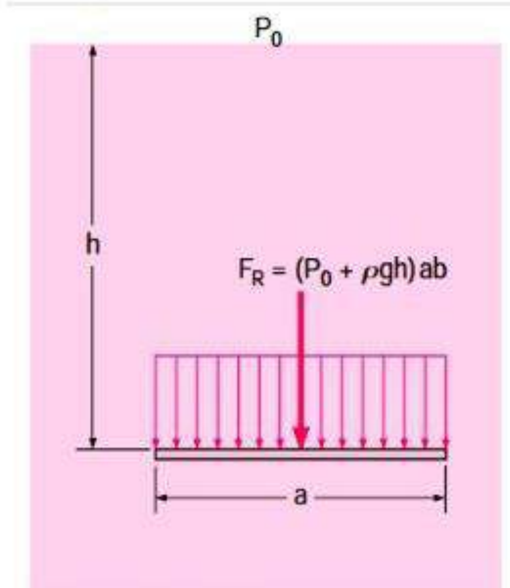


Hydrostatic Forces on Submerged Plane surfaces



(b) Vertical plate

$$F_R = [P_0 + \rho g(s + b/2)]ab$$



(c) Horizontal plate

$$F_R = (P_0 + \rho gh)ab$$