

Dr. Isam M. S. Najar

Ministry of Higher Education, University of Mosul (drisamnajar@gmail.com)
Dr. Najar is a member of teaching staff in Dept. of petroleum reservoir engineering / College of petroleum and mining enginecring / the University of Mosul. He received B.S.. in Geology from the University of Mosul in 1983. He completed M.Sc. in Geophysics (Paleomagnetic dating of recent sediments) from Geology department / University of Baghdad in 1989. He earned Ph.D. in Geophysics (Petroleum reservoir structures exploration study using seismic, gravity \& magnetic methods controlled by boreholes data) from Dept. of Geology/the University of Mosul in 1999. Dr. Najar is very interested in reservoir engineering subjects. He has more than 18 years experience in teaching in the universities inside and outside of Iraq.


Mr. Hameed Mahmood AI-Naser Ministry of Oil, North Oil Company (res.eng.hameed@hotmail.com)
Mr. Hameed Al-Naser is a Reservoir Engineer at North Oil Company; He received B.Sc. in Reservoir Engineering from the University of Mosul. He has worked on several projects in reservoir engineering which are encompassed reservoir evaluation, modeling, management, simulation, and data analysis using the artificial neural network. He has worked as a data analyst and information management specialist at many companies. His passion and experience in reservoir engineering encouraged him to participate in preparing this book.

## FUNDAMENTALS OF Petroleum Reservoir ENGINEERING (FLUID \& ROCK PROPERTIES)

ISAM M. NAJAR \& HAMEED M. AL-NASER

# FUNDAMENTALS OF PETROLEUM RESERVOIR ENGINEERING (FLUID \& ROCK PROPERTIES) 

ISAM MOHAMMED S. NAJAR \&

HAMEED MAHMOOD AL-NASER

This page is internationally left empty

## Preface

Because of lacking a simplified textbook about the fundamentals of petroleum reservoir engineering. The authors wanted to collect Najar's lectures as a book to be a reference for teaching this subjects in the universities of Iraq.

Mr. Al-Naser supported Dr. Najar and helped him with great efforts in feeding the lectures using (Word, Excel, Photoshop, and Petrel), adding more details to the subjects, preparing problems to the chapters in addition to adding some sections to the book. Without Al-Naser efforts this book would not have been come out with the current arrangement.

This textbook contains six chapters. The first three chapters deal with petroleum reservoir fluids properties given as one course. The other three chapters deal with petroleum reservoir rock properties given as another course.

The authors hope that this book to be a very helpful source for students during their study.

## TABLE OF CONTENTS

Some Abbreviation of Units Used ..... VII
Units Conversion ..... X
Chapter One: Basic Concepts and Equations
1.1 What is Reservoir Engineering ..... 1
1.2 Some information about hydrocarbons ..... 1
1.3 Temperature Scales ..... 2
1.4 Equation of State ..... 3
1.4.1 Ideal Gas Law ..... 3
1.4.1.1 Boyle's Law ..... 4
1.4.1.2 Charles's Law ..... 5
1.4.1.3 Gay-Lussac's Law ..... 6
1.4.1.4 The Combined Gas Law ..... 6
1.4.1.5 Avogadro's Law ..... 7
1.4.1.6 Molecular Weight (Molar Mass) ..... 8
1.4.1.7 The Universal Gas Constant (R) ..... 8
1.4.1.8 Dalton's Law ..... 10
1.4.1.9 Amagat's Law ..... 11
1.4.2 Real Gas Law ..... 11
1.4.2.1 Van der Waals Equation ..... 13
Problems ..... 16
References ..... 17
Chapter Two: Natural Gas \& Crude Oil Properties
2.1 Natural Gas Properties ..... 18
2.1.1 Apparent Molecular Weight (Ma) and Mole Fraction (yi) ..... 18
2.1.2 Gas Density, $\rho \mathrm{g}$ ..... 20
2.1.3 Specific Gravity, $\gamma_{\mathrm{g}}$ ..... 21
2.1.4 Specific volume, $v$ ..... 23
2.1.5 Critical Temperature and Pressure. ..... 23
2.1.5.1 Single-Component Systems ..... 23
2.1.5.2 Two-Component Systems ..... 28
2.1.5.3 Three-Component Systems ..... 30
2.1.5.4 Multicomponent Systems ..... 31
2.1.5.5 Principle of Corresponding States ..... 32
2.1.6 Determination of Z-factor Value ..... 32
2.1.7 Isothermal Gas Compressibility, cg ..... 50
2.1.8 Gas Formation Volume Factor, Bg. ..... 54
2.1.9 Gas Expansion Factor, Eg ..... 55
2.1.10 Gas Viscosity, $\mu_{\mathrm{g}}$ ..... 56
2.2 Crude Oil Properties ..... 62
2.2.1 Crude Oil API Gravity. ..... 62
2.2.2 Specific Gravity of the Solution Gas, " $\boldsymbol{\gamma}_{\boldsymbol{g}}$ " ..... 64
2.2.3 Crude Oil Density, " $\boldsymbol{\rho}_{\boldsymbol{o}}$ ". ..... 65
2.2.4 Gas Solubility " $\boldsymbol{R}_{s}$ " ..... 66
2.2.5 Bubble Point Pressure " $\boldsymbol{p}_{\boldsymbol{b}}$ " ..... 68
2.2.6 Oil Formation Volume Factor " $\boldsymbol{B}_{\boldsymbol{o}}$ ". ..... 69
2.2.7 Isothermal Compressibility Coefficient of Crude Oil " $\boldsymbol{c}_{\boldsymbol{o}}$ ". ..... 71
2.2.8 Total Formation Volume Factor " $\boldsymbol{B}_{\mathrm{t}}$ " ..... 73
2.2.9 Crude Oil Viscosity " $\boldsymbol{\mu}_{\boldsymbol{o}}$ ". ..... 75
Problems ..... 79
References ..... 85
Chapter Three: Phase Behavior and Laboratory Analysis of Reservoir Fluids
3.1 Phase Behavior. ..... 87
3.2 Classification of Reservoirs ..... 89
3.2.1 Classification of Oil Reservoirs. ..... 92
3.2.2 Classification of Gas Reservoirs ..... 92
3.3 Classification of Crude Oils ..... 96
3.4 Laboratory Analysis of Reservoir Fluids. ..... 99
3.4.1 Well Conditioning ..... 99
3.4.2 Fluid Sampling ..... 99
3.4.3 PVT Laboratory Tests ..... 101
3.4.4 Compositional Analysis of the Reservoir Fluid (chromatographic Test). ..... 102
3.4.5 Constant-Composition Expansion Tests (Equilibrium Test) ..... 103
3.4.6 Differential Liberation Test (DL) ..... 109
3.4.7 Separator Tests. ..... 119
3.4.7.1 Differential and Flash Separation ..... 121
3.4.7.2 Single-Stage Separation ..... 122
3.4.7.3 Two-Stage Separation ..... 123
3.4.7.4 Relationship of Gasoline Plants in Separation ..... 124
3.4.7.5 Economic Summary ..... 125
Problems. ..... 126
References ..... 129
Chapter Four: Basic Reservoir Rock Properties
4.1 Porosity ..... 130
4.1.1 Factors Affecting Sandstone Porosity ..... 132
4.1.2 Factors Affecting Carbonate Porosity ..... 133
4.1.3 Typical Reservoir Porosity Values ..... 134
4.1.4 Porosity Averaging ..... 134
4.1.5 Laboratory Measurement of Porosity ..... 135
4.1.5.1 Bulk Volume Measurement. ..... 136
4.1.5.2 Pore Volume Measurement ..... 137
4.1.5.3 Grain Volume Measurement ..... 140
4.1.5.4 Indirect Methods of Porosity Measurement (Logging Method) ..... 140
4.1.6 Pore Volume Compressibility ..... 141
4.2 Fluid Saturation ..... 142
4.2.1 Saturation Averaging ..... 144
4.2.2 Laboratory Measurement of Fluid Saturation ..... 145
4.3 Permeability ..... 147
4.3.1 Averaging Absolute Permeabilities ..... 153
4.3.1.1 Parallel Flow ..... 153
4.3.1.2 Series Flow ..... 157
4.3.1.3 Geometric Average Permeability ..... 161
4.3.2 Absolute Permeability Correlations ..... 162
4.3.3 Permeability of Fractures and Channels ..... 163
4.3.4 Factors Affecting Absolute Permeability ..... 164
4.3.5 Laboratory Measurement of Absolute Permeability ..... 164
4.3.6 Klinkenberg Effect. ..... 166
4.4 Surface and Interfacial Tensions ..... 168
4.4.1 Effect of Pressure and Temperature on Interfacial and Surface Tension ..... 171
4.4.2 Laboratory Measurement of Interfacial Tension ..... 172
4.5 Wettability ..... 172
4.5.1 Classification or Types of Wettability ..... 174
4.5.1.1 Water Wet ..... 174
4.5.1.2 Oil Wet ..... 175
4.5.1.3 Intermediate Wet. ..... 175
4.5.1.4 Fractional Wettability ..... 175
4.5.1.5 Mixed Wettability ..... 175
4.5.2 Factors Affecting Wettability ..... 176
4.5.3 Measurement of Reservoir Rock Wettability. ..... 176
Problems ..... 177
References ..... 180
Chapter Five: Capillary Pressure and Relative Permeability
5.1 Capillary Pressure ..... 182
5.1.1 Dependence of Capillary Pressure on Rock and Fluid Properties ..... 186
5.1.2 Laboratory Measurement of Capillary Pressure ..... 187
5.1.3 Averaging Capillary Pressure Data: J-Function ..... 190
5.1.4 Converting Laboratory Capillary Pressure Data to Reservoir Conditions ..... 193
5.1.5 Capillary Pressure and Permeability ..... 194
5.1.6 Zonation, Fluid Contacts, and Initial Saturation Distribution in a Reservoir ..... 195
5.1.6.1 Free Water Level, FWL ..... 197
5.1.6.2 Oil-Water Contact, OWC ..... 198
5.1.6.3 Transition Zone ..... 200
5.2 Relative Permeability ..... 204
5.2.1 Two-Phase Relative Permeability ..... 205
5.2.1.1 Drainage Process ..... 209
5.2.1.2 Imbibition Process. ..... 210
5.2.2 Laboratory Measurement of Tow-Phase Relative Permeability ..... 210
5.2.3 Two-Phase Relative Permeability Correlations ..... 212
5.2.3.1 Wyllie and Gardner Correlation ..... 212
5.2.3.2 Torcaso and Wyllie Correlation ..... 215
5.2.4 Three-Phase Relative Permeability. ..... 215
Problems ..... 219
References ..... 223
Chapter Six: Introduction to Reserve Estimation
6.1 Introduction ..... 224
6.1.1 Reserve Status Categories ..... 225
6.1.2 Reserve Estimation Methods ..... 225
6.2 Oil Recovery Methods ..... 226
6.3 Reservoir Primary Recovery Mechanisms ..... 228
6.3.1 Rock and Liquid Expansion ..... 228
6.3.2 The Depletion-Drive Mechanism. ..... 229
6.3.3 Gas-Cap Drive ..... 229
6.3.4 The Water-Drive Mechanism. ..... 230
6.3.5 The Gravity-Drainage-Drive Mechanism ..... 231
6.3.6 The Combination-Drive Mechanism ..... 232
6.4 Volumetric Methods ..... 233
6.4.1 Oil Reservoirs ..... 233
6.4.2 Gas Reservoirs ..... 235
6.5 Material Balance Equation ..... 236
6.6 Decline Curve analysis and History Matching ..... 246
6.7 Computer Simulation ..... 246
Problems ..... 248
References ..... 250
Index ..... 251

Some Abbreviation of Units Used:

| gal | U.S. liquid gallon $=231$ (inch $)^{3}$ |
| :---: | :---: |
|  | British gallon $=277.42$ (inch) ${ }^{3}$ |
| lb/gal | pound per gallon |
| lb mole | pound mole |
| $\mathrm{lb} / \mathrm{sq}$ in | pound per square inch |
| $\mathrm{lb} / \mathrm{sq} \mathrm{ft}$ | pound per square foot |
| lim | Limit |
| liq | Liquid |
| $\ln$ | natural logarithm (base e) |
| $\log$ | common logarithm (base 10) |
| M | Meter |
| meq | milliequivalent weight |
| mg | Milligram |
| microsip | $\mathrm{Psi}^{-1} \times 10^{6}$ |
| ml | Milliliter |
| mm | Millimeter |
| mm Hg | millimeters of mercury |
| mol wt | molecular weight |
| MMscf | million stander cubic feet |
| MMSTB | million stock-tank barrels |
| Mscf | thousand stander cubic feet |
| Mscf/d | thousand stander cubic feet |
| MSTB | thousand stock-tank barrels |
| MSTB/d | thousand stock-tank barrels per day |
| $\mathrm{oz} / \mathrm{sq}$ in | ounce per square inch |
| ppm | parts per million |
| psi | pounds per square inch |


| psia | puonds per square inch absolute |
| :---: | :---: |
| psig | puonds per square inch gauge |
| res | Reservoir |
| res cu ft | cubic feet of reservoir gas reported at reservoir conditions |
| res bbl | barrels of reservoir liquid or gas (or both) reported at reservoir conditions |
| ${ }^{\circ} \mathrm{R}$ | degree Rankine |
| scf | standard cubic feet, volume of gas reported at standard |
|  | conditions |
| scf/STB | standard cubic feet of gas per barrel of stock-tank liquid |
| sec | Second |
| 1/psi | $\mathrm{psi}^{-1}$ |
| sp. gr. | specific gravity |
| sq | Square |
| sq cm | square centimeter |
| sq ft | square foot |
| sq in | square inch |
| ST | Stock-tank (at conditions of $14.7 \mathrm{psi}-60^{\circ} \mathrm{F}$ ) |
| STO | Stock-tank oil (used only when necessary to distinguish between quantities in same calculation, such as $\rho_{\mathrm{oR}}$ and $\rho_{\mathrm{STO}}$ |
| T | total or two phase |
| T | temperature or temperature dependent |
| V | used in $\mathrm{p}_{\mathrm{v}}$ to indicate vapor pressure and $\mathrm{L}_{\mathrm{v}}$ to indicate |
|  | vaporization |
| W | water (oilfield brine) |
| Wet | wet, with $L_{c}$ to indicate saturated with water vapor prior to |
|  | combustion |
| Wg | wet gas |


| A | angstrom unit $\left(10^{-8} \mathrm{~cm}\right)$ |
| :--- | :--- |
| AGP | additional gas produced, a parameter in reservoir-gas specific <br> gravity equation |
| ${ }^{\circ}$ API | degree (American Petroleum Institute) |
| atm | atmosphere |
| bbl | barrel |
| bbl/d | barrels per day |
| SP | Separator |
| SP bbl | barrels of separator liquid |
| ST | stock tank |
| STB | barrels of stock-tank oil (stock-tank barrel) reported at standard |
| STB/d | conditions, also used for barrels of water at standard conditions |
| STO | stock-tank oil |
| Vs | Versus |
| VEQ | Equivalent gas volume of stock-tank liquid, a parameter in |
| Wt | reservoir-gas specific gravity equation |
| Torr | Weight |
| lbm | pound mass |
| lbf | pound force |

## Units Conversion:

|  | To convert | Into | Multiply by |
| :---: | :---: | :---: | :---: |
| Length (L) | Feet <br> Feet <br> meters <br> inch <br> ft <br> m | inches <br> meters <br> feet <br> cm <br> cm <br> inch | 12 <br> 0.3048 <br> 3.28 <br> 2.54 <br> 30.48 <br> 39.7 |
| Volume (V) | bbl <br> bbl <br> $\mathrm{m}^{3}$ <br> lt <br> bbl <br> $\mathrm{m}^{3}$ | $\begin{aligned} & \mathrm{m}^{3} \\ & \mathrm{ft}^{3} \\ & \mathrm{bbl} \\ & \mathrm{ft}^{3} \\ & \mathrm{lt} \\ & \mathrm{ft}^{3} \end{aligned}$ | 0.159 5.615 6.29 0.03531 159 35.3 |
| Mass (m) | $\begin{array}{\|l\|l\|} \hline \mathrm{lb} \\ \mathrm{~kg} \end{array}$ | $\begin{aligned} & \mathrm{kg} \\ & \mathrm{lb} \end{aligned}$ | $\begin{aligned} & 0.4536 \\ & 2.205 \end{aligned}$ |
| Density ( $\rho$ ) | $\begin{aligned} & \mathrm{g} / \mathrm{cm}^{3} \\ & \mathrm{~kg} / \mathrm{m}^{3} \\ & \mathrm{lb} / \mathrm{ft}^{3} \\ & \mathrm{lb} / \mathrm{ft}^{3} \end{aligned}$ | $\mathrm{lb} / \mathrm{ft}^{3}$ <br> $\mathrm{lb} / \mathrm{ft}^{3}$ <br> $\mathrm{g} / \mathrm{cm}^{3}$ <br> $\mathrm{kg} / \mathrm{m}^{3}$ | $\begin{aligned} & \hline 62.43 \\ & 6.243 \times 10^{-2} \\ & 1.602 \times 10^{-2} \\ & 16.02 \end{aligned}$ |
| Pressure (p) | atm <br> psi <br> bar <br> bar <br> atm <br> psi | psi <br> atm <br> atm <br> Torr <br> pascal <br> pascal | $\begin{aligned} & 14.7 \\ & 6.805 \times 10^{-2} \\ & 0.9869 \\ & 750.061 \\ & 1 \times 10^{5} \\ & 47.88 \end{aligned}$ |

## Chapter One <br> Basic Concepts and Equations

### 1.1 What is Reservoir Engineering:

Reservoir engineering is a branch of petroleum engineering that applies scientific principles to produce oil and gas from the reservoir with high economic recovery. So, reservoir engineer to do that needs:

1- Applied mathematics.
2- Basic laws of physics and chemistry related to behavior of fluids in reservoir rocks.
3- Geology.
4- Experience and practice.
The goal of reservoir engineering is to set up a development project to study the behavior of the reservoir (throughout the life of the field) to derive the information required to reserve estimation, development planning and production operations optimization.

### 1.2 Some information about hydrocarbons:

Petroleum deposits vary widely in chemical composition and those obtained from different localities may have entirely different physical and chemical properties. In spite of this diversity, the bulk of the chemical compound found in petroleum are hydrocarbons (i.e. consisting of hydrogen and carbon only).

However, carbon compounds are known which have carbon chains consisting of hundreds of carbon atoms (see table1.1 below).

Table1.1: components of typical petroleum gases and liquids.

| Hydrocarbon | Natural gas | Gas producing petroleum liquid |
| :---: | :---: | :---: |
| Methane | $70-98 \%$ | $45-92 \%$ |
| Ethane | $1-10 \%$ | $4-21 \%$ |
| Propane | trace-5\% | $1-15 \%$ |
| Butanes | trace- $2 \%$ | $0.5-7 \%$ |
| Pentanes | trace-1\% | trace-3\% |
| Hexanes | trace- $0.5 \%$ | trace-2\% |
| Heptanes + | trace- $0.5 \%$ | trace-1.5\% |

### 1.3 Temperature Scales:

There are three temperature scales in use today, Fahrenheit, Celsius and Kelvin, another scale also used in petroleum industry called as Rankine. Fig. (1.1) shows the total temperature scales. Temperature scales classified as absolute ( ${ }^{\circ} \mathrm{R}$ $\left.\&{ }^{\circ} \mathrm{K}\right)$ and relative $\left({ }^{\circ} \mathrm{F} \&{ }^{\circ} \mathrm{C}\right)$, the absolute scales start from zero value while the relative scales show negative and positive values.

The relationships between the temperature scales are as follow:

$$
\begin{gather*}
T\left({ }^{\circ} C\right)=T\left({ }^{\circ} K\right)-273  \tag{1.1}\\
T\left({ }^{\circ} R\right)=1.8 T\left({ }^{\circ} K\right)  \tag{1.2}\\
T\left({ }^{\circ} F\right)=T\left({ }^{\circ} R\right)-460  \tag{1.3}\\
T\left({ }^{\circ} C\right)=\frac{T\left({ }^{\circ} F\right)-32}{1.8}  \tag{1.4}\\
T\left({ }^{\circ} R\right)=T\left({ }^{\circ} F\right)+460 \tag{1.5}
\end{gather*}
$$



Fig. (1.1): Comparison of temperature scales.

## Example 1.1:

A temperature of body is measured to be $\left(26^{\circ} \mathrm{C}\right)$. Determine the temperature in ${ }^{\circ} \mathrm{R},{ }^{\circ} \mathrm{K}$ and ${ }^{\circ} \mathrm{F}$.

## Solution:

$$
\begin{gathered}
T\left({ }^{\circ} \mathrm{F}\right)=1.8^{\circ} \mathrm{C}+32=1.8(26)+32=78.8^{\circ} \mathrm{F} \\
T\left({ }^{\circ} \mathrm{K}\right)=T\left({ }^{\circ} \mathrm{C}\right)+273=26+273=299^{\circ} \mathrm{K} \\
T\left({ }^{\circ} \mathrm{R}\right)=T\left({ }^{\circ} \mathrm{F}\right)+460=78.8^{\circ} \mathrm{F}+460=538.8^{\circ} \mathrm{R}
\end{gathered}
$$

### 1.4 Equation of State:

An analytic expression relating pressure to temperature and volume is called an "Equation of State" (EoS).

### 1.4.1 Ideal Gas Law:

The ideal gas is defined as the one in which:
1- The Volume occupied by the molecules is small compared to the total volume.
2- All molecular collisions are elastic.
3- There are no attractive or repulsive forces among the molecules.
Ideal gas law is as follow:

$$
\begin{equation*}
p V=n R T \tag{1.6}
\end{equation*}
$$

where, $\mathrm{p}=$ absolute pressure, $p$ sia.
$\mathrm{V}=$ volume, $f t^{3}$.
$\mathrm{T}=$ absolute temperature, ${ }^{\circ} R$.
$\mathrm{n}=$ number of moles of gas, lb-mole.
$\mathrm{R}=$ the universal gas constant, which, for the above units, has the value 10.730 psia.ft $^{3} / l b-m o l e .{ }^{\circ} R$.

The Eq.(1.6) of ideal gas law is a combination of several fundamentals gas laws:

1- Boyle's law.
2- Charles's law.
3- Gay-Lussac's law.
4- Avogadro's law.

### 1.4.1.1 Boyle's Law:

Boyle's law is based on observations made around 1660 that for a fixed mass of gas at a fixed temperature, the volume of gas is inversely proportional to the pressure exerted as shown in Fig. (1.2).

$$
\begin{gather*}
V \propto \frac{1}{p} \quad ; T=\text { constant }  \tag{1.7}\\
p V=\text { constant }  \tag{1.8}\\
p_{1} V_{1}=p_{2} V_{2}  \tag{1.9}\\
V_{2}=\frac{p_{1} V_{1}}{p_{2}} \tag{1.10}
\end{gather*}
$$

## Example 1.2:

A quantity of gas at a pressure ( 1 atm ) has a volume of $(1 \mathrm{~L})$. If the gas is compressed to volume of $(0.5 \mathrm{~L})$. Determine the new pressure assuming the temperature is constant.

## Solution:

$$
\begin{gathered}
p_{1}=1 \mathrm{~atm}, \quad V_{1}=1 \mathrm{lt}, \quad V_{2}=0.5 \mathrm{lt}, \quad p_{2=? ?} \\
p_{2}=\frac{p_{1} V_{1}}{V_{2}}=\frac{(1 \mathrm{~atm})(1 \mathrm{lt})}{(0.5 \mathrm{lt})}=2 \mathrm{~atm}
\end{gathered}
$$



Fig. (1.2): shows Boyle's law principle.

### 1.4.1.2 Charles's Law:

Over a century later (1787), Charles observed that for a fixed mass of gas at constant pressure, the volume varies linearly with temperature, as shown in Fig. (1.3).

$$
\begin{gather*}
V \propto T \quad ; p=\text { constant }  \tag{1.11}\\
\frac{V}{T}=\text { constant }  \tag{1.12}\\
\frac{V_{1}}{T_{1}}=\frac{V_{2}}{T_{2}}  \tag{1.13}\\
V_{2}=\frac{V_{1} T_{2}}{T_{1}} \tag{1.14}
\end{gather*}
$$

## Example 1.3:

A given mass of gas has a volume of (500 ft ${ }^{3}$ ) when the temperature is $\left(50{ }^{\circ} \mathrm{F}\right)$ and the pressure is ( 10 psig ). If the pressure remains constant, but the temperature
is changed to $\left(100^{\circ} \mathrm{F}\right)$, what will be the volume of the gas?

## Solution:

$$
\begin{gathered}
T_{1}=50^{\circ} \mathrm{F}+460=510^{\circ} \mathrm{R} \\
T_{2}=100^{\circ} \mathrm{F}+460=560^{\circ} \mathrm{R} \\
V_{2}=\frac{V_{1} T_{2}}{T_{1}}=\frac{\left(500 f t^{3}\right)\left(560^{\circ} \mathrm{R}\right)}{\left(510^{\circ} \mathrm{R}\right)}=549.02 \mathrm{ft}^{3}
\end{gathered}
$$



Fig. (1.3): Shows Charles law principle.

### 1.4.1.3 Gay-Lussac's Law:

This law was found by Joseph Louis Gay-Lussac in 1809. It states that, for a given mass and constant volume of an ideal gas, the pressure exerted on the sides of its container is directly proportional to its absolute temperature.

$$
\begin{gather*}
p \propto T \quad ; V=\text { constant }  \tag{1.15}\\
\frac{p}{T}=\text { constant }  \tag{1.16}\\
\frac{p_{1}}{T_{1}}=\frac{p_{2}}{T_{2}}  \tag{1.17}\\
p_{2}=\frac{p_{1} T_{2}}{T_{1}} \tag{1.18}
\end{gather*}
$$

## Example 1.4:

A given mass of gas has a volume of ( 500 cu ft ) when the temperature is $\left(50^{\circ} \mathrm{F}\right)$ and the pressure is ( 10 psig ). What would be the new pressure of the gas if the volume remains the same and the temperature is changed from $\left(50^{\circ} \mathrm{F}\right)$ to $(100$ $\left.{ }^{\circ} \mathrm{F}\right)$ ?

## Solution:

$$
\begin{gathered}
p_{1}=10+14.73=24.73 \mathrm{psia} \\
T_{1}=50^{\circ} \mathrm{F}+460=510^{\circ} \mathrm{R} \\
T_{2}=100^{\circ} \mathrm{F}+460=560^{\circ} \mathrm{R} \\
p_{2}=\frac{p_{1} T_{2}}{T_{1}}=\frac{(24.73)(560)}{(510)}=27.15 \mathrm{psia}
\end{gathered}
$$

### 1.4.1.4 The Combined Gas Law:

From the combination of eq.(1.8), eq.(1.12) and eq.(1.16) we get the following equations:

$$
\begin{align*}
& \frac{p V}{T}=\text { constant }  \tag{1.19}\\
& \frac{p_{1} V_{1}}{T_{1}}=\frac{p_{2} V_{2}}{T_{2}} \tag{1.20}
\end{align*}
$$

## Example 1.5:

Find the volume of a gas at STP when ( 2 liters) is collected at ( 745 mmHg ) and $\left(25{ }^{\circ} \mathrm{C}\right.$ ). (note: STP refers to standard temperature and pressure which is $273^{\circ} \mathrm{K}$ and 760 mmHg ).

## Solution:

$\begin{array}{lll}\mathrm{p}_{1}=745 \mathrm{mmHg} & \mathrm{V}_{1}=2 \mathrm{~L} & \mathrm{~T}_{1}=298^{\circ} \mathrm{K} \\ \mathrm{p}_{2}=760 \mathrm{mmHg} & \mathrm{V}_{2}=? ? & \mathrm{~T}_{2}=273^{\circ} \mathrm{K}\end{array}$

$$
\begin{gathered}
\frac{p_{1} V_{1}}{T_{1}}=\frac{p_{2} V_{2}}{T_{2}} \\
V_{2}=\frac{p_{1} V_{1} T_{2}}{p_{2} T_{1}}=\frac{(745)(2)(273)}{(760)(298)}=1.5 \mathrm{lt}
\end{gathered}
$$

(Note: by SPE standard conditions mean $60^{\circ} \mathrm{F}$ and 14.7 psia ).

### 1.4.1.5 Avogadro's Law:

Under the same conditions of P and T , equal volumes of all ideal gases contain the same number of molecules of ideal gas. Or at a given $\mathrm{P} \& \mathrm{~T}$ one molecular weight of any ideal gas occupies the same volume as one molecular weight of another ideal gas. Avogadro's number is $\left(6.023 \times 10^{23} \mathrm{~g}\right.$-mole, $6.023 \times 10^{26} \mathrm{~kg}$ mole or $\mathbf{2 . 7 3 3 \times 1 0} \mathbf{~ 2 6} \mathbf{~ l b}$-mole).
One molecular weight in pounds of any ideal gas at $60^{\circ} \mathrm{F} \& 14.7$ psia occupies a volume of 379.4 SCF, and so on as shown in Fig. (1.4).

$$
\begin{gather*}
V \propto n  \tag{1.21}\\
\frac{V}{n}=\text { constant }  \tag{1.22}\\
\frac{V_{1}}{n_{1}}=\frac{V_{2}}{n_{2}} \tag{1.23}
\end{gather*}
$$


(i)


63

(0)

Fig. (1.4): Shows the idea of Avogadro's law.

## Example 1.6:

If you have a ( 5 lt ) of a gas which contains ( 0.965 mole) of molecules. What will be the new volume of the gas if the quantity is increased to ( 1.80 mole), assuming pressure and temperature are hold constant?

## Solution:

$$
\begin{array}{lcc}
V_{1}=5 L, & n_{1}=0.965 \text { mole }, & n_{2}=1.8 \text { mole } \\
V_{2}=\frac{V_{1} n_{2}}{n_{1}}=\frac{(5)(1.8)}{(0.965)}=9.32 l t
\end{array}
$$

### 1.4.1.6 Molecular Weight (Molar Mass):

It is the summation of all atomic weights of its constituent atoms. And it's also equal to mass divided by number of moles.

$$
\begin{gather*}
M_{w}=\frac{\text { mass }}{\text { number of moles }}=\frac{m}{n}  \tag{1.24}\\
n=\frac{m}{M_{w}} \tag{1.25}
\end{gather*}
$$

Also:

$$
\begin{equation*}
n=\frac{\text { Number of Atoms }}{\text { Avogadro Number }}=\frac{\text { Number of Atoms }}{6.023 \times 10^{23}} \tag{1.26}
\end{equation*}
$$

## Example 1.7:

Find (n) for $(9.6 \mathrm{~g})$ of $\mathrm{SO}_{2}$.

## Solution:

$$
\begin{gathered}
\mathrm{m}=9.6 \mathrm{~g} \quad \mathrm{M}_{\mathrm{w}}=? ? \\
M_{w}=(1 \times 32)+(2 \times 16)=64 \frac{\mathrm{~g}}{\text { mole }} \\
n=\frac{m}{M_{w}}=\frac{9.6(g)}{64\left(\frac{g}{m o l e}\right)}=0.15 \text { mole (in gram case) }
\end{gathered}
$$

### 1.4.1.7 The Universal Gas Constant ( $R$ ):

The ideal gas constant is a universal constant that we use to quantify the relationship between the properties of a gas. From relationships in equations (1.7), (1.11), (1.15) and (1.21), the volume of the ideal gas increases with:

1- Increasing of temperature and mole number of the gas.
2- And, decreasing of pressure.
So:

$$
\begin{align*}
& V \propto n \frac{T}{p}  \tag{1.27}\\
& V=R n \frac{T}{p} \tag{1.28}
\end{align*}
$$

Where $(R)$ is a constant that has the same value for all gases. Due to Avogadro's law any gas occupies $(22.4 \mathrm{~L})$ at $\left(273{ }^{\circ} \mathrm{K}\right)$ and ( $\left.1 \mathrm{~atm}=14.73 \mathrm{psia}\right)$ pressure, Therefor:

$$
R=\frac{p V}{n T}=\frac{(1 \mathrm{~atm})(22.4 \mathrm{~L})}{(1 \mathrm{~mole})\left(273^{\circ} \mathrm{K}\right)}=0.082 \frac{\mathrm{~atm} . \mathrm{L}}{\mathrm{~mole} . \mathrm{K}}
$$

Table 1.1 show the different R values at different units of pressure, temperature, volume and mole number.

Table (1.1): Values of the universal gas constant, $R$.

| p | V | T | N | R |
| :--- | :--- | :--- | :--- | :--- |
| atm | liters | ${ }^{\circ} \mathrm{K}$ | Mole | $0.0821\left(\mathrm{~L} . \mathrm{atm} /{ }^{\circ} \mathrm{K} . \mathrm{mole}\right)$ |
| atm | cc | ${ }^{\circ} \mathrm{K}$ | Mole | $82.14\left(\mathrm{~cm}^{3} \cdot \mathrm{~atm} /{ }^{\circ} \mathrm{K} . \mathrm{g}-\mathrm{mole}\right)$ |
| psia | Cu ft | ${ }^{\circ} \mathrm{R}$ | lb-mole | $10.73\left(\mathrm{ft}^{3} . \mathrm{psia} /{ }^{\circ} \mathrm{R} . \mathrm{lb}-\mathrm{mole}\right)$ |
| $\mathrm{g} / \mathrm{cm}^{3}$ | cc | ${ }^{\circ} \mathrm{K}$ | Mole | $8.314\left(\mathrm{~J} /{ }^{\circ} \mathrm{K} . m o l e\right)$ |

## Example 1.8:

Four pounds of methane are placed in a tank at $\left(60^{\circ} \mathrm{F}\right)$. If the pressure on the tank is (100 psia). What is the volume of the tank? $\left(\mathrm{M}_{\mathrm{w}(\mathrm{CH} 4)}=16 \mathrm{lb} / \mathrm{lb}-\mathrm{mole}\right)$.

## Solution:

$$
\begin{gathered}
V=\frac{n R T}{p} \\
n=\frac{m}{M_{w}}=\frac{4 l b}{16\left(\frac{l b}{l b-m o l e}\right)}=0.25 \mathrm{lb}-\text { mole } \\
T=60^{\circ} \mathrm{F}+460=520^{\circ} \mathrm{R}
\end{gathered}
$$

$$
V=\frac{(0.25 l b-\text { mole })\left(10.73 f^{3} . p s i a . l b-\text { mole }^{-1 \circ} R^{-1}\right)\left(520^{\circ} R\right)}{(100 \text { psia })}=13.94 \mathrm{ft}^{3}
$$

### 1.4.1.8 Dalton's Law:

Each gas in a mixture of gases exerts a pressure equal to that which it would exert if it occupied the same volume as the total mixture. This pressure is called the partial pressure. The total pressure is the sum of the partial pressures, as shown in Fig. (1.5). This law is valid only when the mixture and each component of the mixture obey the ideal gas law. It is sometimes called the law of additive pressures.
According to Dalton's law, the total pressure is the sum of the partial pressures:

$$
\begin{gather*}
p_{T}=p_{1}+p_{2}+p_{3}  \tag{1.29}\\
p_{T}=\frac{n_{1} R T}{V}+\frac{n_{2} R T}{V}+\frac{n_{3} R T}{V}
\end{gather*}
$$

In the same container $\mathrm{R}, \mathrm{V} \& \mathrm{~T}$ are the same. So:

$$
\begin{gather*}
p_{T}=\frac{\left(n_{1}+n_{2}+n_{3}\right) R T}{V} \\
p_{T}=\frac{n_{T} R T}{V} \\
\frac{p_{1}}{p_{T}}=\frac{n_{1}}{n_{T}}  \tag{1.30}\\
p_{j}=p_{T} y_{j} \tag{1.31}
\end{gather*}
$$

$y_{j}$ is defined as the mole fraction of the $\mathrm{j}^{\mathrm{th}}$ component in the gas mixture.

$$
\begin{equation*}
y_{j}=\frac{n_{i}}{n_{T}} \tag{1.32}
\end{equation*}
$$



Fig. (1.5): Shows the idea of Dalton's law.

## Example 1.9:

Calculate the partial pressure exerted by methane in the following gas, when the gas is at a pressure of ( 750 psia ). Assume that the gas mixture is a mixture of ideal gases.

| Component | Composition mole fraction, $y_{j}$ |
| :---: | :---: |
| Methane | 0.85 |
| Ethane | 0.10 |
| Propane | 0.05 |
| Total | 1 |

## Solution:

$$
p_{\text {methane }}=p_{T} y_{\text {methane }}=(750 p \text { sia })(0.85)=638 p \text { sia }
$$

### 1.4.1.9 Amagat's Law:

The total volume of gaseous mixture is the sum of the volumes that each component would occupy at the given $p \& T$. The volumes occupied by the individual components are known as partial volumes. This law is correct only if the mixture and each component obey the ideal gas law.

$$
\begin{gather*}
V_{T}=V_{1}+V_{2}+V_{3}  \tag{1.33}\\
\frac{V_{1}}{V_{T}}=\frac{n_{1}}{n_{T}} \tag{1.34}
\end{gather*}
$$

### 1.4.2 Real Gas Law:

As mentioned earlier, ideal gas law assumes that:
1- Volume of gas molecules is negligible.
2- There is no intermolecular attractive or repulsive forces between gas molecules.

At low pressures and relatively high temperatures, real gases behavior likes ideal gases. But, as the pressure is increased, the ideal gas volume and pressure deduced from the ideal gas law needs correction factor called a compressibility factor (z).

$$
\begin{equation*}
p V=z n R T \tag{1.35}
\end{equation*}
$$

As shown in figure (1.6) we can see the following cases:

1. For one mole of gas at $\mathrm{SC}\left(\mathrm{V}_{\text {ideal }}=22.4 \mathrm{lt}=0.79 \mathrm{ft}^{3}\right)$ and for this case $(\mathrm{Z}$ $=1$ ).
2. If $\mathrm{V}_{\text {real }}$ is less than 22.4 lt , (at low pressure) then $\mathrm{Z}<1$. In low pressure range, a real volume is less than ideal volume because of attractive forces between molecules and the gas will be easy to liquefy.
3. If $\mathrm{V}_{\text {real }}$ is larger than 22.4 lt , (at high pressure) then $\mathrm{Z}>1$. In high pressure range, a real volume is greater than ideal volume because of repulsive forces between molecules and the gas will be difficult to liquefy.
$Z$ also called gas deviation factor, correction factor or simply the z-factor. Numerous equations-of-state have been developed in the attempt to correlate the pressure-volume-temperature variables for real gases with experimental data.

The gas compressibility factor z is a dimensionless quantity and is defined as the ratio of the actual (real) volume of n-moles of gas at $T$ and $p$ to the ideal volume of the same number of moles at the same $T$ and $p$ :

$$
z=\frac{V_{\text {real }}}{V_{\text {ideal }}}=\frac{V}{(n R T) / p}
$$

The Z-factor at standard conditions ( $60 \mathrm{~F}, 14.7 \mathrm{psia}$ ) is equated to (1), this case is for ideal gases also, and can be calculated or measured at other pressures and temperatures. Z-factor is $(<1)$ at low pressures and $(>1)$ at high pressures as shown in Fig. (1.6) for real gases.


Fig.(1.6): Z-factor vs. pressure for different gas components.
(Note: for $\mathrm{H}_{2} \mathrm{Z}>1$ over all pressures)

### 1.4.2.1 Van der Waals Equation:

Various attempts were made in the latter half of the $19^{\text {th }}$ Century to modify the ideal gas law to account for departure from ideal behavior. The first equation of state (EoS) capable of handling the transition from vapour to Liquid states was due to Van der Waales in 1873:

$$
\begin{equation*}
\left(p+\frac{a n^{2}}{V^{2}}\right)(V-n b)=n R T \tag{1.35}
\end{equation*}
$$

Where:
$a=$ attractive factor.
$b=$ bigness factor.
And when $n=1$,

$$
p=\frac{R T}{(V-b)}-\frac{a}{V^{2}}
$$

The parameter " a " was introduced to account for the attractive force between molecules, and the "b" parameters to account for the finite (non-zero) volume of the molecules. The numerical values of the constants " a " and " b " are specific to the gas.

At low pressure attractive force is present so the constant "b" is negligible $(b=0)$, then:

$$
\begin{equation*}
\left(p+\frac{a}{V^{2}}\right) V=R T \tag{1.36}
\end{equation*}
$$

At high pressure repulsive force is present and the constant "a" is negligible $(a=0)$, so:

$$
\begin{equation*}
p(V-b)=R T \tag{1.37}
\end{equation*}
$$

## Example 1.10:

Prove that the value of Z-factor is lower than one at low pressures and larger than one at high pressures.

## Solution:

By using the Van der Waals Eq.

1. at low pressures, equation (1.36).

$$
\left(p+\frac{a}{V^{2}}\right) V=R T
$$

$$
p V+\frac{a}{V}=R T
$$

By dividing the above equation by "RT":

$$
\frac{p V}{R T}+\frac{a}{V R T}=\frac{R T}{R T}
$$

By substituting ( $\mathrm{Z}=\mathrm{pV} / \mathrm{RT}$ ) :

$$
Z=1-\frac{a}{V R T}
$$

So:

$$
\mathrm{Z}<1
$$

## 2. at high pressures:

By using the Van der Waals equation. Eq. (1.37):

$$
\begin{aligned}
& p(V-b)=R T \\
& p V-p b=R T
\end{aligned}
$$

By dividing the above equation by "RT":

$$
\frac{p V}{R T}-\frac{p b}{R T}=\frac{R T}{R T}
$$

By substituting ( $\mathrm{Z}=\mathrm{pV} / \mathrm{RT}$ ):

$$
Z=1+\frac{p b}{R T}
$$

So:

$$
\mathrm{Z}>1
$$

Note:
There are several other equations of state (EoS) were published at the time of Van der Waal equation:

- Berthelot (1878):

$$
\begin{equation*}
\left(p+\frac{a}{T V^{2}}\right)(V-b)=R T \tag{1.38}
\end{equation*}
$$

- Redlich-Kwong (1949):

$$
\begin{equation*}
p=\frac{R T}{(V-b)}-\frac{a}{V(V+b)} \tag{1.39}
\end{equation*}
$$

- Clausius (1980):

$$
\begin{equation*}
\left(p+\frac{a}{T(V+c)^{2}}\right)(V-b)=R T \tag{1.40}
\end{equation*}
$$

## Problems

1. A quantity of gas at pressure of $(50 \mathrm{psig})$ has a volume of $\left(1000 \mathrm{ft}^{3}\right)$. If the gas is compressed to ( 100 psig ), what volume would it occupy? Assume the barometric pressure is ( 14.73 psia ) and the temperature of the gas remains constant.
2. A (600 ml) sample of Nitrogen is heated from $\left(27{ }^{\circ} \mathrm{C}\right)$ to $\left(77{ }^{\circ} \mathrm{C}\right)$ at constant pressure. What is the final volume?
3. A $(20 \mathrm{~L})$ cylinder containing ( 6 atm ) of gas at $\left(27^{\circ} \mathrm{C}\right)$. What would the pressure of the gas be if the gas was heated to $\left(77^{\circ} \mathrm{C}\right)$ ?
4. Calculate the content of a (500 $\mathrm{ft}^{3}$ ) tank of methane at (100 psia) and (100 ${ }^{\circ} \mathrm{F}$ ) in: moles, pounds, molecules and SCF.
5. How many SCF of an ideal gas are required to fill a $\left(100 \mathrm{ft}^{3}\right)$ tank to a pressure of ( 40 psig ) when the temperature of the gas in the tank is $(90$ $\left.{ }^{\circ} \mathrm{F}\right)$ ?
6. An ideal gas exerts ( 100 psig ) in a cylinder at $\left(100^{\circ} \mathrm{F}\right)$. What will the pressure be if the temperature is reduced to $\left(32^{\circ} \mathrm{F}\right)$ ?
7. A container was found to weight ( 80.0 g ) while evacuated. The container then was filled with oxygen and found to weight (81.242 g). When filled with water the container weighed $(1000.0 \mathrm{~g})$. The temperature and pressure for the procedure were ( 14.7 psia ) and ( 60 oF ). What is the gas constant in engineering units?

## REFERENCES

Bradley, H. B.,1987, "Petroleum Engineering Handbook", SPE, USA.
McCain, W. D., 1990. "The Properties of Petroleum Fluids", pennwell books, Tulsa, Oklahoma.

Peng, D. \& Robinson, D., 1976. A new two constant equation of state. Ind. Eng. Chem. Fundam. 15 (1), 59-64.

Redlich, O. \& Kwong, J., 1949. On the thermodynamics of solutions. An equation of state. Fugacities of gaseous solutions. Chem. Rev. 44, 233-247.

Soave, G., 1972. Equilibrium constants from a modified Redlich-Kwong equation of state. Chem. Eng. Sci. 27, 1197-1203.

Van der Waals, J.D., 1873. On the Continuity of the Liquid and Gaseous State. Sijthoff, Leiden, The Netherlands (Ph.D. dissertation).

## Chapter Two

## Natural Gas \& Crude Oil Properties

### 2.1 Natural Gas Properties:

Gas is defined as a homogeneous fluid of low viscosity and density, which has no definite volume but expands to completely fill the vessel in which it is placed. Generally, the natural gas is a mixture of hydrocarbon and nonhydrocarbon gases. The hydrocarbon gases normally found in a natural gas are methane, ethane (light gases), propane, butanes, pentanes, and small amounts of hexanes (intermediate gases) and heavier components (heptane, octane, ...etc). The nonhydrocarbon gases, that is, impurities, include carbon dioxide, hydrogen sulfide, and nitrogen.

Knowledge of pressure-volume-temperature (PVT) relationships and other physical and chemical properties of gases are essential for solving problems in natural gas reservoir engineering. The properties of interest include:

- Apparent molecular weight, $M_{a}$.
- Gas density, $\rho_{g}$.
- Specific gravity, $\gamma g$.
- Specific volume,$v$.
- Compressibility factor, $Z$.
- Isothermal gas compressibility coefficient, $c_{g}$.
- Gas formation volume factor, $B_{g}$.
- Gas expansion factor, $E_{g}$.
- Viscosity, $\mu_{g}$.


### 2.1.1 Apparent Molecular Weight (Ma) and Mole Fraction ( $\mathbf{y}_{\mathrm{i}}$ ):

One of the main gas properties frequently of interest to engineers is the apparent molecular weight. If $y_{i}$ represents the mole fraction of the $i^{\text {th }}$ component in a
gas mixture, the apparent molecular weight is defined mathematically by the following equation:

$$
\begin{equation*}
M_{a}=\sum_{i=1} y_{i} M_{i} \tag{2.1}
\end{equation*}
$$

where:
$M_{a}=$ apparent molecular weight of a gas mixture.
$M_{\mathrm{i}}=$ molecular weight of the $i^{\text {th }}$ component in the mixture.
$y_{i}=$ mole fraction of component $i$ in the mixture.
Conventionally, natural gas compositions can be expressed in three different forms: mole fraction, $y_{i}$, weight fraction, $w_{i}$, and volume fraction, $v_{i}$.

The mole fraction of a particular component, $i$, is defined as the number of moles of that component, $n_{i}$, divided by the total number of moles, $n$, of all the components in the mixture:

$$
\begin{equation*}
y_{i}=\frac{n_{i}}{n}=\frac{n_{i}}{\sum_{i} n_{i}} \tag{2.2}
\end{equation*}
$$

The weight fraction of a particular component, $i$, is defined as the weight of that component, $m_{\mathrm{i}}$, divided by the total weight of $m$, the mixture:

$$
\begin{equation*}
w_{i}=\frac{m_{i}}{m}=\frac{m_{i}}{\sum_{i} m_{i}} \tag{2.3}
\end{equation*}
$$

Similarly, the volume fraction of a particular component, $v_{i}$, is defined as the volume of that component, $V_{i}$, divided by the total volume of $V$, the mixture:

$$
\begin{equation*}
v_{i}=\frac{V_{i}}{V}=\frac{V_{i}}{\sum_{i} V_{i}} \tag{2.4}
\end{equation*}
$$

It is convenient in many engineering calculations to convert from mole fraction to weight fraction and vice versa. The procedure is given in the following steps:

1. As the composition is one of the intensive properties and independent of the quantity of the system, assume that the total number of gas is 1 ; that is, $n=1$.
2. From the definitions of mole fraction and number of moles

$$
\begin{aligned}
& y_{i}=\frac{n_{i}}{n}=\frac{n_{i}}{1}=n_{i} \\
& m_{i}=n_{i} M_{i}=y_{i} M_{i}
\end{aligned}
$$

3. From the above two expressions, calculate the weight fraction to give

$$
w_{i}=\frac{m_{i}}{m}=\frac{m_{i}}{\sum_{i} m_{i}}=\frac{y_{i} M_{i}}{\sum_{i} y_{i} M_{i}}=\frac{y_{i} M_{i}}{M_{a}}
$$

4. Similarly,

$$
y_{i}=\frac{w_{i} / M_{i}}{\sum_{i} w_{i} / M_{i}}
$$

## Example 2.1:

Dry air is a gas mixture consisting essentially of Nitrogen, Oxygen, and small amount of other gases. Compute the apparent molecular weight of air given its approximate composition as:

| Component | Mole Fraction $\left(y_{i}\right)$ | Molecular Weight |
| :---: | :---: | :---: |
| Nitrogen $\left(\mathrm{N}_{2}\right)$ | 0.78 | 28.01 |
| Oxygen $\left(\mathrm{O}_{2}\right)$ | 0.21 | 32 |
| Argon $(\mathrm{A})$ | 0.01 | 39.94 |
|  | $\sum=1.00$ |  |

## Solution:

$$
\begin{gathered}
M_{a}=\left(y_{N_{2}} \times M_{N_{2}}\right)+\left(y_{O_{2}} \times M_{O_{2}}\right)+\left(y_{A} \times M_{A}\right) \\
M_{a}=(0.78 \times 28.01)+(0.21 \times 32)+(0.01 \times 39.94)=28.97 \cong 29
\end{gathered}
$$

### 2.1.2 Gas Density, $\rho_{g}$ :

The density of an ideal gas mixture is calculated by simply replacing the molecular weight, $M$, of the pure component with the apparent molecular weight, $M_{a}$, of the gas mixture to give

$$
\begin{equation*}
\rho_{g}=\frac{p M_{a}}{R T} \tag{2.5}
\end{equation*}
$$

## Example 2.2:

Calculate the density of Methane at 1000 psia and $68^{\circ} \mathrm{F}$ in a cylinder with volume of 3.2 cu ft . assume that Methane is an ideal gas.

## Solution:

$$
\rho_{g}=\frac{p M_{a}}{R T}
$$

$$
\begin{gathered}
\left.\rho_{g}=\frac{(1000 \text { psia })\left(16.04 \frac{l b}{l b . m o l e}\right)}{\left(10.732 \frac{p s i a c u f t}{l b . m o l e}{ }^{\circ} R\right.}\right)\left(528^{\circ} R\right) \\
\rho_{g}=2.83 \frac{l b}{c u f t}
\end{gathered}
$$

### 2.1.3 Specific Gravity, $\gamma_{g}$ :

The specific gravity is defined as the ratio of the gas density to that of the air. Both densities are measured or expressed at the same pressure and temperature. Commonly, the standard pressure, $p_{s c}$, and standard temperature, $T_{s c}$, are used in defining the gas specific gravity.

$$
\begin{equation*}
\gamma_{g}=\frac{\text { gas density @ } 14.7 \text { and } 60^{\circ}}{\text { air density @ } 14.7 \text { and } 60^{\circ}}=\frac{\rho_{g}}{\rho_{\text {air }}} \tag{2.6}
\end{equation*}
$$

Assuming that the behavior of both the gas mixture and the air is described by the ideal gas equation, the specific gravity can be then expressed as

$$
\gamma_{g}=\frac{\frac{p_{s c} M_{a}}{R T_{s c}}}{\frac{p_{s c} M_{a i r}}{R T_{s c}}}
$$

or

$$
\begin{equation*}
\gamma_{g}=\frac{M_{a}}{M_{\text {air }}}=\frac{M_{a}}{28.96} \tag{2.7}
\end{equation*}
$$

where:
$\gamma_{\mathrm{g}}=$ gas specific gravity, $60^{\circ} / 60^{\circ}$.
$\rho_{\text {air }}=$ density of the air.
$\mathrm{M}_{\text {air }}=$ apparent molecular weight of the air $=28.96$.
$\mathrm{M}_{\mathrm{a}}=$ apparent molecular weight of the gas.
$\mathrm{P}_{\mathrm{sc}}=$ standard pressure, psia.
$\mathrm{T}_{\mathrm{sc}}=$ standard temperature, ${ }^{\circ} \mathrm{R}$.

## Example 2.3:

A gas well is producing gas with a specific gravity of (0.65) at a rate of (1.1 MMscf/day). The average reservoir pressure and temperature are ( 1500 psi ) and $\left(150^{\circ} \mathrm{F}\right)$. Calculate:

1. The apparent molecular weight of the gas.
2. The gas density at reservoir conditions.

## Solution:

From Eq. (2.7), solve for the apparent molecular weight:

$$
M_{a}=28.95 \gamma_{g}=(28.96)(0.65)=18.82
$$

Apply Eq. (2.5) to determine gas density:

$$
\rho_{g}=\frac{p M_{a}}{R T}=\frac{(1500)(18.82)}{(10.73)(610)}=4.31 \mathrm{lb} / \mathrm{ft}^{3}
$$

## Example 2.4:

A gas well is producing a natural gas with the following composition:

| Component | $\boldsymbol{y}_{\boldsymbol{i}}$ |
| :--- | ---: |
| CO 2 | 0.05 |
| C 1 | 0.90 |
| C 2 | 0.03 |
| C 3 | 0.02 |

Assuming an ideal gas behavior, calculate the apparent molecular weight, specific gravity, gas density at ( 2000 psia ) and $\left(150^{\circ} \mathrm{F}\right)$, and specific volume at (2000 psia) and ( $150^{\circ} \mathrm{F}$ ).

## Solution:

The table below describes the gas composition.

| Component | $\boldsymbol{y}_{\boldsymbol{i}}$ | $\boldsymbol{M}_{\boldsymbol{i}}$ | $\boldsymbol{y}_{\boldsymbol{i}} \boldsymbol{M}_{\boldsymbol{i}}$ |
| :--- | :--- | :--- | :--- |
| CO 2 | 0.05 | 44.01 | 2.200 |
| C 1 | 0.90 | 16.04 | 14.436 |
| C 2 | 0.03 | 30.07 | 0.902 |
| C 3 | 0.02 | 44.11 | 0.882 |
|  |  | $\boldsymbol{M}_{a}=18.42$ |  |

Apply Eq. (2.1) to calculate the apparent molecular weight:

$$
M_{a}=\sum_{i=1} y_{i} M_{i}=18.42
$$

Calculate the specific gravity by using Eq. (2.7):

$$
\gamma_{g}=\frac{M_{a}}{28.96}=\frac{18.42}{28.96}=0.636
$$

Solve for the density by applying Eq. (2.5):

$$
\rho_{g}=\frac{(2000)(18.42)}{(10.73)(610)}=5.628 \mathrm{lb} / f t^{3}
$$

### 2.1.4 Specific Volume, $\boldsymbol{v}$ :

The specific volume is defined as the volume occupied by a unit mass of the gas. For an ideal gas, this property can be calculated by applying Equation (2.8):

$$
\begin{equation*}
v=\frac{V}{m}=\frac{R T}{P M_{a}}=\frac{1}{\rho} \tag{2.8}
\end{equation*}
$$

### 2.1.5 Critical Temperature and Pressure:

Typical PVT relationships for a pure fluid are illustrated in fig. (2.1). the curve segment B-C-A defines the limits of vapor-liquid co-existence. B-C being the bubble point curve of the liquid and C-A, the dew point curve of the vapor. Any combinations of temperature, pressure and volume above that line segment indicates that the fluid exists in a single phase.

### 2.1.5.1 Single-Component Systems:

Suppose a fixed quantity of a pure component is placed in a cylinder fitted with a frictionless piston at a fixed temperature $T_{l}$. Furthermore, consider the initial pressure exerted on the system to be low enough that the entire system is in the vapor state. This initial condition is represented by point $E$ on the pressurevolume phase diagram ( $p-V$ diagram) as shown in Fig. (2.1). Consider the following sequential experimental steps taking place on the pure component:

1. The pressure is increased isothermally by forcing the piston into the cylinder. Consequently, the gas volume decreases until it reaches point $F$ on the diagram, where the liquid begins to condense. The corresponding pressure is known as the dew point pressure, $\boldsymbol{p}_{d}$, and defined as the pressure at which the first droplet of liquid is formed.
2. The piston is moved further into the cylinder as more liquid condenses from the present gas. This condensation process is characterized by a constant pressure and represented by the horizontal line $F G$. At point $G$, traces of gas remain and the corresponding pressure is called the bubble point pressure, $\boldsymbol{p}_{b}$, and defined as the pressure at which the first sign of
gas formation is detected. A characteristic of a single-component system is that, at a given temperature, the dew point pressure and the bubble point pressure are equal.
3. As the piston is forced slightly into the cylinder, a sharp increase in the pressure (point $H$ ) is noted without an appreciable decrease in the liquid volume. That behavior evidently reflects the low compressibility of the liquid phase.


Fig. (2.1): Typical pressure-volume diagram for pure component.
4. By repeating these steps at progressively increasing temperatures, a family of curves of equal temperatures (isotherms) is constructed as shown in Fig. (3.1). Note that, as the temperature increases, the length of the straight-line portion of the isotherm decreases until it eventually vanishes and the isotherm merely has a horizontal tangent and inflection point at the critical point. This isotherm temperature is called the critical temperature, $T_{c}$, of that single component.
5. The dashed curve connecting the dew points, called the dew point curve (line $F C$ ), represents the states of the "saturated gas." The dashed curve connecting the bubble points, called the bubble point curve (line GC), similarly represents the "saturated liquid." These two curves meet a point $C$, which is known as the critical point. The corresponding pressure and volume are called the critical pressure, $p_{c}$, and critical volume, $V_{c}$, respectively. Referring to Fig. (3.1), the area enclosed by the area $A F C G B$ is called the two-phase region or the phase envelope. Within this defined region, vapor and liquid can coexist in equilibrium. Outside the phase envelope, only one phase can exist.
6. The critical point (point $C$ ) describes the critical state of the pure component and represents the limiting state for the existence of two phases, that is, liquid and gas. In other words, for a single-component system, the critical point is defined as the highest value of pressure and temperature at which two phases can coexist. A more generalized definition of the critical point, which is applicable to a single- or multicomponent system, is that, the critical point is the point at which all intensive properties of the gas and liquid phases are equal.

Another means of presenting the results of this experiment is shown in Fig. (2.2), in which the pressure and temperature of the system are the independent parameters. Fig. (2.2) shows a typical pressure-temperature diagram ( $p-T$ diagram) of a single-component system with solids lines that clearly represent three different phase boundaries: vapor-liquid, vaporsolid, and liquid-solid phase separation boundaries. As shown in the illustration, line $A C$ terminates at the critical point (point $C$ ) and can be thought of as the dividing line between the areas where liquid and vapor exist. The curve is commonly called the vapor pressure curve or the boiling point curve. The corresponding pressure at any point on the curve is called the vapor pressure, $p_{v}$, with a corresponding temperature termed the boiling point temperature.

The lower end of the vapor pressure line is limited by the triple point $A$. This point represents the pressure and temperature at which solid, liquid, and vapor coexist under equilibrium conditions. The line $A B$ is called the sublimation pressure curve of the solid phase, and it divides the area where solid exists from the area where vapor exists. Points above $A B$ represent solid systems, and those below $A B$ represent vapor systems. The line $A D$ is called the melting curve or fusion curve and represents the change of melting-point temperature with pressure. The fusion (melting) curve divides the solid phase area from the liquid phase area, with a corresponding temperature at any point on the curve termed the fusion or melting-point temperature. Note that the solid-liquid curve (fusion curve) has a steep slope, which indicates that the triple point for most fluids is close to their normal melting-point temperatures. For pure hydrocarbons, the melting point generally increases with pressure so the slope of the line $A D$ is positive. Water is the exception in that its melting point decreases with pressure, so in this case, the slope of the line $A D$ is negative.

Each pure hydrocarbon has a $p-T$ diagram similar to that shown in Fig. (2.2). In addition, each pure component is characterized by its own vapor pressures, sublimation pressures, and critical values, which are different for each substance, but the general characteristics are similar. If such a diagram is available for a given substance, it is obvious that it could be used to predict the behavior of the substance as the temperature and pressure are changed.


Fig. (2.2): Typical pressure-temperature diagram for a single-component system.

Critical constants for some of the commonly occurring hydrocarbons and other components of natural gas can be found in Table (2.1).

Table (2.1): Physical Properties for Pure Components

| Compound | Formula | Molar <br> Mass <br> (Molecular Weight) | Critical Constants |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Pressure <br> (psia) | Temperature $\left({ }^{\circ} \mathbf{F}\right)$ | $\begin{aligned} & \text { Volume } \\ & \text { (ft3/lb-m) } \end{aligned}$ |
| Methane | CH 4 | 16.043 | 666.4 | -116.67 | 0.0988 |
| Ethane | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 30.070 | 706.5 | 89.92 | 0.0783 |
| Propane | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 44.097 | 616 | 206.06 | 0.0727 |
| Isobutane | $\mathrm{C}_{4} \mathrm{H}_{10}$ | 58.123 | 527.9 | 274.46 | 0.0714 |


| n-Butane | $\mathrm{C} 4 \mathrm{H}_{10}$ | 58.123 | 550.6 | 305.62 | 0.0703 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| isopentane | $\mathrm{C}_{5} \mathrm{H}_{12}$ | 72.150 | 490.4 | 369.10 | 0.0679 |
| n-Pentane | $\mathrm{C}_{5} \mathrm{H}_{12}$ | 72.150 | 488.6 | 385.8 | 0.0675 |
| Neopentane | $\mathrm{C}_{5} \mathrm{H}_{12}$ | 72.150 | 464.0 | 321.13 | 0.0673 |
| n -Hexane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.177 | 436.9 | 453.6 | 0.0688 |
| 2-Methylpentane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.177 | 436.6 | 435.83 | 0.0682 |
| 3-Methylpentane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.177 | 453.1 | 448.4 | 0.0682 |
| Neohexane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.177 | 446.8 | 420.13 | 0.0667 |
| 2.3-Dimethylbutane | $\mathrm{C}_{6} \mathrm{H}_{14}$ | 86.177 | 453.5 | 440.29 | 0.0665 |
| n-Heptane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 396.8 | 512.7 | 0.0691 |
| 2-Methylhexane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 396.5 | 495.00 | 0.0673 |
| 3-Methylhexane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 408.1 | 503.80 | 0.0646 |
| 3-Ethylpentane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 419.3 | 513.39 | 0.0665 |
| 2.2-Dimethylpetane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 402.2 | 477.23 | 0.0665 |
| 2.4-Dimethylpentane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 396.9 | 475.95 | 0.0668 |
| 3.3-Dimethylpentane | $\mathrm{C} 7 \mathrm{H}_{16}$ | 100.204 | 427.2 | 505.87 | 0.0662 |
| Triptane | $\mathrm{C}_{7} \mathrm{H}_{16}$ | 100.204 | 428.4 | 496.44 | 0.0636 |
| n-Octane | C 8 H 18 | 114.231 | 360.7 | 564.22 | 0.0690 |
| Dilsobutyl | $\mathrm{C}_{8} \mathrm{H}_{18}$ | 114.231 | 360.6 | 530.44 | 0.0676 |
| Isooctane | $\mathrm{C}_{8} \mathrm{H}_{18}$ | 114.231 | 372.4 | 519.46 | 0.0656 |
| n-Nonane | $\mathrm{C}_{\mathrm{C}}^{\mathrm{H}} 2 \mathrm{O}$ | 128.258 | 331.8 | 610.68 | 0.0684 |
| n-Decane | $\mathrm{C}_{8} \mathrm{H}_{22}$ | 14.285 | 305.2 | 652.0 | 0.0679 |
| Cyclopentane | $\mathrm{C}_{5} \mathrm{H}_{10}$ | 70.134 | 653.8 | 461.2 | 0.0594 |
| Methylcyclopentane | $\mathrm{C}_{6} \mathrm{H}_{12}$ | 84.161 | 548.9 | 499.35 | 0.0607 |
| Cyclohexane | $\mathrm{C}_{6} \mathrm{H}_{12}$ | 84.161 | 590.8 | 536.6 | 0.0586 |
| Methyl cyclohexane | $\mathrm{C}_{7} \mathrm{H}_{14}$ | 98.188 | 503.5 | 570.27 | 0.0600 |
| Carbon monoxide | CO | 28.010 | 507.5 | -220.43 | 0.0532 |
| Carbon dioxide | $\mathrm{CO}_{2}$ | 44.010 | 1071 | 87.91 | 0.0344 |
| Hydrogen sulfide | $\mathrm{H}_{2} \mathrm{~S}$ | 34.08 | 1300 | 212.45 | 0.0461 |
| Sulfur dioxide | $\mathrm{SO}_{2}$ | 64.06 | 1143 | 315.8 | 0.0305 |
| Ammonia | NH3 | 17.0305 | 1646 | 270.2 | 0.0681 |
| Air | $\mathrm{N}_{2}+\mathrm{O}_{2}$ | 2.0159 | 546.9 | -221.31 | 0.0517 |
| Hydrogen sulfide | $\mathrm{H}_{2}$ | 2.0159 | 188.1 | -399.9 | 0.5165 |
| Oxygen | $\mathrm{O}_{2}$ | 31.9988 | 731.4 | -181.43 | 0.0367 |
| Nitrogen | N2 | 28.0134 | 493.1 | -232.51 | 0.0510 |
| Chlorine | $\mathrm{Cl}_{2}$ | 70.906 | 1157 | 290.75 | 0.0280 |
| Water | $\mathrm{H}_{2} \mathrm{O}$ | 18.0153 | 3198.8 | 705.16 | 0.0497 s |
| Helium | He | 4.0026 | 32.99 | -450.31 | 0.2300 |
| Hydrogen chloride | HCl | 36.461 | 1205 | 124.77 | 0.0356 |

Source: Courtesy of the Gas Processors Suppliers Association. Published in the GPSA Engineering Data Book, 10th edition, 1987.

### 2.1.5.2 Two-Component Systems:

A unique feature of the single-component system is that, at a fixed temperature, two phases (vapor and liquid) can exist in equilibrium at only the vapor pressure. For a binary system, two phases can exist in equilibrium at various pressures at the same temperature. The following discussion concerning the description of the phase behavior of a two-component system involves many concepts that apply to the more complex multicomponent mixtures of oils and gases.

An important characteristic of binary systems is the variation of their thermodynamic and physical properties with the composition. Therefore, it is necessary to specify the composition of the mixture in terms of mole or weight fractions. It is customary to designate one of the components as the more volatile component and the other the less volatile component, depending on their relative vapor pressure at a given temperature.

Suppose that the examples previously described for a pure component are repeated, but this time we introduce into the cylinder a binary mixture of a known overall composition " $z_{i}$." Consider that the initial pressure $p_{l}$ exerted on the system, at a fixed temperature of $T_{1}$, is low enough that the entire system exists in the vapor state. This initial condition of pressure and temperature acting on the mixture is represented by point 1 on the $p / V$ diagram of Fig. (2.3). As the pressure is increased isothermally, it reaches point 2 , at which point an infinitesimal amount of liquid is condensed. The pressure at this point is called the dew point pressure, $p_{d}$, of the mixture. It should be noted that, at the dew point pressure, the composition of the vapor phase is equal to the overall composition of the binary mixture. As the total volume is decreased by forcing the piston inside the cylinder, a noticeable increase in the pressure is observed as more and more liquid is condensed. This condensation process is continued until the pressure reaches point 3, at which point traces of gas remain. At point 3 , the corresponding pressure is called the bubble point pressure, $p_{b}$. Because, at the bubble point, the gas phase is only of infinitesimal volume, the composition of the liquid phase therefore is identical with that of the whole system. As the piston is forced further into the cylinder, the pressure rises steeply to point 4 with a corresponding decreasing volume.

Repeating the above process at progressively increasing temperatures, a complete set of isotherms is obtained on the $p / V$ diagram of Fig. (2.4) for a binary system consisting of $n$-pentane and $n$-heptane. The bubble point curve, as
represented by line $A C$, represents the locus of the points of pressure and volume at which the first bubble of gas is formed. The dew point curve (line $B C$ ) describes the locus of the points of pressure and volume at which the first droplet of liquid is formed. The two curves meet at the critical point (point $C$ ). The critical pressure, temperature, and volume are given by $p_{c}, T_{c}$, and $V_{c}$, respectively. Any point within the phase envelope (line $A C B$ ) represents a system consisting of two phases. Outside the phase envelope, only one phase can exist.


Fig. (2.3): Pressure-volume isotherm for a two-component system.


Fig. (2.4): Pressure-volume diagram for a binary system.

## Example 2.4:

Consider a mixture 47.6 weight percent of $n$-pentane and 52.4 weight percent $n$ heptane. Estimate the specific volume of the liquid at its bubble point at $400^{\circ} \mathrm{F}$. Also estimate the specific volume of the gas at its dew point at $400^{\circ} \mathrm{F}$.

## Solution:

From $400^{\circ} \mathrm{F}$ isotherm of figure (2.4):
$v$ of bubble point at $400^{\circ} \mathrm{F}$ and $332 \mathrm{psia}=0.036 \mathrm{cu} \mathrm{ft/lb}$
$v$ of dew point at $400^{\circ} \mathrm{F}$ and $275 \mathrm{psia}=0.249 \mathrm{cu} \mathrm{ft} / \mathrm{lb}$

### 2.1.5.3 Three-Component Systems:

The phase behavior of mixtures containing three components (ternary systems) is conveniently represented in a triangular diagram, such as that shown in Fig. (2.5). Such diagrams are based on the property of equilateral triangles that the sum of the perpendicular distances from any point to each side of the diagram is a constant and equal to the length on any of the sides. Thus, the composition $x_{i}$ of the ternary system as represented by point $A$ in the interior of the triangle of Fig. (3.5).


Fig. (2.5): Properties of the ternary diagram.

Typical features of a ternary phase diagram for a system that exists in the twophase region at fixed pressure and temperature are shown in Fig. (2.6). Any mixture with an overall composition that lies inside the binodal curve (phase envelope) will split into liquid and vapor phases. The line that connects the composition of liquid and vapor phases that are in equilibrium is called the tie line. Any other mixture with an overall composition that lies on that tie line will split into the same liquid and vapor compositions. Only the amounts of liquid and gas change as the overall mixture composition changes from the liquid side (bubble point curve) on the binodal curve to the vapor side (dew point curve). The liquid and vapor portions of the binodal curve (phase envelope) meet at the critical point (plait point), where the liquid and vapor phases are identical.


Fig. (2.6): Ternary phase diagram at a constant temperature and pressure for a system that forms a liquid and a vapor.

### 2.1.5.4 Multicomponent Systems:

The phase behavior of multicomponent hydrocarbon systems in the two-phase region, that is, the liquid-vapor region, is very similar to that of binary systems. However, as the system becomes more complex with a greater number of different components, the pressure and temperature ranges in which two phases lie increase significantly.

The conditions under which these phases exist are a matter of considerable practical importance. The experimental or the mathematical determinations of these conditions are conveniently expressed in different types of diagrams, commonly called phase diagrams (see chapter three).

### 2.1.5.5 Principle of Corresponding States:

According to the law of corresponding states, the deviation of a real gas from the ideal gas law is the same for different gases at the same corresponding conditions of reduced temperature $\left(T_{r}\right)$ and reduced pressure $\left(p_{r}\right)$. This law has been useful in correlating the properties of gases. This principle was developed because observers noticed that the behavior of pure gases was qualitatively similar when compared (on p-V plots, for instance as shown in figure (2.1)) even though the quantitative values of $p$ and $V$ were very dissimilar. The idea was advanced that the properties of substances could be correlated if they were all compared at "corresponding" values of $T$ and $p$, which could be referenced easily. In the application of the principle of corresponding states to a single-component gas, the critical state of the gas is used as the reference point. The following terms are used.

$$
\begin{align*}
p_{r} & =\frac{p}{p_{c}}  \tag{2.9}\\
T_{r} & =\frac{T}{T_{c}}  \tag{2.10}\\
V_{r} & =\frac{V}{V_{c}} \tag{2.11}
\end{align*}
$$

where:
$p_{r}=$ reduced pressure.
$T_{r}=$ reduced temperature.
$V_{r}=$ reduced volume.
$P_{c}=$ critical pressure.
$T_{c}=$ critical temperature.
$V_{c}=$ critical volume.

### 2.1.6 Determination of Z-factor Value:

There are two methods of finding Z-factor value:
1- Mathematically by using the law of real gases:

$$
p V=Z n R T
$$

2- by using correlations and charts:
a- Finding $Z$-Factor of known composition (A single component):

Compressibility factors of many pure compounds are available as functions of pressure in most handbooks dealing with gas properties (Standing \& Katz, 1942). Compressibility factor charts for Methane, Ethane and Propane are shown in figures (2.7), (2.8) and (2.9). While the principle of corresponding states is not entirely rigorous, its application has been used widely in the determination of gas volumes for engineering purposes. It also has application in the estimation of gas viscosities.


Fig. (2.7): Compressibility factor for methane. (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.24).


Fig. (2.8): Compressibility factor for ethane (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.26).


Fig. (2.9): Compressibility factor for propane (Brown, Katz, Oberfell, and Alden, Natural Gasoline and Volatile Hydrocarbons, 1948, p.27).

## Example2.5:

Calculate the mass of methane gas contained at 1000 psia and $68^{\circ} \mathrm{F}$ in a cylinder with volume of 3.20 cu ft . Do not assume that methane is an ideal gas.

## Solution:

$$
m=\frac{p M V}{z R T}
$$

$\mathrm{Z}=0.89$ from figure (3.7).

$$
\begin{gathered}
m=\frac{(1000 \mathrm{psia})\left(16.04 \frac{l b}{\mathrm{lb.mole}}\right)(3.20 \mathrm{cu} \mathrm{ft})}{(0.89)\left(10.732 \frac{\text { psia cu ft }}{\text { lb. mole }{ }^{\circ} \mathrm{R}}\right)\left(528^{\circ} \mathrm{R}\right)} \\
m=10.2 \mathrm{lb}
\end{gathered}
$$

Note: that the assumption that methane acts as an ideal gas in Example2.2 resulted in an error in mass of about $11 \%$.

## b- Finding Z-Factor of known composition (Mixture component):

In application of the principle of corresponding states to a mixture of gases (known composition), the true critical temperature and pressure for the gases cannot be used because the paraffinic hydrocarbon series does not strictly follow the principle as stated above. "Pseudocritical" temperature and pressure are defined for use in place of the true critical temperature and pressure to determine the compressibility factor for a mixture. The pseudocritical temperature and pseudocritical pressure normally are defined as the molar average critical temperature and pressure of the mixture components, thus:

$$
\begin{equation*}
p_{p c}=\sum y_{i} p_{c i} \tag{2.12}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{p c}=\sum y_{i} T_{c i} \tag{2.13}
\end{equation*}
$$

where:
$p_{p c}=$ pseudocritical pressure of the gas mixture.
$T_{p c}=$ pseudocritical temperature of the gas mixture.
$p_{c i}=$ critical pressure of Component $i$ in the gas mixture.
$T_{c i}=$ critical temperature of Component $i$ in the gas mixture.
$y_{i}=$ mole fraction of Component $i$ in the gas mixture.

$$
y_{i}=\frac{n_{1}}{n_{1}+n_{2}}+\frac{n_{2}}{n_{1}+n_{2}}
$$

The pseudocritical pressure and temperature are then used to determine the pseudoreduced conditions:

$$
\begin{equation*}
p_{p r}=\frac{p}{p_{p c}} \tag{2.14}
\end{equation*}
$$

where $p_{p r}$ is the pseudoreduced pressure, and:

$$
\begin{equation*}
T_{p r}=\frac{T}{T_{p c}} \tag{2.15}
\end{equation*}
$$

where $T_{p r}$ is the pseudoreduced temperature. These reduced conditions are used to determine the compressibility factor, z, from Fig. (2.10), which was developed by (Standing \& Katz, 1942) from data collected on methane and natural gases. The data used to develop Fig. (2.10) ranged up to 8,200 psia and $250^{\circ} \mathrm{F}$. Compressibility factors of high-pressure natural gases (10,000 to 20,000 psia) may be obtained from Fig. (2.11), which was developed by (Standing \& Katz, 1942). Figs. (2.12) and (2.13) may be used for low pressure applications after (Brown, et. al, 1948).


Fig. (2.10): Standing and Katz compressibility factors chart by reduced temperature and pressure.


Fig. (2.11): Compressibility factor for natural gases at pressures of 10,000 to 20,000 psia, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).


Fig. (2.12): Compressibility factors for natural gases near atmospheric pressures, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).


Fig. (2.13) Compressibility factors for natural gases at low reduced pressures, (from Katz, D.L. et al.: Handbook of Natural Gas Engineering, 1959).

## Example 2.6:

A gas reservoir has the following gas composition: the initial reservoir pressure and temperature are ( $3,000 \mathrm{psia}$ ) and $\left(180^{\circ} \mathrm{F}\right)$, respectively.

| Component | $\boldsymbol{y}_{\boldsymbol{i}}$ |
| :---: | :---: |
| $\mathbf{C O}_{\mathbf{2}}$ | 0.02 |
| $\mathbf{N}_{\mathbf{2}}$ | 0.01 |
| $\mathbf{C}_{\mathbf{1}}$ | 0.85 |
| $\mathbf{C}_{\mathbf{2}}$ | 0.04 |
| $\mathbf{C}_{\mathbf{3}}$ | 0.03 |
| $\mathbf{i}-\mathbf{C}_{\mathbf{4}}$ | 0.03 |
| $\mathbf{n}-\mathbf{C}_{\mathbf{4}}$ | 0.02 |

Calculate the gas compressibility factor under initial reservoir conditions.

## Solution:

Step 1. Finding $p_{c i} \& T_{c i}$ for each component from (Table 2.1 Physical properties for pure component):

Step 2. Converting $T_{c}$ unit from ( ${ }^{\circ} \mathrm{F}$ ) to ( ${ }^{\circ} \mathrm{R}$ ):
Step 3. Finding $p_{p c} \& T_{p c}$ by using the equations below:

$$
\begin{aligned}
p_{p c} & =\sum_{i=1} y_{i} p_{c i} \\
T_{p c} & =\sum_{i=1} y_{i} T_{c i}
\end{aligned}
$$

| Component | $y_{i}$ | $T_{c i}\left({ }^{\circ} R\right)$ | $y_{i} T_{c i}$ | $p_{c i}$ | $y_{i} p_{c i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.02 | 547.91 | 10.96 | 1071 | 21.42 |
| $\mathrm{~N}_{2}$ | 0.01 | 227.49 | 2.27 | 493.1 | 4.93 |
| $\mathrm{C}_{1}$ | 0.85 | 343.33 | 291.83 | 666.4 | 566.44 |
| $\mathrm{C}_{2}$ | 0.04 | 549.92 | 22.00 | 706.5 | 28.26 |
| $\mathrm{C}_{3}$ | 0.03 | 666.06 | 19.98 | 616.4 | 18.48 |
| $\mathrm{i}-\mathrm{C}_{4}$ | 0.03 | 734.46 | 22.03 | 527.9 | 15.84 |
| $\mathrm{n}-\mathrm{C}_{4}$ | 0.02 | 765.62 | 15.31 | 550.6 | 11.01 |
|  |  |  | $\mathrm{~T}_{\mathrm{pc}}=383.38$ |  | $\mathrm{p}_{\mathrm{pc}}=666.38$ |

Step 4. Calculate the pseudo-reduced pressure and temperature by applying the following Equations:

$$
\begin{aligned}
& P_{p r}=\frac{p}{p_{p c}}=\frac{3000}{666.38}=4.5 \\
& T_{p r}=\frac{T}{T_{p c}}=\frac{640}{383.38}=1.67
\end{aligned}
$$

Step 5. Determine the $z$-factor from the chart of (Standing and Katz compressibility factors chart):

$$
\mathrm{z}=0.85
$$

## c- Finding Z-Factor of unknown composition (Mixture component):

In cases where the composition of a natural gas is not available, the pseudocritical properties, $p_{p c}$ and $T_{p c}$, can be predicted solely from the specific gravity of the gas. Brown (Brown et al. 1948) presented a graphical method for a convenient approximation of the pseudocritical pressure and pseudocritical temperature of gases when only the specific gravity of the gas is available, see Fig. (2.14) (Standing, 1977) expressed this graphical correlation in the following mathematical forms.

For Case 1, natural gas systems,

$$
\begin{align*}
& T_{p c}=168+325 \gamma_{g}-12.5 \gamma_{g}^{2}  \tag{2.16}\\
& p_{p c}=677+15 \gamma_{g}-37.5 \gamma_{g}^{2} \tag{2.17}
\end{align*}
$$

For Case 2, gas-condensate systems,

$$
\begin{align*}
& T_{p c}=187+330 \gamma_{g}-71.5 \gamma_{g}^{2}  \tag{2.18}\\
& p_{p c}=706-51.7 \gamma_{g}-11.1 \gamma_{g}^{2} \tag{2.19}
\end{align*}
$$

where:
$T_{p c}=$ pseudocritical temperature, ${ }^{\circ} \mathrm{R}$.
$p_{p c}=$ pseudocritical pressure, psia.
$\gamma_{g}=$ specific gravity of the gas mixture.


Fig. (2.14): Pseudo-critical properties of natural gases, (Courtesy of GPSA and GPA Engineering Data Book, 10th Edition, 1987).

## Example 2.7:

A gas reservoir with unknown gas composition, the initial reservoir pressure and temperature are ( $3,000 \mathrm{psia}$ and $180^{\circ} \mathrm{F}$ ), respectively. Calculate the gas compressibility factor under initial reservoir conditions. Gas Molecular Wight is $\mathrm{Ma}=(20.23)$.

## Solution:

Step 1. Calculate the specific gravity of the gas:

$$
\gamma_{g}=\frac{M_{a}}{M_{\text {air }}}=\frac{20.23}{28.96}=0.699
$$

Step 2. Solve for the pseudo-critical properties:

$$
\begin{aligned}
& T_{p c}=168+325(0.699)-12.5(0.699)^{2}=389.1^{\circ} \mathrm{R} \\
& p_{p c}=677+15(0.699)-37.5(0.699)^{2}=669.2 \mathrm{psia}
\end{aligned}
$$

Also we can use the chart in Fig. (2.14) to find the pseudo-critical properties.
Step 3. Calculate $p_{p r}$ and $T_{p r}$ :

$$
\begin{aligned}
& p_{p r}=\frac{p}{p_{p c}}=\frac{3000}{669.2}=4.48 \\
& T_{p r}=\frac{T}{T_{p c}}=\frac{640}{389.1}=1.64
\end{aligned}
$$

Step 4. Determine the z-factor from the chart of Fig. (2.10):

$$
\mathrm{z}=0.845
$$

## d-Mixture composition with non-hydrocarbon Components:

In this case we use specific gravity $\left(\gamma_{g}\right)$ in chart of Fig. (2.14) to find ( $p_{p c}$ ) and $\left(T_{p c}\right)$ or using Standing correlations ( $2.16 \& 2.17$ ). After that we make corrections to these values because of the effect of non-hydrocarbon gases. It's known that natural gases frequently contain materials other than hydrocarbon components, such as nitrogen, carbon dioxide, and hydrogen sulfide. Hydrocarbon gases are classified as sweet or sour depending on the hydrogen sulfide content. Both sweet and sour gases may contain nitrogen, carbon dioxide, or both. A hydrocarbon gas is termed a sour gas if it contains 1 gram of $\mathrm{H}_{2} \mathrm{~S}$ per $100 \mathrm{ft}^{3}$ or 5.7 mg of $\mathrm{H}_{2} \mathrm{~S} / \mathrm{m}^{3}$ of natural gas.

The common occurrence of small percentages of nitrogen and carbon dioxide, in part, is considered in the correlations previously cited. Concentrations of up to $5 \%$ of these non-hydrocarbon components do not seriously affect accuracy. Errors in compressibility factor calculations as large as $10 \%$ may occur in higher concentrations of non-hydrocarbon components in gas mixtures.

Two methods were developed to adjust the pseudocritical properties of the gases to account for the presence of the n-hydrocarbon components: the Wichert-Aziz method and the Carr-Kobayashi-Burrows method.

## I- Wichert-Aziz's Correction Method:

Natural gases that contain $\mathrm{H}_{2} \mathrm{~S}$ and/or $\mathrm{CO}_{2}$ frequently exhibit different compressibility factors behavior than sweet gases. Wichert and Aziz (1972) developed a simple, easy-to-use calculation to account for these differences. This method permits the use of the Standing-Katz Z-factor chart Fig. (2.10), by using a pseudocritical temperature adjustment factor, $\varepsilon$, which is a function of the concentration of $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{~S}$ in the sour gas. This correction factor then is used to adjust the pseudocritical temperature and pressure according to the following expressions:

$$
\begin{gather*}
T_{p c}^{\prime}=T_{p c}-\varepsilon  \tag{2.20}\\
p_{p c}^{\prime}=\frac{p_{p c} T_{p c}^{\prime}}{T_{p c}+B(1-B) \varepsilon} \tag{2.21}
\end{gather*}
$$

Where:
$T_{p c}=$ pseudo-critical temperature, ${ }^{\circ} \mathrm{R}$.
$p_{p c}=$ pseudo-critical pressure, psia.
$T_{p c}^{\prime}=$ corrected pseudo-critical temperature, ${ }^{\circ} \mathrm{R}$.
$p_{p c}^{\prime}=$ corrected pseudo-critical pressure, psia.
$B=$ mole fraction of H2S in the gas mixture.
$\varepsilon=$ pseudo-critical temperature adjustment factor and is defined mathematically by the following expression.

$$
\begin{equation*}
\varepsilon=120\left[A^{0.9}-A^{1.6}\right]+15\left(B^{0.5}-B^{4.0}\right) \tag{2.22}
\end{equation*}
$$

Where the coefficient $A$ is the sum of the mole fraction $\mathrm{H}_{2} \mathrm{~S}$ and $\mathrm{CO}_{2}$ in the gas mixture:

$$
A=y_{H_{2} S}+y_{C O_{2}}
$$

The computational steps of incorporating the adjustment factor $\varepsilon$ into the Z-factor calculations are summarized next.

Step 1. Calculate the pseudocritical properties of the whole gas mixture by applying Eqs. (2.16), (2.17) or Eqs. (2.18), (2.19).

Step 2. Calculate the adjustment factor, $\varepsilon$, from Eq. (2.22).
Step 3. Adjust the calculated $p_{p c}$ and $T_{p c}$ (as computed in step 1) by applying Eqs. (2.20), (2.21).

Step 4. Calculate the pseudoreduced properties, $p_{p r}$ and $T_{p r}$, from Eqs. (2.12), (2.13).

Step 5. Read the compressibility factor from Fig. (2.10).

## Example 2.8:

A sour natural gas has a specific gravity of (0.7). The compositional analysis of the gas shows that it contains $5 \% \mathrm{CO}_{2}$ and $10 \% \mathrm{H}_{2} \mathrm{~S}$. Calculate the density of the gas at $(3,500 \mathrm{psia})$ and $\left(160^{\circ} \mathrm{F}\right)$.

## Solution:

Step 1. Calculate the uncorrected pseudo-critical properties of the gas:

$$
\begin{aligned}
& T_{p c}=168+325(0.7)-12.5(0.7)^{2}=389.38^{\circ} \mathrm{R} \\
& p_{p c}=677+15(0.7)-37.5(0.7)^{2}=669.1 \mathrm{psia}
\end{aligned}
$$

Step 2. Calculate the pseudo-critical temperature adjustment factor:

$$
\varepsilon=120\left(0.15^{0.9}-0.15^{1.6}\right)+15\left(0.1^{0.5}-0.1^{4}\right)=20.735
$$

Step 3. Calculate the corrected pseudo-critical temperature:

$$
T_{p c}^{\prime}=389.38-20.735=368.64
$$

Step 4. Adjust the pseudo-critical pressure $p_{p c}$ :

$$
p_{p c}^{\prime}=\frac{(669.1)(368.64)}{(389.38)+0.1(1-0.1)(20.635)}=630.44
$$

Step 5. Calculate $p_{p r}$ and $T_{p r}$ :

$$
\begin{gathered}
p_{p r}=\frac{3500}{630.44}=5.55 \\
T_{p r}=\frac{160+460}{36864}=1.68
\end{gathered}
$$

Step 6. Determine the z-factor from chart of Fig. (2.10):

$$
z=0.89
$$

Step 7. Calculate the apparent molecular weight of the gas:

$$
M_{a}=28.96 *\left(\gamma_{g}\right)=28.96 * 0.7=20.27
$$

Step 8. Solve for gas density:

$$
\rho=\frac{p M_{a}}{Z T R}=\frac{(3500)(20.27)}{(0.89)(10.73)(620)}=11.98 \mathrm{Ib} / \mathrm{ft}^{3}
$$

## II- Carr-Kobayashi-Burrows's Correction Method:

Carr et al. (1954) proposed a simplified procedure to adjust the pseudocritical properties of natural gases when nonhydrocarbon components are present. The method can be used when the composition of the natural gas is not available. The proposed procedure is summarized in the following steps.
Step 1. Knowing the specific gravity of the natural gas, calculate the pseudocritical temperature and pressure by applying Eqs. (2.16), (2.17).
Step 2. Adjust the estimated pseudocritical properties by using the following two expressions:

$$
\begin{array}{r}
T_{p c}^{\prime}=T_{p c}-80 y_{C O_{2}}+130 y_{H_{2} S}-250 y_{N_{2}} \\
p_{p c}^{\prime}=p_{p c}+440 y_{C O_{2}}+600 y_{H_{2} S}-170 y_{N_{2}} \tag{2.24}
\end{array}
$$

where:
$T_{p c}^{\prime}=$ the adjusted pseudocritical temperature, ${ }^{\circ} \mathrm{R}$.
$T_{p c}=$ the unadjusted pseudocritical temperature, ${ }^{\circ} \mathrm{R}$.
$y_{\mathrm{CO}_{2}}=$ mole fraction of $\mathrm{CO}_{2}$.
$y_{\mathrm{H}_{2} \mathrm{~S}}=$ mole fraction of $\mathrm{H}_{2} \mathrm{~S}$ in the gas mixture.
$y_{N_{2}}=$ mole fraction of nitrogen.
$p_{p c}^{\prime}=$ the adjusted pseudocritical pressure, psia.
$p_{p c}=$ the unadjusted pseudocritical pressure, psia.
Step 3. Use the adjusted pseudocritical temperature and pressure to calculate the pseudoreduced properties.
Step 4. Calculate the Z-factor from Fig. (2.10).

## Example 2.9:

Using the data in Example (2.8), calculate the density by employing the preceding correction procedure.

## Solution:

Step 1. After calculating the uncorrected $T_{p r}$ and $p_{p r}$ determine the corrected pseudocritical properties from Eqs. (2.23), (2.24):

$$
\begin{gathered}
T_{p c}^{\prime}=389.38-80(0.05)+130(0.10)-250(0)=398.3^{\circ} R \\
p_{p c}^{\prime}=669.1+440(0.05)+600(0.01)-170(0)=751.1 \text { psia }
\end{gathered}
$$

Step 2. Calculate $p_{p r}$ and $T_{p r}$ :

$$
\begin{aligned}
& p_{p r}=\frac{3500}{751.1}=4.56 \\
& T_{p r}=\frac{620}{398.38}=1.56
\end{aligned}
$$

Step 3. Determine the gas compressibility factor from Fig. (2.10):

$$
z=0.820
$$

Step 4. Calculate the gas density:

$$
\rho_{g}=\frac{p M_{a}}{Z R T}=\frac{(3500)(20.27)}{(0.82)(10.73)(620)}=13.0 \mathrm{lb} / f t^{3}
$$

## Note:

It should be noted that the Standing and Katz Z-factor chart result in unsatisfactory Z-factors for high-molecular weight reservoir gases heptanesplus fraction, $\mathrm{C}_{7}$, which their gravity is $\gamma_{g}>0.75$. Sutton (1985) and Stewart et al. (1959), developed a method of correction for high-molecular weight gases.

## $e$ - Direct Calculation of Z-factor (empirical correlations):

After four decades of existence, the Standing-Katz Z-factor chart is still widely used as a practical source of natural gas compressibility factors. As a result, there was an apparent need for a simple mathematical description of that chart. Several empirical correlations for calculating Z-factors have been developed over the years.

Numerous rigorous mathematical expressions have been proposed to accurately reproduce the Standing and Katz Z-factor chart. Most of these expressions are designed to solve for the gas compressibility factor at any $p_{p r}$ and $T_{p r}$ iteratively. Three of these methods are:

- Hall and Yarborough method (1973).
- Dranchuk, Purvis and Robinson (1974).
- Dranchuk and Abu-Kassem (1975).

For more details see (Ahmed, 2019)

### 2.1.7 Isothermal Gas Compressibility, $\boldsymbol{c}_{g}$ :

The coefficient of isothermal compressibility is defined as the fractional change of volume as pressure is changed at constant temperature. The defining equation is:

$$
\begin{equation*}
c_{g}=\frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_{T} \tag{2.25}
\end{equation*}
$$

where, $\mathrm{c}_{\mathrm{g}}=$ isothermal gas compressibility, $1 /$ psi.
Negative sign in the equation above refers to volume decreasing with pressure increasing.

This coefficient normally is referred to simply as compressibility or gas compressibility, whereas, the term compressibility factor refers to z-factor.

1- Isothermal compressibility factor for ideal gas given by driving the ideal gas law at constant temperature:

$$
\begin{gather*}
p V=n R T ; \quad T=\text { constant } \\
p \partial V+V \partial p=0 \\
p \partial V=-V \partial p \Rightarrow \frac{\partial V}{\partial p}=\frac{-V}{p} \tag{2.26}
\end{gather*}
$$

by substituting into Eq. (2.25):

$$
\begin{equation*}
c_{g}=\frac{-1}{V}\left(\frac{-V}{p}\right) \Rightarrow c_{g}=\frac{1}{p}\left(p s i^{-1}\right) \tag{2.27}
\end{equation*}
$$

2- Isothermal compressibility factor for real gas given by driving the real gas law at constant temperature:

$$
\begin{gathered}
p V=Z n R T ; \quad T=\text { constant } \\
p \partial V+V \partial p=n R T \partial Z
\end{gathered}
$$

by dividing the equation above by $(\partial p)$ :

$$
\begin{gather*}
p \frac{\partial V}{\partial p}+V \frac{\partial p}{\partial p}=n R T \frac{\partial Z}{\partial p} \\
p \frac{\partial V}{\partial p}=n R T \frac{\partial Z}{\partial p}-V \\
\frac{\partial V}{\partial p}=\frac{n R T}{p} \frac{\partial Z}{\partial p}-\frac{V}{p} \tag{2.28}
\end{gather*}
$$

by substituting ( $V=Z n R T / p$ ) from real gas law and Eq.(2.28) into Eq. (2.25):

$$
\begin{gathered}
c_{g}=\frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_{T} \\
c_{g}=\frac{-p}{Z n R T}\left(\frac{n R T}{p} \frac{\partial Z}{\partial p}-\frac{V}{p}\right)_{T} \\
c_{g}=\frac{-p}{Z n R T} \frac{n R T}{p} \frac{\partial Z}{\partial p}-\frac{-p}{Z n R T} \frac{V}{p} \\
c_{g}=\frac{V}{Z n R T}-\frac{1}{Z} \frac{\partial Z}{\partial p}
\end{gathered}
$$

By by substituting ( $V=Z n R T / p$ ) from real gas law in the last equation:

$$
\begin{gather*}
c_{g}=\frac{Z n R T}{p Z n R T}-\frac{1}{Z} \frac{\partial Z}{\partial p} \\
c_{g}=\frac{1}{p}-\frac{1}{Z} \frac{\partial Z}{\partial p} \tag{2.29}
\end{gather*}
$$

Equation (2.29) is the isothermal compressibility factor for real gas.

From the law of corresponding state, we know that:

$$
p_{p r}=\frac{p}{p_{p c}} \Rightarrow p=p_{p r} p_{p c}
$$

By substituting $p=p_{p r} p_{p c}$ from the law of corresponding state into Eq. (2.29):

$$
c_{g}=\frac{1}{p_{p r} p_{p c}}-\frac{1}{z}\left[\frac{\partial Z}{\partial\left(p_{p r} p_{p c}\right)}\right]_{T_{p r}}
$$

Multiplying this equation by $p_{p c}$ yields:

$$
\begin{equation*}
c_{g} p_{p c}=c_{p r}=\frac{1}{p_{p r}}-\frac{1}{Z}\left[\frac{\partial Z}{\partial p_{p r}}\right]_{T_{p r}} \tag{2.30}
\end{equation*}
$$

The term $c_{p r}$ is called the isothermal pseudoreduced compressibility, defined by the relationship:

$$
\begin{equation*}
c_{p r}=c_{g} p_{p c} \tag{2.31}
\end{equation*}
$$

where:
$c_{p r}=$ isothermal pseudoreduced compressibility.
$c_{g}=$ isothermal gas compressibility, $\mathrm{psi}^{-1}$.
$p_{p c}=$ pseudoreduced pressure, psi.

Trube (1957a,b) presented graphs from which the isothermal compressibility of natural gases may be obtained. The graphs, Fig. (2.15), (2.16) give the isothermal pseudoreduced compressibility as a function of pseudoreduced pressure and temperature.


Fig. (2.15): Trube's pseudoreduced compressibility for natural gases. (from Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355-357).


Fig. (2.16): Trube's pseudoreduced compressibility for natural gases, (from Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355-357).

## Example 2.10:

A hydrocarbon gas mixture has a specific gravity of (0.72). Calculate the isothermal gas compressibility coefficient at ( 2000 psia ) and $\left(140^{\circ} \mathrm{F}\right)$ assuming, first, an ideal gas behavior, then a real gas behavior.

## Solution:

Assuming an ideal gas behavior, determine $c_{g}$ by applying Eq. (2.27):

$$
c_{g}=\frac{1}{2000}=500 \times 10^{-6} p s i^{-1}
$$

Assuming a real gas behavior, use the following steps.
Step 1. Calculate $T_{p c}$ and $p_{p c}$ by applying Eqs. (2.16), (2.17):

$$
\begin{gathered}
T_{p c}=168+325(0.72)-12.5(0.72)^{2}=395.5^{\circ} R \\
p_{p c}=677+15(0.72)-37.5(0.72)^{2}=668.4 \text { psia }
\end{gathered}
$$

Step 2. Compute $p_{p r}$ and $T_{p r}$ from Eqs. (2.14), (2.15):

$$
\begin{aligned}
p_{p r} & =\frac{2000}{668.4}=2.99 \\
T_{p r} & =\frac{600}{395.5}=1.52
\end{aligned}
$$

Step 3. From Fig. (2.15), find $c_{p r}$ to give $\left(c_{p r}=0.36\right)$.
Step 4. Solve for $c_{g}$ by applying Eq. (2.31):

$$
c_{g}=\frac{c_{p r}}{p_{p c}}=\frac{0.36}{668.4}=539 \times 10^{-6} p s i^{-1}
$$

### 2.1.8 Gas Formation Volume Factor, $B_{g}$ :

The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions; that is, $60^{\circ} \mathrm{F}$ and 14.7 psia . This gas property is then defined as the actual volume occupied by a certain amount of gas at a specified pressure and temperature, divided by the volume occupied by the same amount of gas at standard conditions. In equation form, the relationship is expressed as:

$$
\begin{equation*}
B_{g}=\frac{(V)_{p, T}}{V_{s c}} \tag{2.32}
\end{equation*}
$$

where:
$B_{g}=$ gas formation volume factor, $f t^{3} / s c f$.
$V_{p, T}=$ volume of gas at pressure $p$ and temperature $T, f t^{3}$.
$V_{s c}=$ volume of gas at standard conditions.

Applying the real gas equation of state, $(\mathrm{pV}=\mathrm{ZnRT})$ and substituting for the volume V, gives:

$$
B_{g}=\frac{\frac{Z n R T}{p}}{\frac{Z_{s c} n R T_{s c}}{p_{s c}}}=\frac{p_{s c} Z T}{T_{s c} p}
$$

where:
$Z_{s c}=$ Z-factor at standard conditions is equal to (1.0).
$p_{s c}, T_{s c}=$ standard pressure and temperature.
Assuming that the standard conditions are represented by $p_{s c}=14.7 \mathrm{psia}$ and $T_{s c}=520$, the preceding expression can be reduced to the following relationship:

$$
\begin{equation*}
B_{g}=0.02827 \frac{Z T}{p}\left(f t^{3} / s c f\right) \tag{2.33}
\end{equation*}
$$

where:
$B_{g}=$ gas formation volume factor, $f t^{3} / s c f$.
$Z=$ gas compressibility factor.
$T=$ temperature, ${ }^{\circ} \mathrm{R}$.

In other field units, the gas formation volume factor can be expressed in $b b l / s c f$, by dividing it on $5.614 \mathrm{ft}^{3} /$ bbl to give:

$$
\begin{equation*}
B_{g}=0.005035 \frac{Z T}{p}(b b l / s c f) \tag{2.34}
\end{equation*}
$$

And also:

$$
\begin{equation*}
c_{g}=\frac{-1}{B_{g}}\left(\frac{\partial B_{g}}{\partial p}\right)_{T} \tag{2.35}
\end{equation*}
$$

### 2.1.9 Gas Expansion Factor, $\boldsymbol{E}_{g}$ :

The reciprocal of the gas formation volume factor, called the gas expansion factor, is designated by the symbol $E_{g}$ :

$$
E_{g}=\frac{1}{B_{g}}
$$

In terms of $s c f / f t^{3}$, the gas expansion factor is:

$$
\begin{equation*}
E_{g}=35.37 \frac{p}{Z T}, s c f / f t^{3} \tag{2.36}
\end{equation*}
$$

In other units:

$$
\begin{equation*}
E_{g}=198.6 \frac{p}{Z T}, s c f / b b l \tag{2.37}
\end{equation*}
$$

## Example 2.11:

A gas well is producing from a gas reservoir at an average pressure of (2000 psia) and a temperature of $\left(140^{\circ} \mathrm{F}\right)$. The specific gravity is $(0.72)$. Calculate the gas formation volume factor and gas expansion factor in $s c f / f t^{3}$ and $s c f / b b l$.

## Solution:

Step 1. Calculate the pseudocritical properties from Eqs. (2.16), (2.17), to give:

$$
\begin{aligned}
& T_{p c}=168+325(0.72)-12.5(0.72)^{2}=395.5^{\circ} R \\
& p_{p c}=677+15(0.72)-37.5(0.72)^{2}=668.4 p s i a
\end{aligned}
$$

Step 2. Calculate the $p_{p r}$ and $T_{p r}$ :

$$
\begin{aligned}
p_{p r} & =\frac{p}{p_{p c}}=\frac{2000}{668.4}=2.99 \\
T_{p r} & =\frac{T}{T_{p c}}=\frac{600}{395.5}=1.52
\end{aligned}
$$

Step 3. Determine the Z-factor from Fig. (2.10):

$$
Z=0.78
$$

Step 4. Calculate the gas formation volume factor from Eqs. (2.33), (2.34):

$$
\begin{aligned}
& B_{g}=0.02827 \frac{(0.78)(140+460)}{(2000)}=0.006615\left(f t^{3} / s c f\right) \\
& B_{g}=0.005035 \frac{(0.78)(140+460)}{(2000)}=0.001178(\mathrm{bbl} / \mathrm{scf})
\end{aligned}
$$

Step 5. Calculate the gas expansion factor from Eqs. (2.36), (2.37):

$$
\begin{aligned}
& E_{g}=35.37 \frac{(2000)}{(0.78)(600)}=151.15 \mathrm{scf} / \mathrm{ft}^{3} \\
& E_{g}=198.6 \frac{(2000)}{(0.78)(600)}=848.69 \mathrm{scf} / \mathrm{bbl}
\end{aligned}
$$

### 2.1.10 Gas Viscosity, $\mu_{g}$ :

The viscosity of a fluid is a measure of the internal fluid friction (resistance) to flow. If the friction between layers of the fluid is small, that is, low viscosity, an applied shearing force will result in a large velocity gradient. As the viscosity
increases, each fluid layer exerts a larger frictional drag on the adjacent layers and velocity gradient decreases.

The viscosity of a fluid is generally defined as the ratio of the shear force per unit area to the local velocity gradient. Viscosities are expressed in terms of poises, centipoises, or micropoises. One poise equals a viscosity of $1 \mathrm{dyn}-\mathrm{s} / \mathrm{cm}^{2}$ and can be converted to other field units by the following relationships:

$$
\begin{aligned}
& 1 \text { centipoise }=\mathrm{g} \text { mass } / 100 \mathrm{sec} \mathrm{~cm} \\
& 1 \text { poise }=100 \text { centipoises } \\
&=1 \times 10^{6} \text { micropoises } \\
&=6.72 \times 10^{-2}(\text { lb.mass } / \text { ft.sec }) \\
&=20.9 \times 10^{-3}\left(\text { lb.sec } / f t^{2}\right)
\end{aligned}
$$

The gas viscosity is not commonly measured in the laboratory because it can be estimated precisely from empirical correlations. Like all intensive properties, viscosity of a natural gas is completely described by the following function:

$$
\mu_{g}=\left(p, T, y_{i}\right)
$$

where, $\mu_{g}=$ the viscosity of the gas phase. This relationship simply states that the viscosity is a function of pressure, temperature, and composition. Many of the widely used gas viscosity correlations may be viewed as modifications of that expression.

There are two popular methods are commonly used to calculate the viscosity of natural gases in the petroleum industry:

1- Carr-Kobayashi-Burrows correlation.
2- Lee-Gonzalez-Eakin method.

## 1- Carr-Kobayashi-Burrows's Method:

Carr et al. (1954) developed graphical correlations for estimating the viscosity of natural gas as a function of temperature, pressure, and gas gravity. The computational procedure of applying the proposed correlations is summarized in the following steps:

Step 1. Calculate the pseudocritical pressure, pseudocritical temperature, and apparent molecular weight from the specific gravity or the composition of the natural gas. Corrections to these pseudocritical properties for the presence of the nonhydrocarbon gases $\left(\mathrm{CO}_{2}, \mathrm{~N}_{2}\right.$, and $\left.\mathrm{H}_{2} \mathrm{~S}\right)$ should be made if they are present in concentration greater than $5 \mathrm{~mol} \%$.

Step 2. Obtain the viscosity of the natural gas at one atmosphere and the temperature of interest from Fig. (2.17). This viscosity, as denoted by $\mu_{1}$, must be corrected for the presence of nonhydrocarbon components using the inserts of Fig. (2.17). The nonhydrocarbon fractions tend to increase the viscosity of the gas phase. The effect of nonhydrocarbon components on the viscosity of the natural gas can be expressed mathematically by the following relationship:

$$
\begin{equation*}
\mu_{1}=\left(\mu_{1}\right)_{\text {uncorrected }}+(\Delta \mu)_{N_{2}}+(\Delta \mu)_{C O_{2}}+(\Delta \mu)_{H_{2} S} \tag{2.38}
\end{equation*}
$$

where:
$\mu_{1}=$ "corrected" gas viscosity at 1 atm and reservoir temperature, cp .
$(\Delta \mu)_{N_{2}}=$ viscosity corrections due to the presence of $N_{2}$.
$(\Delta \mu)_{\mathrm{CO}_{2}}=$ viscosity corrections due to the presence of $\mathrm{CO}_{2}$.
$(\Delta \mu)_{H_{2} S}=$ viscosity corrections due to the presence of $\mathrm{H}_{2} \mathrm{~S}$.
$\left(\mu_{1}\right)_{\mathrm{uncorrected}}=$ uncorrected gas viscosity, cp .

Step 3. Calculate the pseudoreduced pressure and temperature.
Step 4. From the pseudoreduced temperature and pressure, obtain the viscosity ratio $\left(\mu_{g} / \mu_{I}\right)$ from Fig. (2.18). The term $\mu_{g}$ represents the viscosity of the gas at the required conditions.
Step 5. The gas viscosity, $\mu_{g}$, at the pressure and temperature of interest, is calculated by multiplying the viscosity at 1 atm and system temperature, $\mu_{1}$, by the viscosity ratio. The following examples illustrate the use of the proposed graphical correlations.

## Example 2.12:

A gas well is producing from a gas reservoir at an average pressure of (2000 psia) and a temperature of $\left(120^{\circ} \mathrm{F}\right)$. The specific gravity is ( 0.72 ). Calculate the gas viscosity.

## Solution:

Step 1. Calculate the apparent molecular weight of the gas:

$$
M_{a}=(0.72)(28.96)=20.85
$$

Step 2. Determine the viscosity of the gas at 1 atm and $140^{\circ} \mathrm{F}$ from Fig. (2.17):

$$
\mu_{1}=0.0113
$$

Step 3. Calculate $p_{p r}$ and $T_{p r}$ :

$$
\begin{aligned}
p_{p r} & =2.99 \\
T_{p r} & =1.52
\end{aligned}
$$

Step 4. Determine the viscosity rates from Fig. (2.18):

$$
\frac{\mu_{g}}{\mu_{1}}=1.5
$$

Step 5. Solve for the viscosity of the natural gas:

$$
\mu_{g}=\frac{\mu_{g}}{\mu_{1}}\left(\mu_{1}\right)=(1.5)(0.0113)=0.01695 c p
$$




Fig. (2.18): Carr et al.'s viscosity ratio correlation. (Carr, N., Kobayashi, R., Burrows, D., 1954. Viscosity of hydrocarbon gases under pressure, Trans. AIME).

Standing (1977) proposed a convenient mathematical expression for calculating the viscosity of the natural gas at atmospheric pressure and reservoir temperature, $\mu_{1}$. Standing also presented equations for describing the effects of $\mathrm{N}_{2}, \mathrm{CO}_{2}$, and $\mathrm{H}_{2} \mathrm{~S}$ on $\mu_{1}$. The proposed relationships are:

$$
\mu_{1}=\left(\mu_{1}\right)_{\text {uncorrected }}+(\Delta \mu)_{N_{2}}+(\Delta \mu)_{C O_{2}}+(\Delta \mu)_{H_{2} S}
$$

For details see (Ahmed T., 2019, p.66).

## 2- Lee-Gonzalez-Eakin's Method:

Lee et al. (1966) presented a semiempirical relationship for calculating the viscosity of natural gases. The authors expressed the gas viscosity in terms of the reservoir temperature, gas density, and the molecular weight of the gas. Their proposed equation is given by:

$$
\begin{align*}
\mu_{g} & =10^{-4} K \exp \left[X\left(\frac{\rho_{g}}{62.4}\right)^{Y}\right]  \tag{2.39}\\
K & =\frac{\left(9.4+0.02 M_{a}\right) T^{1.5}}{209+19 M_{a}+T} \tag{2.40}
\end{align*}
$$

$$
\begin{gather*}
X=3.5+\frac{986}{T}+0.01 M_{a}  \tag{2.41}\\
Y=2.4-0.2 X \tag{2.42}
\end{gather*}
$$

where:
$\rho=$ gas density at reservoir pressure and temperature, $l b / f t^{3}$.
$T$ = reservoir temperature, ${ }^{\circ} \mathrm{R}$.
$M_{a}=$ apparent molecular weight of the gas mixture.
The proposed correlation can predict viscosity values with a standard deviation of $2.7 \%$ and a maximum deviation of $8.99 \%$. The correlation is less accurate for gases with higher specific gravities. The authors pointed out that the method cannot be used for sour gases.

## Example 2.11:

Rework Example (2.10) and calculate the gas viscosity by using the Lee Gonzales-Eakin method.

## Solution:

Step 1. Calculate the gas density from Eq. (2.6):

$$
\rho_{g}=\frac{p M_{a}}{Z R T}=\frac{(2000)(20.85)}{(10.73)(600)(0.78)}=8.3 \mathrm{lb} / \mathrm{ft}^{3}
$$

Step 2. Solve for the parameters K, X, and Y using Eqs. (2.40), (2.41), (2.42), respectively:

$$
\begin{gathered}
K=\frac{\left(9.4+0.02 M_{a}\right) T^{1.5}}{209+19 M_{a}+T} \\
K=\frac{[9.4+(0.02)(20.85)](600)^{1.5}}{209+(19)(20.85)+600}=119.72 \\
X=3.5+\frac{986}{T}+0.01 M_{a} \\
X=3.5+\frac{986}{600}+(0.01)(20.85)=5.35 \\
Y=2.4-0.2 X \\
Y=2.4-(0.2)(5.35)=1.33
\end{gathered}
$$

Step 3. Calculate the viscosity from Eq. (2.39):

$$
\begin{gathered}
\mu_{g}=10^{-4} \operatorname{Kexp}\left[X\left(\frac{\rho_{g}}{62.4}\right)^{Y}\right] \\
\mu_{g}=10^{-4}(119.72) \exp \left[5.35\left(\frac{8.3}{62.4}\right)^{1.33}\right]=0.0173 \mathrm{cp}
\end{gathered}
$$

### 2.2 Crude Oil Properties:

Petroleum (an equivalent term is crude oil) is a complex mixture consisting predominantly of hydrocarbons and containing sulfur, nitrogen, oxygen, and helium as minor constituents. The physical and chemical properties of crude oils vary considerably and depend on the concentration of the various types of hydrocarbons and minor constituents present.

An accurate description of physical properties of crude oils is of a considerable importance in the fields of both applied and theoretical science and especially in the solution of petroleum reservoir engineering problems. Physical properties of primary interest in petroleum engineering studies include:

1. Crude oil gravity " $\gamma_{0}$ ".
2. Specific gravity of solution gas " $\gamma_{g}$ ".
3. Crude oil density " $\rho_{o}$ ".
4. Gas solubility " $R_{s}$ ".
5. Bubble point pressure " $p_{b}$ ".
6. Oil formation volume factor " $B_{o}$ ".
7. Isothermal compressibility coefficient of crude oil " $c_{o}$ ".
8. Total formation volume factor " $B_{t}$ ".
9. Crude oil viscosity.

Data on most of these fluid properties is usually determined by laboratory experiments performed on samples of actual reservoir fluids. In the absence of experimentally measured properties of crude oils, it is necessary for the petroleum engineer to determine the properties from empirically derived correlations.

### 2.2.1 Crude Oil API Gravity:

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature. It is usually expressed in pounds per cubic foot. The specific gravity of a crude oil is defined as the ratio of the density of
the oil to that of water. Both densities are measured at $60^{\circ} \mathrm{F}$ and atmospheric pressure:

$$
\begin{equation*}
\gamma_{o}=\frac{\rho_{o}}{\rho_{w}} \tag{2.43}
\end{equation*}
$$

where:
$\gamma_{o}=$ specific gravity of the oil.
$\rho_{o}=$ density of the crude oil, $l b / f t^{3}$.
$\rho_{w}=$ density of the water, $l b / f t^{3}$.
It should be pointed out that the liquid specific gravity is dimensionless but traditionally is given the units $60^{\circ} / 60^{\circ}$ to emphasize that both densities are measured at standard conditions (SC). The density of the water is approximately (62.4 lb/ft ${ }^{3}$ ), or:

$$
\gamma_{o}=\frac{\rho_{o}}{62.4}, 60^{\circ} / 60^{\circ}
$$

Although the density and specific gravity are used extensively in the petroleum industry, the API gravity is the preferred gravity scale. This gravity scale is precisely related to the specific gravity by the following expression:

$$
\begin{equation*}
A P I=\frac{141.5}{\gamma_{o}}-131.5 \tag{2.44}
\end{equation*}
$$

The API gravities of crude oils usually range from $47^{\circ} \mathrm{API}$ for the lighter crude oils to $10^{\circ} \mathrm{API}$ for the heavier asphaltic crude oils.

## Example 2.12:

Calculate the specific gravity and the API gravity of a crude oil system with a measured density of (53 lb/ft ${ }^{3}$ ) at SC.

## Solution:

Step 1. Calculate the specific gravity from Eq. (2.43):

$$
\gamma_{o}=\frac{\rho_{o}}{\rho_{w}}=\frac{53}{62.4}=0.849
$$

Step 2. Solve for the API gravity from Eq. (2.44):

$$
A P I=\frac{141.5}{0.849}-131.5=35.2^{\circ} A P I
$$

### 2.2.2 Specific Gravity of the Solution Gas, $\gamma_{g}$ ":

The specific gravity of the solution gas, $\gamma_{g}$, is described by the weighted average of the specific gravities of the separated gas from each separator. This weighted average approach is based on the separator gas-oil ratio (GOR):

$$
\begin{equation*}
\gamma_{g}=\frac{\sum_{i=1}^{n}\left(R_{s e p}\right)_{i}\left(\gamma_{s e p}\right)_{i}+R_{s t} \gamma_{s t}}{\sum_{i=1}^{n}\left(R_{s e p}\right)_{i}+R_{s t}} \tag{2.45}
\end{equation*}
$$

where:
$\mathrm{n}=$ number of separators.
$\mathrm{R}_{\text {sep }}=$ separator GOR, scf/STB.
$\gamma_{\text {sep }}=$ separator gas gravity.
$\mathrm{R}_{\text {st }}=$ GOR from the stock-tank, scf/STB.
$\gamma_{\mathrm{st}}=$ gas gravity from the stock-tank.

## Example 2.13:

Separator tests were conducted on a crude oil sample. Results of the test in terms of the separator GOR and specific gravity of the separated gas are given in the table below. Calculate the specific gravity of the separated gas.

| Separator | Pressure <br> $(\mathrm{psig})$ | Temperature <br> $\left({ }^{\circ} \mathrm{F}\right)$ | Gas-Oil Ratio <br> $(\mathrm{scf} / \mathrm{STB})$ | Gas Specific <br> Gravity |
| :---: | :---: | :---: | :---: | :---: |
| Primary | 660 | 150 | 724 | 0.743 |
| Intermediate | 75 | 110 | 202 | 0.956 |
| Stock-tank | 0 | 60 | 58 | 1.296 |

## Solution:

Calculate the solution specific gravity by using Eq. (2.45):

$$
\begin{gathered}
\gamma_{g}=\frac{\sum_{i=1}^{n}\left(R_{\text {sep }}\right)_{i}\left(\gamma_{s e p}\right)_{i}+R_{s t} \gamma_{s t}}{\sum_{i=1}^{n}\left(R_{\text {sep }}\right)_{i}+R_{s t}} \\
\gamma_{g}=\frac{(724)(0.743)+(202)(0.965)+(58)(1.296)}{724+202+58}=0.819
\end{gathered}
$$

### 2.2.3 Crude Oil Density, $\rho_{o}$ ":

The crude oil density is defined as the mass of a unit volume of the crude at a specified pressure and temperature, mass/volume. The density usually is expressed in pounds per cubic foot and it varies from $30 \mathrm{lb} / \mathrm{ft}^{3}$ for light volatile oil to $60 \mathrm{lb} / \mathrm{t}^{3}$ for heavy crude oil with little or no gas solubility. It is one of the most important oil properties, because its value substantially affects crude oil volume calculations. This vital oil property is traditionally measured in the laboratory as part of routine pressure-volume-temperature (PVT) tests. When laboratory crude oil density measurement is not available, correlations can be used to generate the required density data under reservoir pressure and temperature.

Numerous empirical correlations for calculating the density of liquids have been proposed over the years. Based on the available limited measured data on the crude, the correlations can be divided into the following two categories:

- Correlations that use the crude oil composition to determine the saturated oil density and temperature.
- Correlations that use limited PVT data, such as gas gravity, oil gravity, and GOR, as correlating parameters.


## I. Density Correlations Based on the Oil Composition:

Several reliable methods are available for determining the density of saturated crude oil mixtures from their compositions. The best known and most widely used calculation methods are the following two:

- Standing-Katz (1942).
- Alani-Kennedy (1960).

These two methods were developed to calculate the saturated oil density.

## II. Density Correlations Based on Limited PVT Data:

Several empirical correlations for calculating the density of liquids of unknown compositional analysis have been proposed. The correlations employ limited PVT data such as gas gravity, oil gravity, and gas solubility as correlating parameters to estimate liquid density at the prevailing reservoir pressure and temperature. Two methods are presented as representatives of this category:

- Katz's method (1942).
- Standing's method (1981).


## III. Standing's Method for Determining " $\rho_{o}$ ":

Standing (1981) proposed an empirical correlation for estimating the oil FVF as a function of the gas solubility, $R_{s}$; the specific gravity of stock-tank oil, $\gamma_{o}$; the specific gravity of solution gas, $\gamma_{g}$; and the system temperature, T. By coupling the mathematical definition of the oil FVF (as discussed in a later section) with Standing's correlation, the density of a crude oil at a specified pressure and temperature can be calculated from the following expression:

$$
\begin{gather*}
\rho_{o}=\frac{62.4 \gamma_{o}+0.0136 R_{s} \gamma_{g}}{B o} \\
\rho_{o}=\frac{62.4 \gamma_{o}+0.0136 R_{s} \gamma_{g}}{0.9759+0.000120\left[R_{s}\left(\frac{\gamma_{g}}{\gamma_{o}}\right)^{0.5}+1.25(T-460)\right]^{1.2}} \tag{2.46}
\end{gather*}
$$

where:

$$
\begin{aligned}
& T=\text { system temperature, }{ }^{\circ} \mathrm{R} . \\
& \gamma_{o}=\text { specific gravity of the stock-tank oil, } 60^{\circ} / 60^{\circ} . \\
& \gamma_{g}=\text { specific gravity of the gas. } \\
& R_{s}=\text { gas solubility, scf } / \mathrm{STB} . \\
& \rho_{o}=\text { oil density, } \mathrm{lb} / f t^{3} . \\
& B o=\text { oil formation volume factor, bbl/STB }
\end{aligned}
$$

### 2.2.4 Gas Solubility " $R_{s}$ ":

The gas solubility, $R_{s}$, is defined as the number of standard cubic feet of gas that dissolved in one stock-tank barrel of crude oil at certain pressure and temperature. The solubility of a natural gas in a crude oil is a strong function of the pressure, the temperature, the API gravity, and the gas gravity.

For a particular gas and crude oil to exist at a constant temperature, the solubility increases with pressure until the saturation pressure is reached. At the saturation pressure (bubble point pressure), all the available gases are dissolved in the oil and the gas solubility reaches its maximum value. Rather than measuring the amount of gas that dissolves in a given stock-tank crude oil as the pressure is increased, it is customary to determine the amount of gas that comes out of a sample of reservoir crude oil as pressure decreases.

A typical gas solubility curve, as a function of pressure for an undersaturated crude oil, is shown in Fig. (2.19). As the pressure is reduced from the initial
reservoir pressure, $p_{i}$, to the bubble point pressure, $p_{b}$, no gas evolves from the oil and consequently the gas solubility remains constant at its maximum value of $R_{s b}$. Below the bubble point pressure, the solution gas is liberated and the value of $R_{s}$ decreases with pressure.

In the absence of experimentally measured gas solubility on a crude oil system, it is necessary to determine this property from empirically derived correlations. Five empirical correlations for estimating the gas solubility are:

- Standing's correlation.
- Vasquez-Beggs's correlation.
- Glaso's correlation.
- Marhoun's correlation.
- Petrosky-Farshad's correlation.


Fig. (2.19): Typical gas solubility-pressure relationship.

## I. Standing's Correlation for Determining " $\boldsymbol{R}_{\boldsymbol{s}}$ ":

Standing (1947) proposed a graphical correlation for determining the gas solubility as a function of pressure, gas specific gravity, API gravity, and system temperature. The correlation was developed from a total of 105 experimentally determined data points on 22 hydrocarbon mixtures from California crude oils and natural gases. The proposed correlation has an average error of $4.8 \%$. In a mathematical form, Standing (1981) expressed his proposed graphical correlation in the following more convenient mathematical form:

$$
\begin{equation*}
R_{S}=\gamma_{g}\left[\left(\frac{p}{18.2}+1.4\right) 10^{x}\right]^{1.2048} \tag{2.47}
\end{equation*}
$$

with:

$$
\begin{equation*}
x=0.0125 A P I-0.00091(T-460) \tag{2.48}
\end{equation*}
$$

where:
$R_{s}=$ gas solubility, scf/STB.
$T=$ temperature, ${ }^{\circ} \mathrm{R}$.
$p=$ system pressure, psia.
$\gamma_{g}=$ solution gas specific gravity .
API = oil gravity, ${ }^{\circ}$ API.
It should be noted that Standing's equation is valid for applications at and below the bubble point pressure of the crude oil.

### 2.2.5 Bubble Point Pressure " $p_{b}$ ":

The bubble point pressure, $p_{b}$, of a hydrocarbon system is defined as the highest pressure at which a bubble of gas is first liberated from the oil. This important property can be measured experimentally for a crude oil system by conducting a constant composition expansion (CCE) test.

In the absence of the experimentally measured bubble point pressure, it is necessary for the engineer to make an estimate of this crude oil property from the readily available measured producing parameters. Several graphical and mathematical correlations for determining $p_{b}$ have been proposed during the last four decades. These correlations are essentially based on the assumption that the bubble point pressure is a strong function of gas solubility, $R_{s}$; gas gravity, $\gamma_{s}$; oil gravity, API; and temperature "T":

$$
p_{b}=f\left(R_{s}, \gamma_{g}, A P I, T\right)
$$

Techniques of combining these parameters in a graphical form or a mathematical expression were proposed by several authors, including:

- Standing.
- Vasquez and Beggs.
- Glaso.
- Marhoun.
- Petrosky and Farshad.


## I. Standing's Correlation for Determining " $\boldsymbol{p}_{\boldsymbol{b}}$ ":

Based on 105 experimentally measured bubble point pressures on 22 hydrocarbon systems from California oil fields, Standing (1947) proposed a graphical correlation for determining the bubble point pressure of crude oil systems. The correlating parameters in the proposed correlation are the gas solubility, $R_{s}$; gas gravity, $\gamma_{g}$; oil API gravity; and the system temperature. The reported average error is $4.8 \%$.

In a mathematical form, Standing (1981) expressed the graphical correlation by the following expression:

$$
\begin{equation*}
p_{b}=18.2\left[\left(R_{s} / \gamma_{g}\right)^{0.83}(10)^{a}-1.4\right] \tag{2.49}
\end{equation*}
$$

with:

$$
\begin{equation*}
a=0.00091(T-460)-0.0125(A P I) \tag{2.50}
\end{equation*}
$$

where:
$R_{s}=$ gas solubility, scf/STB.
$p_{b}=$ bubble point pressure, psia.
$T=$ system temperature, ${ }^{\circ} \mathrm{R}$.

Standing's correlation should be used with caution if nonhydrocarbon components are known to be present in the system.

### 2.2.6 Oil Formation Volume Factor " $\boldsymbol{B}_{\boldsymbol{o}}$ ":

The oil $\mathrm{FVF}, B_{o}$, is defined as the ratio of the volume of oil (plus the gas in solution) at the reservoir temperature and pressure to the volume of oil at SC. Evidently, Bo always is greater than or equal to unity. The oil FVF can be expressed mathematically as:

$$
\begin{equation*}
B_{o}=\frac{\left(V_{o}\right)_{p, T}}{\left(V_{o}\right)_{s c}} \tag{2.51}
\end{equation*}
$$

where:
$B_{o}=$ oil FVF, bbl/STB.
$\left(V_{o}\right)_{p, T}=$ volume of oil, in $b b l$, under reservoir pressure, $p$, and temperature, $T$.
$\left(V_{o}\right)_{s c}=$ volume of oil is measured under SC, STB.

A typical oil formation factor curve, as a function of pressure for an undersaturated crude oil $\left(p_{i}>p_{b}\right)$, is shown in Fig. (2.20). As the pressure is reduced below the initial reservoir pressure, $p_{i}$, the oil volume increases due to the oil expansion. This behavior results in an increase in the oil FVF and continues until the bubble point pressure is reached. At $p_{b}$, the oil reaches its maximum expansion and consequently attains a maximum value of $B_{o b}$ for the oil FVF. As the pressure is reduced below $p_{b}$, volume of the oil and $B_{o}$ are decreased as the solution gas is liberated. When the pressure is reduced to atmospheric pressure and the temperature to $60^{\circ} \mathrm{F}$, the value of $B_{o}$ is equal to 1 .

Most of the published empirical $B_{o}$ correlations utilize the following generalized relationship:

$$
B_{o}=f\left(R_{s}, \gamma_{g}, \gamma_{o}, T\right)
$$

Six methods of predicting the oil FVF are presented, including:

- Standing's correlation
- Vasquez and Beggs's correlation
- Glaso's correlation
- Marhoun's correlation
- Petrosky and Farshad's correlation
- The material balance equation

It should be noted that all the correlations could be used for any pressure equal to or below the bubble point pressure.


Fig. (2.20): Typical oil formation volume factor-pressure relationship.

## I. Standing's Correlation for Determining " $\boldsymbol{B}_{\boldsymbol{o}}$ ":

Standing (1947) presented a graphical correlation for estimating the oil FVF with the gas solubility, gas gravity, oil gravity, and reservoir temperature as the correlating parameters. This graphical correlation originated from examining 105 experimental data points on 22 California hydrocarbon systems. An average error of $1.2 \%$ was reported for the correlation.

Standing (1981) showed that the oil FVF can be expressed more conveniently in a mathematical form by the following equation:

$$
\begin{equation*}
B_{o}=0.9759+0.000120\left[R_{S}\left(\frac{\gamma_{g}}{\gamma_{o}}\right)^{0.5}+1.25(T-460)\right]^{1.2} \tag{2.52}
\end{equation*}
$$

where:

$$
\begin{aligned}
& T=\text { temperature },{ }^{\circ} \mathrm{R} . \\
& \gamma_{o}=\text { specific gravity of the stock-tank oil, } 60^{\circ} / 60^{\circ} . \\
& \gamma_{g}=\text { specific gravity of the solution gas. }
\end{aligned}
$$

### 2.2.7 Isothermal Compressibility Coefficient of Crude Oil " $\boldsymbol{c}_{\boldsymbol{o}}$ ":

The isothermal compressibility coefficient is defined as the rate of change in volume with respect to pressure increase per unit volume, all variables other than pressure being constant, including temperature. Mathematically, the isothermal compressibility, $c$, of a substance is defined by the following expression:

$$
\begin{gathered}
c=-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T} \\
c_{o}=-\frac{1}{V}\left(\frac{V_{1}-V_{2}}{p_{1}-p_{2}}\right)_{T}
\end{gathered}
$$

For example, table (2.2) shows the variation in the volume of a reservoir fluid relative to the volume at the bubble point 2695 psig $\left(V_{r}\right)$ as measured in the laboratory.

Table2.2: Relative volume data

| Pressure (psig) | Relative volume factor $\left(V_{r}\right)$ |
| :---: | :---: |
| 5000 | 0.9739 |
| 4700 | 0.9768 |
| 4400 | 0.9799 |


| 4100 | 0.9829 |
| :---: | :---: |
| 3800 | 0.9862 |
| 3000 | 0.996 |
| 2800 | 0.9985 |
| 2695 | 1.000 |

Between 5000 psig and 4100 psig, $c_{o}$ value will be:

$$
c_{o}=\frac{0.9829-0.9739}{0.9739(5000-4100)}=10.27 \times 10^{-6} \mathrm{psi}^{-1}
$$

A compressibility of $10.27 \times 10^{-6} p s i^{-1}$ means that the volume of 1 million barrels of reservoir fluid will increase by 10.27 bbls for a reduction of 1 psi in pressure.

Isothermal compressibility coefficients are required in solving many reservoir engineering problems, including transient fluid flow problems; also, they are required in the determination of the physical properties of the undersaturated crude oil.

For a crude oil system, the isothermal compressibility coefficient of the oil phase, $c_{o}$, is categorized into the following two types based on reservoir pressure:

- At reservoir pressures that are greater than or equal to the bubble point pressure $\left(p \geq p_{b}\right)$, the crude oil exists as a single phase with all its dissolved gas still in solution. The isothermal compressibility coefficient of the oil phase, $c_{o}$, above the bubble point reflects the changes in the volume associated with oil expansion or compression of the single-phase oil with changing the reservoir pressure. The oil compressibility in this case is termed undersaturated isothermal compressibility coefficient.
- Below the bubble point pressure, the solution gas is liberated with decreasing reservoir pressure or redissolved with increasing the pressure. The changes of the oil volume as the result of changing the gas solubility must be considered when determining the isothermal compressibility coefficient. The oil compressibility in this case is termed saturated isothermal compressibility coefficient.

The following four correlations for estimating the undersaturated oil compressibility are presented below:

- Trube's correlation (1957).
- Vasquez-Beggs's correlation (1980).
- Petrosky-Farshad's correlation (1993).
- Standing's correlation (1974).


## I. Standing's Correlation for Determining " $\boldsymbol{c}_{\boldsymbol{o}}$ ":

Standing (1974) proposed a graphical correlation for determining the oil compressibility for undersaturated hydrocarbon systems. Whitson and Brule expressed his relationship in the following mathematical form:

$$
\begin{equation*}
c_{o}=10^{-6} \exp \left[\frac{\rho_{o b}+0.004347\left(p-p_{b}\right)-79.1}{0.0007141\left(p-p_{b}\right)-12.938}\right] \tag{2.53}
\end{equation*}
$$

where:
$\rho_{o b}=$ oil density at the bubble point pressure, $l b / f t^{3}$.
$p_{b}=$ bubble point pressure, psia.
$c_{o}=$ oil compressibility, $\mathrm{psia}^{-1}$.

## II. The Vasquez-Beggs Correlation

From a total of 4,036 experimental data points used in a linear regression model, Vasquez and Beggs (1980) correlated the isothermal oil compressibility coefficients with Rs, T, ${ }^{\circ} \mathrm{API}, \gamma \mathrm{g}$, and p . They proposed the following expression:

$$
\begin{equation*}
c_{o}=\frac{-1433+5 R_{s b} 17.2(T-460)-1180 \gamma_{g s}+12.61^{\circ} A P I}{10^{5} p} \tag{2.54}
\end{equation*}
$$

where:
$T=$ temperature,${ }^{\circ} \mathrm{R}$
$p=$ pressure above the bubble-point pressure, psia.
$R_{s b}=$ gas solubility at the bubble-point pressure.
$\gamma_{g s}=$ corrected gas gravity.

See (Ahmed T., 2019) for more details.

### 2.2.8 Total Formation Volume Factor " $\boldsymbol{B}_{\boldsymbol{t}}$ ":

To describe the pressure-volume relationship of hydrocarbon systems below their bubble point pressure, it is convenient to express this relationship in terms
of the total FVF as a function of pressure. This property defines the total volume of a system regardless of the number of phases present.

The total FVF, denoted $B_{t}$, is defined as the ratio of the total volume of the hydrocarbon mixture; that is, oil and gas, if present, at the prevailing pressure and temperature per unit volume of the stock-tank oil. Because naturally occurring hydrocarbon systems usually exist in either one or two phases, the term two-phase FVF has become synonymous with the total formation volume.

Mathematically, $B_{t}$ is defined by the following relationship:

$$
\begin{equation*}
B_{t}=\frac{\left(V_{o}\right)_{p, T}+\left(V_{g}\right)_{p, T}}{\left(V_{o}\right)_{s c}} \tag{2.55}
\end{equation*}
$$

where:

$$
\begin{aligned}
& B_{t}=\text { total FVF, bbl/STB. } \\
& \left(V_{o}\right)_{p, T}=\text { volume of the oil at } p \text { and } T, \mathrm{bbl} . \\
& \left(V_{g}\right)_{p, T}=\text { volume of the liberated gas at } p \text { and } T, \mathrm{bbl} . \\
& \left(V_{o}\right)_{s c}=\text { volume of the oil at SC, STB. }
\end{aligned}
$$

A typical plot of $B_{t}$ as a function of pressure for an undersaturated crude oil is shown in Fig. (2.21). The oil FVF curve is also included in the illustration. As pointed out, $B_{o}$ and $B_{t}$ are identical at pressures above or equal to the bubble point pressure because only one phase, the oil phase, exists at these pressures. It should also be noted that at pressures below the bubble point pressure, the difference in the values of the two oil properties represents the volume of the evolved solution gas as measured at system conditions per stock-tank barrel of oil.

From the definition of the two-phase FVF,

$$
\begin{equation*}
B_{t}=B_{o}+\left(R_{s b}-R_{s}\right) B_{g} \tag{2.56}
\end{equation*}
$$

where:
$R_{s b}=$ gas solubility at the bubble point pressure, scf/STB.
$R_{s}=$ gas solubility at any pressure, $\mathrm{scf} / \mathrm{STB}$.
$B_{o}=$ oil FVF at any pressure, bbl/STB.
$B_{g}=$ gas $\mathrm{FVF}, \mathrm{bbl} / \mathrm{scf}$.

Several correlations can be used to estimate the two-phase FVF when the experimental data are not available; three of these methods are detailed below:

- Standing's correlation (1947).
- Glaso's correlation (1980).
- Marhoun's correlation (1988).


Fig. (2.21): $B_{o}$ and $B_{t}$ vs. pressure.

### 2.2.9 Crude Oil Viscosity " $\mu_{o}$ ":

Crude oil viscosity is an important physical property that controls the flow of oil through porous media and pipes. The viscosity, in general, is defined as the internal resistance of the fluid to flow. It ranges from 0.1 cp for near critical to over 100 cp for heavy oil. It is considered the most difficult oil property to calculate with a reasonable accuracy from correlations.

Oil's viscosity is a strong function of the temperature, pressure, oil gravity, gas gravity, gas solubility, and composition of the crude oil. Whenever possible, oil viscosity should be determined by laboratory measurements at reservoir temperature and pressure. The viscosity usually is reported in standard PVT analyses. If such laboratory data are not available, engineers may refer to published correlations, which usually vary in complexity and accuracy, depending on the available data on the crude oil. Based on the available data on the oil mixture, correlations can be divided into the following two types: correlations based on other measured PVT data, such as API or $R_{s}$, and correlations based on oil composition. Depending on the pressure, p, the viscosity of crude oils can be classified into three categories:

- Dead oil viscosity, $\mu_{o d}$. The dead oil viscosity (oil with no gas in the solution) is defined as the viscosity of crude oil at atmospheric pressure and system temperature, $T$.
- Saturated oil viscosity, $\mu_{o b}$. The saturated (bubble point) oil viscosity is defined as the viscosity of the crude oil at any pressure less than or equal to the bubble point pressure.
- Undersaturated oil viscosity, $\boldsymbol{\mu}_{o}$. The undersaturated oil viscosity is defined as the viscosity of the crude oil at a pressure above the bubble point and reservoir temperature.

The definition of the three categories of oil viscosity is illustrated conceptually in Fig. (2.22).

Crude oil viscosity


Fig. (2.22) Oil viscosity as a function of $R_{s}$ and $p$.

## I. Dead Oil Viscosity " $\mu_{o d}$ ":

Several empirical methods are proposed to estimate the viscosity of the dead oil that are based on the API gravity of the crude oil and system temperature. These correlations, as listed below, calculate the viscosity of the dead oil (ie, no dissolved gas) at reservoir temperature " $T$ ":

- Beal's correlation (1946).
- Beggs-Robinson's correlation (1975).
- Glaso's correlation (1980).


## II. Glaso's Correlation for Determining " $\mu_{o d}$ ":

Glaso (1980) proposed a generalized mathematical relationship for computing the dead oil viscosity. The relationship was developed from experimental measurements on 26 crude oil samples. The correlation has the following form:

$$
\begin{equation*}
\mu_{o d}=\left[3.141\left(10^{10}\right)\right](T-460)^{-3.444}[\log (A P I)]^{A} \tag{2.57}
\end{equation*}
$$

The temperature T is expressed in ${ }^{\circ} \mathrm{R}$ and the coefficient A is given by:

$$
\begin{equation*}
A=10.313[\log (T-460)]-36.447 \tag{2.58}
\end{equation*}
$$

This expression can be used within the range of $50-300^{\circ} \mathrm{F}$ for the system temperature and $20-48^{\circ}$ for the API gravity of the crude. Sutton and Farshad (1984) concluded that Glaso's correlation showed the best accuracy of the three previous correlations.

## III. Saturated Oil Viscosity, " $\mu_{o b}$ ":

Having calculated the viscosity of the dead crude oil at reservoir temperature using one of the three above correlations, the dead oil viscosity must be adjusted to account for the solution gas (ie, "Rs"). The following three empirical methods are proposed to estimate the viscosity of the saturated oil:

- Chew-Connally correlation (1959).
- Beggs-Robinson correlation (1975).
- Abu-Khamsin and Al-Marhoun (1991).


## IV. Abu-Khamsin and Al-Marhoun Correlation for Determining " $\mu_{o b}$ "

An observation by Abu-Khamsin and Al-Marhoun (1991) suggests that the saturated oil viscosity, $\mu_{o b}$, correlates very well with the saturated oil density, $\rho_{o b}$, as given by:

$$
\begin{equation*}
\ln \left(\mu_{o b}\right)=8.484462 \rho_{o b}^{4}-2.652294 \tag{2.59}
\end{equation*}
$$

where, the saturated oil density, $\rho_{o b}$, is expressed in $\mathrm{g} / \mathrm{cm}^{3}$; that is, $\rho_{o b} / 62.4$.

## V. Undersaturated Oil Viscosity " $\boldsymbol{\mu}_{\boldsymbol{o}}$ ":

Oil viscosity at pressures above the bubble point is estimated by first calculating the oil viscosity at its bubble point pressure and adjusted to higher pressures. Three methods for estimating the oil viscosity at pressures above saturation pressure are presented next:

- Beal's correlation (1946).
- Khan's correlation (1987).
- Vasquez-Beggs correlation (1980).


## IV. Khan's Correlation for Determining " $\mu_{o}$ ":

From 1500 experimental viscosity data points on Saudi Arabian crude oil systems, Khan et al. (1987) developed the following equation with a reported absolute average relative error of $2 \%$ :

$$
\begin{equation*}
\mu_{o}=\mu_{o b} \exp \left[9.6\left(10^{-6}\right)\left(p-p_{b}\right)\right] \tag{2.60}
\end{equation*}
$$

## Problems

1. Assuming an ideal gas behavior, calculate the density of n-butane at $\left(220^{\circ} \mathrm{F}\right)$ and (50 psia).
2. Given the following gas:

| Component | Weight Fraction |
| :---: | :---: |
| $\mathrm{C}_{1}$ | 0.65 |
| $\mathrm{C}_{2}$ | 0.15 |
| $\mathrm{C}_{3}$ | 0.10 |
| $\mathrm{n}-\mathrm{C}_{4}$ | 0.06 |
| $\mathrm{n}-\mathrm{C}_{5}$ | 0.04 |

Calculate:
a. Mole fraction of the gas.
b. Apparent molecular weight.
c. Specific gravity.
3. An ideal gas mixture has a density of ( $1.92 \mathrm{lb} / \mathrm{ft}^{3}$ ) at ( 500 psia ) and $\left(100^{\circ} \mathrm{F}\right)$. Calculate the apparent molecular weight of the gas mixture.
4. Using the gas composition as given in Problem 2, and assuming real gas behavior, calculate:
a. Gas density at $\left(2,000 \mathrm{psia}\right.$ and $\left.150^{\circ} \mathrm{F}\right)$.
b. Gas formation volume factor in $\mathrm{scf} / \mathrm{ft}^{3}$.
5. Compute the composition in weight fraction of a gas with the following composition:

| Component | Composition, <br> mole fraction |
| :---: | :---: |
| Methane | 0.870 |
| Ethane | 0.049 |
| Propane | 0.036 |
| i-Butane | 0.025 |
| n-Butane | 0.020 |
|  | 1.00 |

6. A ( $20 \mathrm{cu}-\mathrm{ft}$ ) tank at at ( 100 oF ) is pressured to ( 200 psia ) with a pure paraffin gas. Ten pounds of ethane are added, and the specific gravity of the gas mixture is measured to be (1.68). Assume that the gases act as ideal gases. What was the gas originally in the tank?
7. Compute the apparent molecular weight and specific gravity of the gas of problem 5.
8. A gas composed of methane and ethane has a specific gravity of (0.75). What is the weight percent and volume percent of the methane in the gas mixture?
9. Determine the partial pressure of each component of the gas in problem 5 if the total pressure on the gas is ( 350 psia ) and the temperature is $\left(90^{\circ} \mathrm{F}\right)$.
10.What is the specific gravity of the following gas?

| Component | Partial pressure, psia |
| :---: | :---: |
| Methane | 17.8 |
| Ethane | 1.0 |
| Propane | 0.4 |
| i-Butane | 0.2 |
| n-Butane | 0.6 |
|  | 20.0 |

11. (2.4 cu-ft) cylinder of ethane shows a pressure of (1600 psia) at $\left(90^{\circ} \mathrm{F}\right)$. What is the mass in pounds of ethane contained in the cylinder? Do not assume ethane is an ideal gas.
12. The gas in problem 5 was produced from a reservoir at ( 4650 psia ) and $\left(180^{\circ} \mathrm{F}\right)$. What was the density of the gas at reservoir conditions?
13.A tank contains methane at ( 1000 psia ) and $\left(140{ }^{\circ} \mathrm{F}\right)$. Another tank of equal volume contains ethane at ( 500 psia ) and ( $140{ }^{\circ} \mathrm{F}$ ). The two tanks are connected, the gases are allowed to mix, and the temperature is restored to $\left(140^{\circ} \mathrm{F}\right)$. Calculate the final pressure, the composition of the
mixture, and the partial pressure of the components at final conditions. Do not assume that ideal gas equation apply.
13. A cylinder has a volume of ( $0.5 \mathrm{cu}-\mathrm{ft}$ ) and contains a gas at a pressure of ( 2000 psia ) and ( $120^{\circ} \mathrm{F}$ ). The pressure drops to ( 1000 psia ) after ( 0.0923 lb moles) of gas are removed. The temperature is constant. The z -factor was ( 0.90 ) at ( 2000 psia ). What is the z -factor at ( 1000 psia )?
15.A tank with volume of ( $2.4 \mathrm{cu}-\mathrm{ft}$ ) is filled with methane to a pressure of ( 1500 psia ) at ( $104{ }^{\circ} \mathrm{F}$ ). Determine the molecular weight and specific gravity of the gas and the pound moles, pounds, and density of the gas in the tank.
16.A cylinder with an initial pressure of ( 14.7 psia ) and volume of ( 75000 cc) is held at a constant temperature of ( 200 oF ). The volume of the cylinder is reduced by insertion of mercury. The corresponding volumes (cc) and pressures (psia) of the gas inside the cylinder are recorded as follows:

| Press. | 400 | 800 | 1200 | 2000 | 2500 | 3000 | 4000 | 5000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Vol. | 2448 | 1080 | 648.6 | 350.6 | 292.9 | 266.4 | 234.6 | 206.2 |

Calculate the ideal volumes for the gas at each pressure and use them to calculate the z -factor. Plot the z -factor against pressure.
17.Determine the value of z-factor for the gas given below at ( 5420 psig ) and ( $257^{\circ} \mathrm{F}$ ):

| Component | Composition, <br> mole fraction |
| :---: | :---: |
| Hydrogen sulfide | 0.100 |
| Carbon dioxide | 0.050 |
| Nitrogen | 0.021 |
| Methane | 0.703 |
| Ethane | 0.062 |
| Propane | 0.037 |
| n-Butane | 0.027 |
| ${1.000} \\ {\hline}$ |  |

18.The reservoir gas in a wet gas reservoir has a specific gravity of (1.295), a hydrogen sulfide concentration of ( 20.9 mole percent), and a carbon dioxide concentration of ( 44.7 mole percent). Determine a value of zfactor for use at reservoir conditions of ( 5720 psig ) and ( $268^{\circ} \mathrm{F}$ ).
19.A wet gas has composition as given below. Determine a value of z-factor for this gas at reservoir conditions of ( 5720 psig ) and ( $268{ }^{\circ} \mathrm{F}$ ):

| Component | Composition, <br> mole percent |
| :---: | :---: |
| Hydrogen sulfide | 20.90 |
| Carbon dioxide | 44.69 |
| Nitrogen | 1.22 |
| Methane | 21.80 |
| Ethane | 3.68 |
| Propane | 2.05 |
| i-Butane | 0.58 |
| n-Butane | 1.09 |
| i-Pentane | 0.46 |
| n-Pentane | 0.56 |
| Hexanes | 0.72 |
| Heptanes plus | 2.25 |
| Properties of heptanes plus |  |
| 100.00 |  |
| Specific gravity | 0.844 |
| Molecular weight | $115 \mathrm{lb} / \mathrm{lb}$ mole |

Compare your answer with laboratory measured z-factor of (0.914) at ( 5720 psig ) and ( $268{ }^{\circ} \mathrm{F}$ ) for this gas.
20.A natural gas with a specific gravity of ( 0.75 ) has a gas formation volume factor of ( $0.00529 \mathrm{ft}^{3} / \mathrm{scf}$ ) at the prevailing reservoir pressure and temperature. Calculate the density of the gas.
21.A natural gas has the following composition:

| Component | Mole Fraction, $\boldsymbol{y}_{\boldsymbol{i}}$ |
| :---: | :---: |
| $C_{1}$ | 0.75 |
| $C_{2}$ | 0.10 |
| $C_{3}$ | 0.05 |


| $i-C_{4}$ | 0.04 |
| :---: | :--- |
| $n-C_{4}$ | 0.03 |
| $i-C_{5}$ | 0.02 |
| $n-C_{5}$ | 0.01 |

Reservoir conditions are ( 3500 psia ) and ( $200{ }^{\circ} \mathrm{F}$ ). Calculate:
a. Isothermal gas compressibility coefficient
b. Gas viscosity by using the

1. Carr-Kobayashi-Burrows method.
2. Lee-Gonzales-Eakin method.
3. Given the following gas composition:

| Component | Mole Fraction, $\boldsymbol{y}_{\boldsymbol{i}}$ |
| :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.06 |
| $\mathrm{~N}_{2}$ | 0.03 |
| $C_{1}$ | 0.75 |
| $C_{2}$ | 0.07 |
| $C_{3}$ | 0.04 |
| $n-C_{4}$ | 0.03 |
| $n-C_{5}$ | 0.02 |

If the reservoir pressure and temperature are ( 2500 psia ) and $\left(175^{\circ} \mathrm{F}\right)$, respectively, calculate:
a. Isothermal gas compressibility coefficient.
b. Gas viscosity by using the:

1. Carr-Kobayashi-Burrows method.
2. Lee-Gonzales-Eakin method.
23.A crude oil system exists at its bubble-point pressure of (1708.7 psia) and a temperature of $\left(131^{\circ} \mathrm{F}\right)$. Given the following data:
$\mathrm{API}=40^{\circ}$
Average specific gravity of separator gas $=0.85$
Separator pressure $=100 \mathrm{psig}$
a. Calculate $R_{s b}$ by using Standing correlation.
b. Calculate $B_{o b}$ by using Standing correlation.
24.Estimate the bubble-point pressure of a crude oil system by using Standing correlation with the following limited PVT data:

$$
\mathrm{API}=35^{\circ} \quad \mathrm{T}=160^{\circ} \mathrm{F} \quad R_{s b}=700 \mathrm{scf} / \mathrm{STB} \quad \gamma_{\mathrm{g}}=0.75
$$

25.A crude oil system exists at an initial reservoir pressure of ( 4500 psi ) and $\left(85^{\circ} \mathrm{F}\right)$. The bubble-point pressure is estimated at ( 2109 psi ). The oil properties at the bubble-point pressure are as follows:
$B_{o b}=1.406 \mathrm{bbl} / \mathrm{STB}$
$R_{s b}=692 \mathrm{scf} / \mathrm{STB}$
$\gamma_{\mathrm{g}}=0.876$
$\mathrm{API}=41.9^{\circ}$

Calculate:
a. Oil density at the bubble-point pressure.
b. Oil density at 4500 psi .
c. $B_{o}$ at 4500 psi .
26.A hydrocarbon gas mixture with a specific gravity of (0.7) has a density of ( $9 \mathrm{lb} / \mathrm{ft}^{3}$ ) at the prevailing reservoir pressure and temperature. Calculate the gas formation volume factor in $\mathrm{bbl} / \mathrm{scf}$.
27.A high-pressure cell has a volume of ( $0.33 \mathrm{ft}^{3}$ ) and contains gas at ( 2,500 psia) and $\left(130^{\circ} \mathrm{F}\right)$, at which conditions its z -factor is ( 0.75 ). When (43.6 scf ) of the gas are bled from the cell, the pressure dropped to ( $1,000 \mathrm{psia}$ ), the temperature remaining at $\left(130^{\circ} \mathrm{F}\right)$. What is the gas deviation factor at ( $1,000 \mathrm{psia}$ ) and $\left(130^{\circ} \mathrm{F}\right)$ ?
28.A gas reservoir exists at a $\left(150^{\circ} \mathrm{F}\right)$. The gas has the following composition:

| Component | Mole \% |
| :---: | :---: |
| $C_{1}$ | 89 |
| $C_{2}$ | 7 |
| $C_{3}$ | 4 |

The gas expansion factor $E_{g}$ was calculated as (204.648 scf/ft ${ }^{3}$ ) at the existing reservoir pressure and temperature. Calculate the viscosity of the gas.
29.A ( $20 \mathrm{ft}^{3}$ ) tank at a pressure of ( 2500 psia ) and $\left(212^{\circ} \mathrm{F}\right)$ contains ethane gas. How many pounds of ethane are in the tank?

## REFERENCES

Ahmed, T.,2019, "Reservoir Engineering Handbook", Gulf Professional Publishing.
Ahmed, T., 2016, "Equation of State and PVT Analysis Application for Improved Reservoir Modelling". Elsevier Inc, USA.

Alani, G.H., Kennedy, H.T., 1960. Volume of liquid hydrocarbons at high temperatures and pressures. Trans. AIME 219, 288-292.

Brown, G.G., et al., 1948. Natural Gasoline and The Volatile Hydrocarbons. National Gas Association of America, Tulsa, OK.

Carr, N., Kobayashi, R., Burrows, D., 1954. Viscosity of hydrocarbon gases under pressure. Trans. AIME 201, 270-275.

Craft, B. C., and Hawkins, M. (Ronald E. Terry, J. Brandon Rogers.), 2015, "Applied Petroleum Reservoir Engineering", Third edition. Englewood Cliffs, NJ: Prentice Hall.

Dranchuk, P.M., Abu-Kassem, J.H., 1975. Calculate of Z-factors for natural gases using equations-of-state. J. Can. Pet. Technol. 34-36.

Dranchuk, P.M., Purvis, R.A., Robinson, D.B., 1974. Computer Calculations of Natural Gas Compressibility Factors Using the Standing and Katz Correlation. Technical Series, no. IP 74-008. Institute of Petroleum, Alberta, Canada.

Glaso, O., 1980. Generalized pressure-volume-temperature correlations. J. Pet. Technol. 32 (5), 785-795.

Hall, K.R., Yarborough, L., 1973. A new equation of state for Z-factor calculations. Oil Gas J. 82-92.

Katz, D.L. et al, 1959, "Handbook of Natural Gas Engineering", McGraw-Hill Book Co. Inc., New York City.

Katz, D., 1942, "Drilling and Production Practice". American Petroleum Institute, Dallas, TX.

Lee, A.L., Gonzalez, M.H., Eakin, B.E., 1966. The viscosity of natural gases. J. Petrol. Technol. 997-1000.

Mattar, L.G., Brar, S., Aziz, K., 1975. Compressibility of natural gases. J. Can. Pet. Technol. 77-80.

Petrosky, G.E., Farshad, F., 1993. Pressure-volume-temperature correlations for Gulf of Mexico crude oils. SPE paper 26644, In: Presented at the 68th Annual Technical Conference of the Society of Petroleum Engineers, Houston, 3-6 October 1993.

Standing, M., 1974. Petroleum Engineering Data Book. Norwegian Institute of Technology, Trondheim.

Standing, M.B., 1977. Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems. Society of Petroleum Engineers, Dallas. pp. 125-126.

Standing, M.B., Katz, D.L., 1942. Density of natural gases. Trans. AIME 146, 140-149.

Standing, M.B., 1981. Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems, ninth ed. Society of Petroleum Engineers, Dallas, TX.

Standing, M.B., 1947. A pressure-volume-temperature correlation for mixtures of California oils and gases. Drilling and Production Practice. API, Washington, D.C, pp. 275-287.

Stewart, W.F., Burkhard, S.F., Voo, D., 1959. Prediction of Pseudo-Critical Parameters for Mixtures. Paper Presented at the AIChE Meeting, Kansas City, MO.

Sutton, R.P.M., 1985. Compressibility factors for high-molecular-weight reservoir gases. In: Paper SPE 14265, Presented at the 60th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Las Vegas.

Sutton, R.P., Farshad, F.F., 1984. Evaluation of empirically derived PVT properties for Gulf of Mexico crude oils. Paper SPE 13172, In: Presented at the 59th Annual Technical Conference, Houston.
Trube, A.S., 1957a. Compressibility of undersaturated hydrocarbon reservoir fluids. Trans. AIME 210, 341-344.

Trube, A.S., 1957b. Compressibility of natural gases. Trans. AIME 210, 355-357.
Vasquez, M., Beggs, H.D., 1980. Correlations for fluid physical property prediction. J. Pet. Technol. 32, 968-970.

Wichert, E., Aziz, K., 1972. Calculation of Z's for sour gases. Hydrocarb. Process. 51 (5), 119-122.

## Chapter Three

## Phase Behavior and Laboratory Analysis of Reservoir Fluids

### 3.1 Phase Behavior and Pressure-Temperature Diagram:

A PHASE is defined as any homogeneous part of a system that is physically distinct and separated from other parts of the system by definite boundaries. For example, ice, liquid water, and water vapor constitute three separate phases of the pure substance $\mathrm{H}_{2} \mathrm{O}$, because each is homogeneous and physically distinct from the others; moreover, each is clearly defined by the boundaries existing between them. Whether a substance exists in a solid, liquid, or gas phase is determined by the temperature and pressure acting on the substance. It is known that ice (solid phase) can be changed to water (liquid phase) by increasing its temperature and, by further increasing the temperature, water changes to steam (vapor phase). This change in phases is termed phase behavior.

Hydrocarbon systems found in petroleum reservoirs are known to display multiphase behavior over wide ranges of pressures and temperatures. The most important phases that occur in petroleum reservoirs are a liquid phase, such as crude oils or condensates, and a gas phase, such as natural gases, depending on the nature of crude oil composition (light $\left(\mathrm{C}_{1} \& \mathrm{C}_{2}\right)$, intermediate $\left(\mathrm{C}_{3}-\mathrm{C}_{6}\right)$ or heavy components $\left(\mathrm{C}_{7}+\right)$ ).

The conditions under which these phases exist are a matter of considerable practical importance. The experimental or the mathematical determinations of these conditions are conveniently expressed in different types of diagrams, commonly called phase diagrams. Phase diagram can be defined as a map of the phase of a substance as a function of temperature (X-axis) and pressure (Yaxis). Phase diagram is also called the pressure-temperature diagram.

Fig. (3.1) shows a typical pressure-temperature diagram ( $p-T$ diagram) of a multicomponent system with a specific overall composition. Although a different hydrocarbon systems would have a different phase diagrams, the general configuration is similar.

These multicomponent $p-T$ diagrams are essentially used to:

- Classify reservoirs.
- Specify the naturally occurring hydrocarbon systems.
- Describe the phase behavior of the reservoir fluid.


Fig. (3.1): Typical p-T diagram for a multicomponent system.

To fully understand the significance of the $p-T$ diagrams, it is necessary to identify and define the following key points on the p-T diagram:

* Cricondentherm $\left(\boldsymbol{T}_{c t}\right)$ : The cricondentherm is the maximum temperature above which liquid cannot be formed regardless of pressure (point $E$ ), or simply it is the maximum temperature of the tow phase region. The corresponding pressure is termed the cricondentherm pressure, $p_{c t}$.
* Cricondenbar $\left(\boldsymbol{p}_{\boldsymbol{c b}}\right)$ : The cricondenbar is the maximum pressure above which no gas can be formed regardless of temperature (point $D$ ), or simply it is the maximum pressure of the tow phase region. The corresponding temperature is called the cricondenbar temperature, $T_{c b}$.
* Critical point: The critical point for a multicomponent mixture is referred to as the state of pressure and temperature at which all intensive properties of the gas and liquid phases are equal (point $C$ ). At the critical
point, the corresponding pressure and temperature are called the critical pressure, $p_{c}$, and critical temperature, $T_{c}$, of the mixture.
* Phase envelope (two-phase region): The region enclosed by the bubble point curve and the dew point curve (line $B C A$ ), where gas and liquid coexist in equilibrium, is identified as the phase envelope of the hydrocarbon system.
* Quality lines: The lines within the phase diagram are called quality lines. They describe the pressure and temperature conditions for equal liquid percentage by volumes (\% of liquid). Note that the quality lines converge at the critical point (point $C$ ).
* Bubble point curve: The bubble point curve (line $B C$ ) is defined as the line separating the liquid phase region from the two-phase region.
* Dew point curve: The dew point curve (line $A C$ ) is defined as the line separating the vapor phase region from the two-phase region.


### 3.2 Classification of Reservoirs:

Petroleum reservoirs are broadly classified as oil or gas reservoirs. As shown in Fig. (3.2) these broad classifications are further subdivided depending on:

1. The composition of the reservoir hydrocarbon mixture.
2. Initial reservoir pressure and temperature.
3. Pressure and temperature of the surface production.
4. Location of the reservoir temperature with respect to the critical temperature and the cricondentherm.

In general, reservoirs are conveniently classified on the basis of the reservoir temperature " $T$ " as compared with the critical temperature of the hydrocarbon mixture " $\left(T_{c}\right)_{m i x}$." It should be noted that perhaps the most important components are the Methane and the Plus-Fraction, e.g., C7+. In general, these two components impact the size of the phase envelope as well as the critical temperature of the mixture. Accordingly, reservoirs can be classified into basically three types:

1. Oil reservoirs: If the reservoir temperature, $T$, is less than the critical temperature, $T_{c}$, of the reservoir fluid, the reservoir is classified as an oil reservoir.
2. Gas reservoirs: If the reservoir temperature is greater than the critical temperature of the hydrocarbon fluid, the reservoir is considered a gas reservoir.
3. Near-critical reservoirs: If the reservoir temperature is very close to the critical temperature of the hydrocarbon fluid, the reservoir is classified as near-critical reservoir.


Fig. (3.2): Main reservoir classifications.
Below the flow diagram summarizes the three reservoir classifications. The impact of fluid composition on the shape and size of the phase envelope as well as the reservoir type identification are illustrated in Fig. (3.3).


| Composition defines the size of phase envelop |  |  |  |
| :---: | :---: | :---: | :---: |
| Camp. | Max 2 | Popria | $T_{5} \cdot \mathrm{R}$ |
| $\mathrm{CO}_{2}$ | 0.000 | 1071 | 547.9 |
| $\mathrm{N}_{2}$ | a,tos | 493 | 237818 |
| $\mathrm{C}_{1}$ | A 350 | 667.8 | 343.37 |
| $\mathrm{C}_{7}$ | 0.100 | 70\%: 8 | 650.09 |
| $c_{2}$ | 6.605 | 6403 | bte it |
| $)_{4}$ | 2600 | 5291 | 734.93 |
| $\mathrm{SCH}_{4}$ | 80.00 | 580 T | 785.65 |
| $H_{5}$ | 8.501 | 400.4 | Emal |
| ${ }_{*} \mathrm{C}_{2}$ | 0.002 | 4896 | 845.7 |
| $\mathrm{C}_{2}$ | 0000 | 436.8 | 9187 |
| $5_{7+}$ | 0.417 | 3203 | 1138.4 |




| Comp | M $\times 1$ | Popas | $T_{5} \cdot \mathrm{R}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{CO}_{\mathrm{r}}$ | 0.600 | t677 | 507.8 |
| $\mathrm{N} /$ | 0.000 | 491 | 2275 |
| $C_{1}$ | 0.888 | 667.8 | 34337 |
| $c_{2}$ | Q.t16 | 707 B | 95009 |
| $\mathrm{C}_{3}$ | 3200 | 8163 | 686.01 |
| $\mathrm{HC}_{4}$ | 0.623 | 5881 | 73488 |
| < $C_{4}$ | a60a | 5507 | 765.85 |
| $\mathrm{H}_{5}$ | 20.55 | 4504 | 029.1 |
| ${ }^{-6} \mathrm{C}_{2}$ | 0.016 | 4868 | (045.) |
| $c_{0}$ | 2.016 | 4369 | 918.7 |
| C\% | 0.158 | 3203 | 1139.4 |

Fig. (3.3): Impact of composition on the size of phase envelope.

### 3.2.1 Classification of Oil Reservoirs:

Depending on initial reservoir pressure, $p_{i}$, oil reservoirs can be subclassified into the following categories:

1. Undersaturated oil reservoir: If the initial reservoir pressure, $p_{i}$ (as represented by point 1 in Fig. (3.1)), is greater than the bubble point pressure, $p_{b}$, of the reservoir fluid, the reservoir is an undersaturated oil reservoir.
2. Saturated oil reservoir: When the initial reservoir pressure is equal to the bubble point pressure of the reservoir fluid, as shown in Fig. (3.1) by point 2 , the reservoir is a saturated oil reservoir.
3. Gas-cap reservoir: When two phases (gas and oil) initially exit in equilibrium in a reservoir; the reservoir is defined as a gas-cap reservoir. (as represented by point 3 in Fig. (3.1)), in which the gas or vapor phase is underlain by an oil phase.

### 3.2.2 Classification of Gas Reservoirs:

On the basis of their phase diagrams and the prevailing reservoir conditions, natural gases can be classified into four categories:

1. Retrograde gas-condensate reservoir: If the reservoir temperature $T$ lies between the critical temperature $T_{c}$ and cricondentherm $T_{c t}$ of the reservoir fluid, the reservoir is classified as a retrograde gas-condensate reservoir. This category of gas reservoir is a unique type of hydrocarbon accumulation in that the special thermodynamic behavior of the reservoir fluid is the controlling factor in the development and the depletion process of the reservoir. When the pressure is decreased on these mixtures, instead of expanding (if a gas) or vaporizing (if a liquid) as might be expected, they vaporize instead of condensing. So retrograde condensation is defined as the formation of liquid by an isothermal decrease in pressure or an isobaric increase in temperature. Consider that the initial condition of a retrograde gas reservoir is represented by point 1 on the pressure-temperature phase diagram of Fig. (3.4) because the reservoir pressure is above the upper dew point pressure, the hydrocarbon system exists as a single phase (i.e., vapor phase) in the reservoir. As the reservoir pressure declines isothermally during production from the initial pressure (point 1) to the upper dew point pressure (point 2), the attraction between the molecules of the light and heavy components move farther
apart. As this occurs, attraction between the heavy component molecules becomes more effective; therefore, liquid begins to condense. This retrograde condensation process continues with decreasing pressure until the liquid dropout (LDO) reaches its maximum at (point 3). Further reduction in pressure permits the heavy molecules to commence the normal vaporization process. This is the process whereby fewer gas molecules strike the liquid surface and more molecules leave than enter the liquid phase. The vaporization process continues until the reservoir pressure reaches the lower dew point pressure. This means that all the liquid that formed must vaporize because the system essentially is all vapor at the lower dew point.


Fig. (3.4): A typical phase diagram of a retrograde system.
2. Near-Critical Gas-Condensate Reservoirs: If the reservoir temperature is near the critical temperature, as shown in Fig. (3.5), the hydrocarbon mixture is classified as a near-critical gas-condensate. The volumetric behavior of this category of natural gas is described through the isothermal pressure declines, as shown by the vertical line 1-3 in Fig. (3.5) and the corresponding LDO curve of Fig. (3.6) because all the quality lines converge at the critical point as shown in Fig. (3.6), a rapid
liquid buildup immediately occurs below the dew point as the pressure is reduced to (point 2).


Temperature

Fig. (3.5): A typical phase diagram for a near-critical gas-condensate reservoir.


Fig. (3.6): Liquid-shrinkage (dropout) curve for a near-critical gas-condensate system.
3. Wet Gas Reservoirs: A typical phase diagram of a wet gas is shown in Fig. (3.7), where the reservoir temperature is above the cricondentherm of the hydrocarbon mixture. Because the reservoir temperature exceeds the
cricondentherm of the hydrocarbon system, the reservoir fluid always remains in the vapor phase region as the reservoir is depleted isothermally, along the vertical line $A B$. However, as the produced gas flows to the surface, the pressure and temperature of the gas decline. The gas in this case is composed of intermediate and heavy molecules $\left(\mathrm{C}_{3}\right.$ and more) i.e. the attractive force between molecules will be strong if the gas enters the two-phase region, a liquid phase condenses out of the gas and is produced from the surface separators.


Fig. (3.7): Phase diagram for a wet gas and production process.
4. Dry Gas Reservoirs: The hydrocarbon mixture is composed of methane and ethane (light components) which exists as a gas both in the reservoir and the surface because of weak attractive force between these small molecules. The only liquid associated with the gas from a dry gas reservoir is water. Fig. (3.8) is a phase diagram of a dry gas reservoir.


Fig. (3.8): Phase diagram for a dry gas.

### 3.3 Classification of Crude Oils:

Crude oils cover a wide range in physical properties and chemical compositions, and it is often important to be able to group them into broad categories of related oils. The classification of crude oils essentially is based on the physical and PVT properties exhibited by the crude oil, including:

* Physical properties, such as API gravity of the stock-tank liquid.
* Composition.
* Initial producing gas-oil ratio (GOR).
* Appearance, such as the color of the stock-tank liquid.
* Pressure-temperature phase diagram.

In general, crude oils are commonly classified into the following types:

1. Ordinary black oil: usually yield GORs between 200 and 700 scf/STB and oil gravities of 15-40 API. The stock-tank oil usually is brown to dark green in color.
2. Low-shrinkage oil: The associated properties of this type of crude oil are:

- Oil formation volume factor less than $1.2 \mathrm{bbl} /$ STB.
- GOR less than 200 scf/STB.
- Oil gravity less than $35^{\circ} \mathrm{API}$.
- Black or deeply colored.

3. Volatile crude oil: The other characteristic properties of this oil include:

- Oil formation volume factor greater than $1.5 \mathrm{bbl} / \mathrm{STB}$.
- GORs between 2000 and 3000 scf/STB.
- Oil gravities between $45^{\circ}$ and $55^{\circ} \mathrm{API}$.
- Lower liquid recovery of separator conditions.
- Greenish to orange in color.

4. Near-critical crude oil: The near-critical crude oil is characterized by a high GOR, in excess of $3000 \mathrm{scf} / \mathrm{STB}$, with an oil formation volume factor of $2.0 \mathrm{bbl} / \mathrm{STB}$ or higher. The compositions of near-critical oils usually are characterized by $12.5-20 \mathrm{~mol} \%$ heptanes-plus, $35 \%$ or more of ethane through hexanes, and the remainder methane. It should be pointed out that nearcritical oil systems essentially are considered the borderline to very rich gas-condensates on the phase diagram.

The liquid-shrinkage curves, given in Fig. (3.9), shows the shrinkage characteristics of all categories of crude oils.

How near are the quality lines to the bubblepoint curve will determine the type of crude oil


Fig. (3.9): Liquid shrinkage curves for crude oil systems.

The ternary diagram shown in Fig. (3.10) with equilateral triangles can be conveniently used to roughly define the compositional phase boundaries that separate different types of hydrocarbon systems.


Fig. (3.10) Compositions of various reservoir fluid systems.

### 3.4 Laboratory Analysis of Reservoir Fluids:

Accurate laboratory studies of PVT and phase-equilibria behavior of reservoir fluids are necessary for characterizing these fluids and evaluating their volumetric performance at various pressure levels by conducting laboratory tests on a reservoir fluid sample. The amount of data desired determines the number of tests performed in the laboratory; however, for a successful PVT analysis it requires fluid samples that represent the original hydrocarbon in the reservoir. Fluid sampling for laboratory PVT analysis must be collected immediately after the exploration wells are drilled to properly characterize the original hydrocarbon in place. Sampling techniques depend on the reservoir.

### 3.4.1 Well Conditioning:

Proper well conditioning is essential to obtain representative samples from the reservoir. The following procedure is recommended in conditioning of an oil well for subsurface sampling. Before collecting fluid samples, the following guidelines and well condition consecutive steps must be considered:

1. The tested well should be allowed to produce for a sufficient amount of time to remove the drilling fluids, acids, and other well stimulation materials.
2. After the cleaning period, the flow rate should be reduced to one-half the flow rate used during the cleaning period.
3. The well should be allowed to flow at this reduced rate for at least 24 h . However, for tight formation or viscous oil, the reduced flow-rate period must be extended to 48 h or higher to stabilize the various parameters monitored.
4. The pressure drawdown should be controlled and remain low to ensure that bottom-hole pressure does not fall below the saturation pressure.
5. During the reduced flow-rate period, the well should be closely monitored to establish when the wellhead pressure, production rate, and GOR have stabilized.

### 3.4.2 Fluid Sampling:

The main objective of a successful sampling process is to obtain representative fluid samples for determining PVT properties. For proper definition of the type of reservoir fluid and to perform a proper fluid study, the collection of reservoir fluid samples must occur before the reservoir pressure is allowed to deplete
below the saturation pressure of the reservoir fluid. Therefore, it is essential that the reservoir fluid samples be collected immediately after the hydrocarbon discovery, and the only production before sampling be as a result of well cleanout and to remove contamination. The critical steps in any successful sampling program include:

- Avoiding two-phase flow in the reservoir; that is, bottom-hole pressure below the saturation pressure.
- Minimizing fluid contamination introduced by drilling and completion fluids.
- Obtaining adequate volumes.
- Preserving sample integrity.

There are three basic methods of sample collection:

- Subsurface (bottom-hole) sampling.
- Surface (separator) sampling.
- Wellhead sampling.

1. Subsurface sampling uses a sampler tool (eg, Schlumberger Repeat Formation Testing "RFT" or Modular Dynamic Testing tool "MDT") that is run on a wireline to the reservoir depth. After lowering the sampling chamber to the top of the producing zone, the chamber is opened hydraulically or via an electronic signal from the surface. The crude oil is allowed to flow at a very low flow rate into the tool at constant pressure to avoid the liberation of the solution gas. A piston seals the chamber and the tool is brought to the surface.


Fig. (3.11): Subsurface collection tool of PVT sample.
2. Surface sampling involves taking samples of the gas and liquid flowing through the surface separator, as conceptually shown in Fig. (3.12), and recombining the two fluids in an appropriate ratio in such a way that the recombined sample is representative of the overall reservoir fluid stream. The oil and gas samples are sent separately to the laboratory, where they are recombined before the PVT analysis is performed. Recombining separator samples requires thorough understanding of the relationship between gas solubility " $R_{S}$ " and GOR.
3. Wellhead sampling involves the collection of a fluid sample at the surface from the wellhead itself or from the flow line or the upstream side of the choke manifold, provided that the fluid is still in one-phase condition. This option is restricted to wells producing dry gas, very low GOR oils (ie, heavy oils), and highly undersaturated reservoir fluids.


Fig. (3.12): Separator samples.

### 3.4.3 PVT Laboratory Tests:

PVT tests are conducted in a "PVT CELL" and designed to study and quantify the phase behavior and properties of a reservoir fluid for the use in a numerical simulator or performing material balance applications. In general, there are two
types of laboratory tests that are traditionally performed on hydrocarbon samples:

1. Routine laboratory tests: Several laboratory tests are routinely conducted to characterize the reservoir hydrocarbon fluid, including:
a. Compositional analysis of the hydrocarbon system (chromatographic test), (see section 3.4.4).
b. Constant composition expansion (equilibrium test), (see section 3.4.5).
c. Differential liberation test, (see section 3.4.6).
d. Separator tests, (see section 3.4.7).
e. Constant volume depletion.

The $b$ and $c$ tests are very important in distinguishing between the high shrinkage oil from the low shrinkage oil and retrograde condensation oil as it will be shown later (see sections 3.4.6 \& 3.4.7).
2. Special laboratory PVT tests: These types of tests are performed for very specific applications. If a reservoir is to be depleted under miscible gas injection or gas cycling scheme, the following tests may be performed:
a. Slim-tube test.
b. Swelling test.
c. Multiple forward contact experiment.
d. Multiple backward contact experiment.
e. Flow assurance tests.

### 3.4.4 Compositional Analysis of the Reservoir Fluid (chromatographic Test):

An important test on all reservoir samples is the determination of the fluid composition. Compositional analysis generally refers to the measurement of the distribution of hydrocarbons and other components present in oil and gas samples. The components of petroleum and petroleum products number in the tens of thousands. They range in molecular weight from methane (16) to very large uncharacterized components with molecular weight in the thousands. Analysis of the hydrocarbon mixture is complicated not only by the enormous number of components, but also by the large variation in compound concentrations. Using modern chromatography techniques (as shown in figure (3.13)), samples are analyzed to determine the breakdown of the components in the sample. Gas chromatography (GC) is a technique for separating and identifying components of hydrocarbon mixtures.


Carrier gas

Fig. (3.13): Schematic of GC main components.

### 3.4.5 Constant-Composition Expansion Tests (Equilibrium Test):

Equilibrium tests are conventionally performed on all types of reservoir fluids to obtain a better understanding of the pressure-volume relationship of these hydrocarbon systems. The equilibrium test can also be referred to by different names, including pressure-volume relations, constant mass expansion, flash liberation, flash vaporization, or flash expansion. The equilibrium test is performed on crude oil systems for the purposes of determining:

- Saturation pressure.
- Relative volume " $V_{\text {rel }}$ ".
- Isothermal compressibility coefficient " $C_{o}$ " above $p_{b}$.
- Oil density " $\rho_{o}$ " at and above $p_{b}$.
- The traditional Y-function.
- The extended Y-function " $Y_{E X T}$ ".
- Oil FVF " $B_{o}$ " above $p_{b}$.
- Two-phase formation factor $B_{t}$ below $p_{b}$.

The experimental procedure involves placing a hydrocarbon fluid sample in a visual PVT cell at reservoir temperature that is held constant during the experiment. To ensure that the hydrocarbon sample exits in a single phase, the sample is pressurized to a much higher pressure than the initial reservoir
pressure. The pressure is reduced isothermally in steps by removing mercury from the PVT cell and the total hydrocarbon volume " $V_{t}$ " is measured at each pressure (see Fig. (3.14)). The combination of successive reductions in the pressure and the measurement of total fluid volume " $V_{t}$ " are performed at a equilibrium; that is, no gas or liquid is removed from the PVT cell at any time throughout the experiment. As the pressure approaches the saturation pressure, the total volume continuously increases due to the expansion of the single-phase crude oil.


Fig. (3.14): Constant composition expansion test.
Reaching the bubble point pressure marks the appearance and start of liberation of the solution gas. The saturation pressure value is identified by visual inspection and by the point of discontinuity on the plot "pressure-volume" curve, as shown in Fig. (3.15). The saturation pressure and corresponding volume are recorded and designated as " $p_{\text {sat }}$ " and " $V_{\text {sat }}$ ", respectively. The volume at the saturation pressure " $V_{\text {sat }}$ " is used as a reference volume and the total volume " $\mathrm{V}_{\mathrm{t}}$ " as a function of pressure is reported relative to $V_{\text {sat }}$ as a ratio of " $V_{t} / V_{\text {sat. }}$." This volume is termed the relative volume and is expressed mathematically by the following equation:

$$
\begin{equation*}
V_{r e l}=\frac{V_{t}}{V_{s a t}} \tag{3.1}
\end{equation*}
$$

where:
$V_{\text {rel }}=$ relative volume.
$V_{t}=$ total hydrocarbon volume.
$V_{\text {sat }}=$ volume at the saturation pressure.


Fig. (3.15): Equilibrium test pressure-volume relationship.

It should be pointed out the change in the slope is related and attributed to the fluid compressibility resulting from the transition from the single phase to the two-phase state. In the single liquid phase region, the liquid compressibility is relatively small resulting in a steep decline in the pressure with a small increase in the liquid volume. As the pressure approaches the bubble point pressure and initiation of liberation solution gas, the evolved gas is characterized by a high compressibility that causes less reduction in the pressure resulting in a sharp break in the slope between the single- and two-phase regions.

## - Oil density from the equilibrium test:

The density is determined directly from mass " $m$ " of the fluid sample and fluid volume " $V$ " measurements. Designated $\rho_{\text {sat }}$ for the fluid density at the saturation pressure, it can be calculated from:

$$
\begin{equation*}
\rho_{s a t}=\frac{m}{V_{s a t}} \tag{3.2}
\end{equation*}
$$

Above the bubble point pressure, the density of the oil can be calculated by using the recorded relative volume at any pressure, to give:

$$
\begin{equation*}
\rho=\frac{\left(m / V_{\text {sat }}\right)}{\left(V_{p, T} / V_{\text {sat }}\right)}=\frac{\rho_{\text {sat }}}{V_{\text {rel }}} \tag{3.3}
\end{equation*}
$$

where:
$\rho=$ density at any pressure above the saturation pressure.
$\rho_{\text {sat }}=$ density at the saturation pressure.
$V_{r e l}=$ relative volume at the pressure of interest.
$V_{p, T}=$ volume at the pressure " $p$ " and reservoir (cell) temperature.

- Isothermal compressibility coefficient from equilibrium test:

The instantaneous isothermal compressibility coefficient " $c_{o}$ " is defined by the following expression:

$$
\begin{gather*}
c_{o}=-\frac{1}{V} \frac{\Delta V}{\Delta p} \\
c_{o}=\frac{-1}{\left(V_{\text {rel }}\right)_{2}} \frac{\left(V_{\text {rel }}\right)_{1}-\left(V_{\text {rel }}\right)_{2}}{p_{1}-p_{2}} \tag{3.4}
\end{gather*}
$$

## Example 3.1:

Using the following data of CCE test to:

| Pressure (psig) | Total Volume (cc) |
| :---: | :---: |
| 5000 | 61.030 |
| 4500 | 61.435 |
| 4000 | 61.866 |
| 3500 | 62.341 |
| 3000 | 62.866 |
| 2900 | 62.974 |
| 2800 | 63.088 |
| 2700 | 63.208 |
| 2605 | 63.455 |
| 2591 | 63.576 |
| 2516 | 64.291 |
| 2401 | 65.532 |
| 2253 | 67.400 |
| 2090 | 69.901 |
| 1897 | 73.655 |
| 1698 | 78.676 |

1. Find the saturation pressure.
2. Find the relative volume.
3. Calculate $\rho_{\text {sat }}$ if $\mathrm{m}=0.094 \mathrm{lb}$.

## Solution:

1. Plot pressure vs. volume to find the saturation pressure:


From the pressure volume relationship, the saturation pressure is ( 2570 psig) and the saturation volume is ( 63.776 cc ).
2. The values of relative volume are calculated using Eq. (3.1):

$$
V_{\text {rel }}=\frac{V_{t}}{V_{\text {sat }}}=\frac{V_{t}}{63.776}
$$

the results are shown in the table below:

| Pressure $(\mathrm{psig})$ | Total Volume $(\mathrm{cc})$ | $\mathrm{V}_{\text {rel }}$ |
| :---: | :---: | :---: |
| 5000 | 61.030 | 0.95694305 |
| 4500 | 61.435 | 0.963293402 |
| 4000 | 61.866 | 0.97005143 |
| 3500 | 62.341 | 0.977499373 |
| 3000 | 62.866 | 0.98573131 |
| 2900 | 62.974 | 0.987424737 |
| 2800 | 63.088 | 0.989212243 |
| 2700 | 63.208 | 0.991093828 |


| 2605 | 63.455 | 0.994966759 |
| :---: | :---: | :---: |
| 2591 | 63.576 | 0.996864024 |
| 2570 | 63.776 | 1 |
| 2516 | 64.291 | 1.008075138 |
| 2401 | 65.532 | 1.027533869 |
| 2253 | 67.400 | 1.056823884 |
| 2090 | 69.901 | 1.096039262 |
| 1897 | 73.655 | 1.15490153 |
| 1698 | 78.676 | 1.233630206 |

3. $V_{\text {sat }}=63.776 \mathrm{cc}$

$$
\rho_{s a t}=\frac{m}{V_{s a t}}=\frac{0.094 \mathrm{lb}}{63.776 c c\left(35.516 \times 10^{-6}\right) \frac{c u \mathrm{ft}}{c c}}=41.78 \mathrm{lb} / \mathrm{cu} \mathrm{ft}
$$

## - Traditional Y-Function:

The relative volume data frequently require smoothing to correct for laboratory inaccuracies in measuring the total hydrocarbon volume, particularly below the saturation pressure and also at lower pressures. A dimensionless compressibility function, commonly called the Y-function, is used to smooth the values of the relative volume " $V_{\text {rel }}$ " below the saturation pressure. The $Y$-function in its mathematical expression is ONLY defined below the saturation pressure; that is, $p<p_{b}$, and given by the following equation:

$$
Y=\frac{p_{b}-p}{p\left[\frac{V_{t}}{V_{b}}-\mathbf{1}\right]}
$$

where:
$p_{b}=$ bubble point or saturation pressure.
$p=$ reservoir pressure which Y is being calculated.
$\frac{V_{t}}{V_{b}}=$ relative total volume at the pressure $p$.

The Y-function is computed and plotted as a function of the pressure $p$, as shown in the figure (3.16) below. For more details see (Amyx, et al, 1960, p384).


Figure (3.16): smoothing of relative total volume data.

### 3.4.6 Differential Liberation Test (DL):

The differential liberation (DL) test is perhaps the most common laboratory test that is conducted on crude oil samples. The DL process is summarized in the following steps and is illustrated schematically in Fig. (3.17):

- A crude oil sample is placed in a visual PVT cell at its bubble point pressure and reservoir temperature.
- The cell pressure is reduced in stages, usually $10-15$ pressure increments, from the saturation pressure to atmospheric pressure.
- The liberated gas at each pressure stage is allowed to reach equilibrium with the cell remaining oil.
- The volume of the two phases (ie, remaining oil volume " $V_{L}$ " and liberated gas volume " $V_{\text {gas }}$ ") are measured and recorded at each pressure level.
- The liberated gas is then displaced at constant cell pressure to a metering device (e.g., gasometer) and the measured volume is corrected to SC and designated to as " $\left(V_{g a s}\right)_{s c}$ ".
- The above depletion process is repeated at constant reservoir temperature until a pressure close to atmospheric pressure is reached. Having arrived at the last stage, the volume of the residual oil is first measured at last cell
conditions, then corrected to SC (ie, 14.7 psia and $60^{\circ} \mathrm{F}$ ), and designated as " $V_{L s c}$ ".


Fig. (3.17): Differential vaporization test at constant temperature " $T$."
The following nomenclatures are used to describe the various hydrocarbon volume measurements associated with each pressure-depletion stage during the DL experiment:
$P=$ cell pressure.
$T=$ cell temperature.
$V_{t}=$ total combined volume of the liberated gas and remaining oil at $p \& T$.
$V_{L}=$ remaining oil volume at $p \& T$.
$V_{L s c}=$ remaining oil volume at $p_{s c} \& T_{s c}$.
$V_{g a s}=$ liberated gas volume at $p \& T$.
$\left(V_{g a s}\right)_{s c}=$ liberated gas volume at $p_{s c} \& T_{s c}$.
It should be noted that the remaining oil at each depletion stage is subjected to continual compositional changes as it becomes progressively richer in the heavier components. In addition, the above described type of DL is characterized by a varying composition of the total hydrocarbon system; whereas the total system composition is kept unchanged during the equilibrium test.

The experimental data obtained from the DL test include:

- The differential oil FVF " $B_{o d}$ ".
- The differential gas solubility; that is, solution GOR, " $R_{s d}$ ".
- Gas FVF " $B_{g}$ " in $f t^{3} / \mathrm{scf}$.
- Gas compressibility factor " $Z$ ".
- Total FVF " $B_{t d}$ ".
- Composition of the liberated gas.
- Gas specific gravity.
- Density of the remaining oil as a function of pressure.

The combined data from the DL and equilibrium tests are very important to distinguish between low-shrinkage oil and high-shrinkage oil as be shown in the figures (3.18) \& (3.19). In Fig. (3.18) under equilibrium conditions, a greater quantity of heavy components are progressively pulled into gas by the light gas components; thus, oil shrinkage is greater. Lower shrinkage by DL under these conditions is caused by gas being removed as it's formed, thereby preventing further attraction of light gas molecules to heavy liquid molecules and minimizing the quantity of liquid pulled into the gas phase.

A comparison between equilibrium and DL of a very high shrinkage oil is presented in Fig. (3.19). In this case the difference is reversed from that of low shrinkage oil. More gas is formed in the high pressure range than with low shrinkage oil. This is caused by a large quantity of intermediates being pulled into the gas phase (through great attraction to the densely spaced light gas molecules and through high kinetic energy of the intermediates). Once the large quantity of intermediates is removed from the system by DL, they have no further influence on gas liberation and oil shrinkage; therefore, oil shrinkage remains high through lower pressure ranges. If the gas is not removed, as in flash or equilibrium liberation, some intermediate gas molecules may condense because there attraction to light gas molecules is reduced as pressure drops and the light gas molecules become farther apart; thus, vaporizing tendency of high shrinkage oil may be reversed until shrinkage is less under equilibrium conditions than under the differential process.

Shrinkage curves of both low and high shrinkage oil illustrate that, near atmospheric temperature and low pressure conditions, shrinkage of oil greatly increases and presents a problem if maximum liquid volume is to be produced to the stock tank as saleable material.

Oil of all degrees of shrinkage will shrink less to the stock tank if the material is first passed to a separator at some elevated pressure and free gas, formed down to that pressure and temperature, is separated. Also, there is an optimum separator pressure condition where minimum shrinkage of oil is obtained.


Fig. (3.18): Differential and equilibrium shrinkage of low shrinkage oil, (Clark, 1969).


Fig. (3.19): Differential and equilibrium shrinkage of high shrinkage oil, (Clark, 1969).

## Differential Gas Solubility " $\boldsymbol{R s}_{\text {sd }}$ ":

The gas solubility (i.e., differential solution GOR, " $R_{s d}{ }^{\prime}$ ") at any pressure is defined as the volume of gas that is dissolved (remaining) in solution at the pressure per stock-tank barrel of the corrected residual oil. Mathematically, the differential solution GOR, $R_{s d}$, is calculated by dividing the volume of gas remaining (dissolved) in solution at the specified pressure by the residual oil volume.

$$
\begin{gathered}
R_{s d}=\frac{\text { volume of remaining solution gas }}{\text { volume of residual oil }} \\
R_{s d}=\frac{\text { vol.of total gas removed }- \text { vol. of gas removed till the cell wanted }}{\text { vol.of residual oil }}
\end{gathered}
$$

## Differential Oil FVFs "Bod ${ }_{\text {od }}$ :

The differential oil FVFs, Bod (commonly called the relative oil volume factors), are calculated at all pressure levels by dividing the recorded oil volumes, $V_{L}$, by the volume of residual oil, $V_{L s c}$ :

$$
\begin{equation*}
B_{o d}=\frac{\text { volume of oil at any }(p, T)}{\text { volume of residual oil }\left(\text { at } p_{s c}, T_{s c}\right)}=\frac{V_{L}}{V_{L s c}} \tag{3.5}
\end{equation*}
$$

## Gas Deviation Factor "Z":

Using the volume of the free gas at any pressure and the corresponding volume at SC, the gas deviation factor can be calculated by applying the following equation:

$$
\begin{equation*}
Z=\frac{p\left(V_{\text {gas }}\right)_{p, T}}{T} \frac{T_{s c}}{p_{s c}\left(V_{g a s}\right)_{s c}}=\frac{V_{R} p_{R} T_{s c}}{V_{s c} p_{s c} T_{R}} \tag{3.6}
\end{equation*}
$$

## Gas FVF "Bg":

The gas FVF is the defined ratio of the volume " $\left(V_{g a s}\right)_{p, T}$ " occupied by n moles of gas at a specified pressure and temperature to the volume " $\left(V_{g a s}\right)_{s c}$ " occupied by the same number of moles (i.e., $n$ moles) at SC. Mathematically, this is defined by

$$
\begin{equation*}
B_{g}=\frac{\left(V_{\text {gas }}\right)_{p, T}}{\left(V_{\text {gas }}\right)_{s c}}=\frac{0.02827 \mathrm{ZT}}{p} ; f t^{3} / s c f \tag{3.7}
\end{equation*}
$$

## Differential gas oil ratio at the bubble point pressure " $\boldsymbol{R}_{\text {sdb }}$ ":

$R_{s d b}$ value is constant in all steps of calculations.

$$
R_{s d b}=\frac{\text { vol.of total gas removed }}{\text { vol.of residual oil }}
$$

## Total FVF "Btd" From DL Test:

The two-phase (total) FVF from DL test is determined by applying the definition of this property as expressed mathematically by Eq. (3.8). In terms of differential data, $B_{t d}$ is given by

$$
\begin{equation*}
B_{t d}=B_{o d}+\left(R_{s d b}-R_{s d}\right) B_{g} \tag{3.8}
\end{equation*}
$$

where:
$R_{s d b}=$ gas solubility at the bubble point pressure, scf/STB.
$R_{s d}=$ gas solubility at any pressure, $\mathrm{scf} / \mathrm{STB}$.
$B_{o d}=$ oil FVF at any pressure, bbl/STB.
$B_{g}=$ gas $\mathrm{FVF}, \mathrm{bbl} / \mathrm{scf}$.

## Example 3.2:

The data from a differential vaporization on a black oil at $\left(220^{\circ} \mathrm{F}\right)$ are given below. Prepare a table of solution gas- oil ratios, relative oil volumes, and relative total volumes by this differential process. Also include Z- factors and formation volume factors of the increments of gas removed.

| Pressure <br> $(\mathbf{p s i g})$ | Gas <br> removed,* <br> $(\mathbf{c c})$ | Gas <br> removed,** <br> $(\mathbf{s c f})$ | Oil volume, <br> $(\mathbf{c c})$ | Incremental <br> gas gravity |
| :---: | :---: | :---: | :---: | :---: |
| 2620 | - | - | 63.316 | - |
| 2350 | 4.396 | 0.02265 | 61.496 | 0.825 |
| 2100 | 4.292 | 0.01966 | 59.952 | 0.818 |
| 1850 | 4.478 | 0.01792 | 58.528 | 0.797 |
| 1600 | 4.960 | 0.01693 | 57.182 | 0.791 |
| 1350 | 5.705 | 0.01618 | 55.876 | 0.794 |
| 1100 | 6.891 | 0.01568 | 54.689 | 0.809 |
| 850 | 8.925 | 0.01543 | 53.462 | 0.831 |
| 600 | 12.814 | 0.01543 | 52.236 | 0.881 |
| 350 | 24.646 | 0.01717 | 50.771 | 0.988 |
| 159 | 50.492 | 0.01643 | 49.228 | 1.213 |
| 0 |  | 0.03908 | 42.540 | 2.039 |
| 0 |  | 0.21256 | 39.572 at60${ }^{\circ} \mathrm{F}$ |  |

* at $220^{\circ} \mathrm{F}$ and cell pressure.
**at $60^{\circ} \mathrm{F}$ and $14.65 \mathrm{psia}(\cong 14.7)$.


## Solution:

Calculations will be at ( 2100 and 1600 psig) as samples.
Step 1. calculation of differential solution GOR " $R_{s d}$ " at ( 2100 psig ):

$$
R_{s d}=\frac{(0.21256-0.02265-0.019966) s c f}{(39.572 c c, \text { residual oil })\left(6.29 \times 10^{-6} \mathrm{bbl} / c c\right)}=684 \mathrm{scf} / \mathrm{bbl}
$$

Step 2. calculation of differential solution GOR " $R_{\text {sd }}$ " at ( 1600 psig ):
$R_{s d}=\frac{(0.21256-0.02265-0.01966-0.01792-0.01693) s c f}{(39.572 \text { cc, residual oil })\left(6.29 \times 10^{-6} \mathrm{bbl} / \mathrm{cc}\right)}=544 \mathrm{scf} / \mathrm{bbl}$

Step 3. calculation of " $B_{o d}$ " at ( 2100 psig ):

$$
B_{o d}=\frac{59.952(c c, \text { at reservoir } p, T)}{39.572(c c, \text { residual oil at standard conditions })}=1.515 \mathrm{bbl} / \mathrm{STB}
$$

Step 4. calculation of " $B_{o d}$ " at ( 1600 psig ):

$$
B_{o d}=\frac{57.182(c c, \text { at reservoir } p, T)}{39.572(c c, \text { residual oil at standard conditions })}=1.445 \mathrm{bbl} / \mathrm{STB}
$$

Step 5. calculation of Z-factor at ( 2100 psig ):

$$
\begin{gathered}
Z=\frac{V_{\text {res }} p_{\text {res }} T_{s c}}{V_{s c} p_{s c} T_{\text {res }}} \\
Z=\frac{\left(4.292 c c \times 35.315 \times 10^{-6} f t^{3} / c c\right)(2114.7 p s i a)\left(520^{\circ} \mathrm{R}\right)}{(0.01966 s c f)(14.65 p s i a)\left(680^{\circ} \mathrm{R}\right)}=0.851
\end{gathered}
$$

Step 6. calculation of Z-factor at ( 1600 psig ):

$$
Z=\frac{\left(4.960 c c \times 35.315 \times 10^{-6} f t^{3} / c c\right)(1614.7 p \operatorname{sia})\left(520^{\circ} R\right)}{(0.01693 s c f)(14.65 p s i a)\left(680^{\circ} R\right)}=0.872
$$

Step 7. calculation of " $B_{g}$ " at ( 2100 psig ):

$$
\begin{gathered}
B_{g}=0.0282 \frac{Z T}{p} \\
B_{g}=\frac{(0.0282)(0.851)(680)}{(2114.7)}=0.00771 \mathrm{ft}^{3} / \mathrm{scf}
\end{gathered}
$$

Step 8. calculation of " $B_{g}$ " at ( 1600 psig ):

$$
B_{g}=\frac{(0.0282)(0.872)(680)}{(1614.7)}=0.01035 \mathrm{ft}^{3} / \mathrm{scf}
$$

Step 9. calculation of solution GOR at bubble point pressure " $R_{\text {sdb }}$ ":

$$
\begin{gathered}
R_{s d b}=\frac{\text { vol.of total gas removed }}{\text { vol.of residual oil }} \\
R_{s d b}=\frac{(0.21256 s c f)\left(5.615 f t^{3} / b b l\right)}{(39.572 c c)\left(35.315 \times 10^{-6} f t^{3} / c c\right)}=854 \mathrm{scf} / \mathrm{bbl}
\end{gathered}
$$

Step 10. calculation of " $B_{t}$ " at ( 2100 psig ):

$$
\begin{gathered}
B_{t}=B_{o d}+B_{g}\left(R_{s d b}-R_{s d}\right) \\
B_{t}=1.515 \frac{b b l}{S T B}+\frac{0.00771 f t^{3} / s c f}{5.615 f^{3} / b b l}(854-684) \frac{s c f}{b b l}=1.748 \mathrm{bbl} / \mathrm{STB}
\end{gathered}
$$

Step 11. calculation of " $B_{t}$ " at ( 1600 psig ):

$$
B_{t}=1.445 \frac{b b l}{S T B}+\frac{0.01035 f t^{3} / s c f}{5.615 f t^{3} / b b l}(854-544) \frac{s c f}{b b l}=2.0164 \mathrm{bbl} / \mathrm{STB}
$$

The table below show the total calculations of all pressure values:

| Pressure <br> $(\mathbf{p s i g})$ | $\boldsymbol{R}_{\text {sd }}$ <br> $(\boldsymbol{s c f} / \boldsymbol{b} \boldsymbol{b} \boldsymbol{l})$ | $\boldsymbol{B}_{\text {od }}$ <br> $(\boldsymbol{b} \boldsymbol{b l} / \boldsymbol{S T B})$ | Z-factor | $\boldsymbol{B}_{\boldsymbol{g}}$ <br> $\left(\boldsymbol{f t ^ { 3 }} / \boldsymbol{s c} \boldsymbol{f}\right)$ | $\boldsymbol{B}_{\boldsymbol{t}}$ <br> $(\boldsymbol{b} \boldsymbol{b l} / \boldsymbol{S T B})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2620 | $854=R_{\text {sdb }}$ | 1.600 |  |  | 1.60002 |
| 2350 | 763 | 1.554 | 0.846 | 0.00686 | 1.66523 |
| 2100 | 684 | 1.515 | 0.851 | 0.00772 | 1.748665 |
| 1850 | 612 | 1.479 | 0.859 | 0.00883 | 1.859753 |
| 1600 | 544 | 1.445 | 0.872 | 0.01036 | 2.016848 |
| 1350 | 479 | 1.412 | 0.887 | 0.01246 | 2.244538 |
| 1100 | 416 | 1.382 | 0.903 | 0.01553 | 2.593997 |
| 850 | 354 | 1.351 | 0.922 | 0.02045 | 3.17193 |
| 600 | 292 | 1.320 | 0.941 | 0.02936 | 4.258547 |
| 350 | 223 | 1.283 | 0.965 | 0.05074 | 6.985549 |
| 159 | 157 | 1.244 | 0.984 | 0.10863 | 14.7302 |
| 0 | 0 | 1.075 |  |  |  |
| $0 @ 60^{\circ} \mathrm{F}$ |  | 1.000 |  |  |  |

## Example 3.3:

A DL test was conducted on a crude oil sample at $\left(180^{\circ} \mathrm{F}\right)$. The sample exhibited a bubble point pressure of ( 3565 psig ). The residual (remaining) oil volume was corrected to SC of 14.7 psia and $60^{\circ} \mathrm{F}$ to give $\left(60 \mathrm{~cm}^{3}\right)$. The following measurements were obtained from a differential liberation (DL) test:

| Pressure <br> (psig) | Total Volume <br> (cc) | Liquid volume <br> (cc) | Gas removed <br> (scf) |
| :---: | :---: | :---: | :---: |
| 3565 | 99.29514779 | 99.29514779 | 0 |
| 3000 | 105.1180611 | 92.66229773 | 0.086792948 |
| 2400 | 100.8688861 | 87.02573049 | 0.07660421 |
| 1800 | 98.43652432 | 82.03645568 | 0.067170194 |
| 1200 | 100.8219751 | 77.70667102 | 0.061887145 |
| 600 | 118.801706 | 73.45739134 | 0.059245621 |
| 200 | 168.9280437 | 70.05558226 | 0.043396474 |
| 0 | 63.85313132 | 63.85313132 | 0.061509785 |
| 0 psig, $60^{\circ} \mathrm{F}$ |  | 60 | $\sum 0.456606$ |

Using the above reported DL measurements data to calculate:

1. Oil FVF " $B_{o d}$ ".
2. Solution GOR " $R_{s d}$ ".
3. Gas FVF " $B_{g}$ ".
4. Z-factor.

## Solution:

1. As a sample, calculating $B_{o d}$ at 3000 psig:

$$
B_{o d}=\frac{92.66229773}{60}=1.544371629
$$

2. Calculate the differential solution GOR " $R_{s d}$.":

Step 1. Convert the volume of the corrected residual oil volume as expressed in cubic centimeter to stock-tank barrel:
Stock - tank volume $=\frac{60 \mathrm{~cm}^{3}}{(30.48 \mathrm{~cm} / \mathrm{ft})^{3}\left(5.615 \mathrm{ft}^{3} / \mathrm{bbl}\right)}=0.000377$ STB
Step 2. Calculate $R_{s d}$ by dividing the volume of the dissolved gas at each pressure by the volume of residual oil ( 0.000377 STB ), as a sample calculating $R_{s d}$ at 3000 psig :

$$
R_{s d}=\frac{(0.456606-0.086792948) s c f}{0.000377 \mathrm{STB}}=980 s c f / S T B
$$

3. As a sample, calculating $B_{g}$ at 3000 psig and $180^{\circ} \mathrm{F}$ :

Step 1. volume of the free gas at $3000 \mathrm{psig}=$ total volume - volume of the liquid:

$$
\left(V_{\text {gas }}\right)_{3000,180}=\frac{(105.1180611)-(92.66229773)}{(30.48)^{3}}=0.00044 \mathrm{ft}^{3}
$$

Step 2. Calculate $B_{g}$ at 3000 psig and $180^{\circ} \mathrm{F}$ by applying the definition of this gas property:

$$
B_{g}=\frac{0.00044}{0.086792948}=0.005068 \mathrm{ft}^{3} / s c f
$$

4. As a sample calculating Z-factor at 3000 psig :

$$
\begin{gathered}
Z=\frac{V_{\text {res }} p_{\text {res }} T_{\text {sc }}}{V_{s c} p_{s c} T_{\text {res }}} \\
Z=\frac{(3000+14.7)(0.00044)(520)}{(180+460)(14.7)(0.08679)}=0.8445
\end{gathered}
$$

The table below show the total calculations of all pressure values:

| Pressure (psig) | $\begin{gathered} R_{s d} \\ (s c f / b b l) \end{gathered}$ | $\begin{gathered} B_{\text {od }} \\ (b b l / S T B) \end{gathered}$ | Z-factor | $\begin{gathered} B_{g} \\ \left(f t^{3} / s c f\right) \end{gathered}$ | $\begin{gathered} B_{t} \\ (b b l / S T B) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3565 | 1211 | 1.6549 |  |  | 1.655 |
| 3000 | 980 | 1.5443 | 0.84448 | 0.005068 | 1.752 |
| 2400 | 777 | 1.4504 | 0.85174 | 0.006382 | 1.943 |
| 1800 | 599 | 1.3672 | 0.864839 | 0.008622 | 2.306 |
| 1200 | 435 | 1.2951 | 0.885583 | 0.01319 | 3.116 |
| 600 | 278 | 1.2242 | 0.918313 | 0.027028 | 5.711 |
| 200 | 163 | 1.1675 | 0.954804 | 0.080459 | 16.17 |
| 0 | 0 | 1.0642 |  |  |  |
| $0 @ 60{ }^{\circ} \mathrm{F}$ |  | 1.000 |  |  |  |

### 3.4.7 Separator Tests:

Separator tests are conducted to determine the changes in the volumetric behavior of the reservoir fluid as the fluid passes through the separator (or separators) and then into the stock tank (as shown in Fig. (3.20)). The resulting volumetric behavior is influenced to a large extent by the operating conditions,
i.e., pressures and temperatures, of the surface separation facilities. The primary objective of conducting separator tests, therefore, is to provide the essential laboratory information necessary for determining the optimum surface separation conditions, which in turn will maximize the stock-tank oil production. In addition, the results of the test, when appropriately combined with the differential liberation test data, provide a means of obtaining the PVT parameters ( $B_{o}, R_{s}$, and $B_{t}$ ) required for petroleum engineering calculations. These separator tests are performed only on the original oil at the bubble point.


Fig. (3.20): Single-stage separation system (one separator), (Clark, 1960).
The test involves placing a hydrocarbon sample at its saturation pressure and reservoir temperature in a PVT cell. The volume of the sample is measured as $V_{\text {sat }}$. The hydrocarbon sample is then displaced and flashed through a laboratory multistage separator system-commonly one to three stages. The pressure and temperature of these stages are set to represent the desired or actual surface separation facilities. The gas liberated from each stage is removed and its specific gravity and volume at standard conditions are measured. The volume of the remaining oil in the last stage (representing the stock-tank condition) is measured and recorded as $\left(V_{o}\right)_{\mathrm{st}}$. These experimental measured data can then be used to determine the oil formation volume factor and gas solubility at the bubble-point pressure as follows:

$$
\begin{align*}
& B_{o f b}=\frac{V_{s a t}}{\left(V_{o}\right)_{s t}}  \tag{3.9}\\
& R_{s f b}=\frac{\left(V_{g}\right)_{s c}}{\left(V_{o}\right)_{s t}} \tag{3.10}
\end{align*}
$$

where:

$$
\begin{aligned}
B_{o f b}= & \text { bubble-point oil formation volume factor, as measured by flash } \\
& \text { liberation, bbl of the bubble-point oil/STB. }
\end{aligned}
$$

$R_{s f b}=$ bubble-point solution gas-oil ratio as measured by flash liberation, scf/STB.
$\left(V_{g}\right)_{\mathrm{sc}}=$ total volume of gas removed from separators, scf.
Almost without exception, oil production is attended by gas that comes from solution in the oil at reservoir conditions. This gas, when liberated from solution in the oil, cause the oil to shrinkage in volume. The term "shrinkage factor" is given to the ratio of the amount of oil recovered in the stock tank from one unit volume of produced reservoir oil. The greater the amount of gas evolving from the oil, the greater will be the shrinkage of the oil and the smaller will be the volume of saleable oil at the surface from a given volume of produced reservoir oil. Both the amount of gas which a given reservoir oil will liberate and the resulting shrinkage factor will depend upon the composition of the reservoir oil and the temperature and pressure under which the oil and gas exist at separation.

### 3.4.7.1 Differential and Flash Separation:

To best explain the various separation processes, it is convenient to consider the component hydrocarbon as "lights", "intermediates" and "heavies" (as explained earlier in the section (3.1)). Mechanically, there are two types (or methods) of gas-oil separation: "differential" separation and "flash" or "equilibrium" separation. The light components may be removed little by little from the remaining oil mixture as pressure on the oil is reduced; this is differential separation. If the gas is separated in this manner, a maximum amount of heavy and intermediate components will remain in the liquid and minimum shrinkage will result. This occurs because gas liberated earlier at higher pressures is not present at lower pressures to attract the intermediates and heavies and pull them into the gas phase in excessive quantities. Flash, or equilibrium separation, is accomplished by keeping all liberated gas in contact with the liquid until its instantaneous removal at the final separation pressure. A
maximum proportion of intermediate and heavy components are attracted into the gas phase by this process and result in maximum oil shrinkage. Less oil shrinkage and, thus, greater stock tank oil recovery will normally occur by differential separation processes than will occur by flash separation.

### 3.4.7.2 Single-Stage Separation

It can be seen from the foregoing that the best mechanical separation system for retaining absolute maximum recovery of stock tank liquid would be one which would include a large number of separators. In such a system, the oil would be produced to a separator set at the highest possible producible pressure. The separator liquid obtained therein would be passed to successively lower pressure separators until the remaining liquid finally would emerge from a very low pressure separator into the stock tank. This would result in maximum oil recovery because such a procedure would approach the differential liberation process. Quite obviously, such a large separation system would require an extremely high investment, and for that reason, it is impractical. In practice, however, with most average shrinkage oils, only one separator is utilized in the system. Such a system is termed "single-stage separation" (see Fig. (3.18)).

Single-stage separation of low shrinkage oils generally will recover an amount of oil very near to that which might be expected under conditions of many stages of separation.

In a single-stage separation system, the gas is removed in two batches from the original liquid as pressure drops. The first batch of gas is removed in the separator and the second in the stock tank. By removing some gas in a separator the quantity of intermediate and heavy components attracted to the lights of the gas are minimized and final oil shrinkage is lessened so that more oil is retained in the tank from a given amount of reservoir oil.

It has been found that separator pressure can be adjusted to some particular point which will result in less shrinkage to the oil than would occur at other separator pressures.

1. If separator pressure is set high, a small amount of gas will be removed in the first step at the separator, and a large amount will be removed in the second step at the stock tank. Large quantities of intermediates and heavier components will be lost from the liquid to gas in the stock tank where a large quantity of gas is removed as a second "batch"; therefore, high shrinkage with low stock tank oil recovery will be obtained.
2. If the separator pressure is set very low, a large amount of gas will be separated in the first step at the separator, with a small amount of gas being separated in the second step from the stock tank. Here again, large quantities of intermediates and heavier components will be lost from the liquid to the gas in the separator where a large quantity of gas is removed as a first batch; therefore, high shrinkage with low stock tank oil recovery again will result.
3. If the separator pressure is adjusted to more properly balance the amounts of gas removed, at the separator and at the stock tank, a minimum of intermediates and heavier components will be lost from the liquid to the gas; therefore, lowest shrinkage will occur to the oil and the highest stock tank oil recovery will be obtained from a particular reservoir liquid. The separator pressure found to accomplish this is termed "optimum separator pressure" (see Fig. (3.21)).


Fig. (3.21): Optimum single-stage oil separation for a given oil, (Clark, 1960).

### 3.4.7.3 Two-Stage Separation:

Some reservoir oils are classified high shrinkage oils. Although this classification is arbitrary, these oils have higher shrinkage than usual because in
the original mixture there are greater quantities of lighter or intermediate components present which vaporize to greater extent when pressure on the mixture is lowered. Therefore, it may be desirable to employ two separator in the system to obtain optimum economic separation. Such a system is known as "two-stage separation" (see Fig. (3.22)). In the case of high shrinkage oil, the two stages of separation merely permit the gas to be removed at three points: first in the high pressure separator, second in the low pressure separator and third in the stock tank. With the gas divided into three batches, there is less tendency for these smaller amounts, separated under their respective conditions of temperature and pressure, to attract and remove the intermediate and heavier (and more valuable) components from the liquid.

In this system, there are optimum pressure conditions for both high and low pressure separators just as there are for one separator system.


Fig. (3.22): Two-stage separation system (two separators), (Clark, 1960).

### 3.4.7.4 Relationship of Gasoline Plants in Separation:

It is not always economical to separate oil and gas by optimum separation. In many cases (for example, in high shrinkage oils and especially for oils where large reserves are concerned), it may be most economical to produce the separated gas to a gasoline plant and remove the last remaining quantities of intermediate and heavy materials in the form of natural gasoline (even after
considerable investment for gasoline plant equipment has been made). If such is true, it may be economically justifiable to operate the separators under vacuum conditions to further vaporize intermediate and heavy materials into the gas separated at the separator. This results in removing from the liquid at the separator a maximum amount of intermediate and heavier components which would be passed as gas to the gasoline plant, providing higher volumes of gasoline plant products.

### 3.4.7.5 Economic Summary:

The over-all economic problems involving oil and gas separation is summarized in Fig. (3.23) which schematically shows a producing well connected to two types of separation systems: one to an optimum and the other to a low pressure separation. In the case of optimum separation, maximum oil will be recovered in the stock tank; less intermediate and heavier components will be produced to the gasoline plant; and plant product yield will be small. In the case of low pressure separation system, a minimum of stock tank oil will be recovered in the stock tank; more intermediate and heavier components will be produced to the gasoline plant; and plant produce yield will be large. Therefore, the over-all economics of project would include such factors as prices of gasoline plant products and stock tank oil, cost of investment in gasoline plant equipment and separators, degree of oil shrinkage, analysis of separated gas, availability and uniformity of markets, and size of oil and gas reserves.


Fig. (3.23): Separation and gasoline plant yield, (Clark, 1960).

## Problems

1. The average gas-oil ratio produced from the upper WashitaFredericksburg formation of the Summerland Field is ( $275 \mathrm{scf} / \mathrm{STB}$ ). The gravity of the produced oil is $\left(26^{\circ} \mathrm{API}\right)$. The color of the stock-tank oil is black. What type of reservoir fluid is in this formation?
2. Laborotary analysis of a sample from the Summerland (Upper WashitaFredericksburg) Field indicates a ratio of volume oil leaving the reservoir to volume of oil arriving at the stock-tank of ( $1.10 \mathrm{resbbl} / \mathrm{STB}$ ). Does this information confirm your answer to the problem 1? Why or why not?
3. A field in north Louisiana discovered in 1953 and developed by 1956 had an initial producing gas-oil ratio of ( $2000 \mathrm{scf} / \mathrm{STB}$ ). The stock-tank liquid was 'medium orange' and had a gravity of ( $51.2{ }^{\circ} \mathrm{API}$ ). Classify this reservoir fluid.
4. Answer the following questions:
a. Cricondentherm of a black oil is always higher than the cricondentherm of a dry gas. True or False
b. Dry gases have the widest phase envelope among all five reservoir fluids. True or False
c. In a saturated oil reservoir, the bubble-point pressure and dewpoint pressure of the oil column and gas cap fluids, respectively, are both equal to the reservoir pressure at the reservoir temperature. True or False
d. A retrograde condensate is produced in a dry gas reservoir. True or False
e. Gas-to-condensate (oil) ratio and the API gravity of the produced condensate remain constant throughout the producing life of a wetgas reservoir. True or False
f. In gas-condensate reservoirs, gas-to-condensate (oil) ratio and the API gravity of the produced condensate remain constant as long as the reservoir pressure stays above the dew-point pressure. True or False
g. The production characteristics of an Alaska North Slope reservoir include a GOR of ( $548 \mathrm{scf} / \mathrm{STB}$ ), stock tank oil of ( $26.9^{\circ} \mathrm{API}$ ), and
a formation volume factor of (1.29 res. $\mathrm{Bbl} / \mathrm{STB}$ ). What type of fluid is in this reservoir?
h. The initial reservoir pressure and temperature in a North Sea reservoir is ( 5000 psia ) and $\left(260^{\circ} \mathrm{F}\right)$. The PVT analysis indicated the bubble-point pressure of the oil at ( 3500 psia ). Is the reservoir fluid saturated or undersaturated? How do you know?
5. Compositional analysis of a reservoir fluid from a field in India reported a $\mathrm{C}_{7+}$ of ( $15.0 \mathrm{~mol} \%$ ), while the PVT analysis of this fluid indicated a formation volume factor of ( 2.5 res. $\mathrm{bbl} / \mathrm{STB}$ ). What type of reservoir fluid exists in this field?
6. A PVT cell contains a natural gas mixture at ( 1400 psia ) and $\left(190^{\circ} \mathrm{F}\right)$. The cell volume is $\left(10.0 \mathrm{ft}^{3}\right)$ and the total lb-mol of gas are determined to be (2.0). Calculate the gas deviation factor.
7. The following PV data from a CCE test at $220^{\circ} \mathrm{F}$ are available for a reservoir oil. Determine the bubble-point pressure of this oil:

| Pressure (psia) | Total Cell Volume (cc) |
| :---: | :---: |
| 5000 | 144.6 |
| 4100 | 146.3 |
| 3500 | 147.7 |
| 2900 | 149.2 |
| 2700 | 149.8 |
| 2605 | 150.3 |
| 2516 | 152.4 |
| 2253 | 159.7 |
| 1897 | 174.4 |
| 1477 | 204.2 |
| 1040 | 267.6 |
| 640 | 414.0 |

8. The following data are available from a laboratory test carried out on a black oil at $\left(225^{\circ} \mathrm{F}\right)$. What is the bubble-point pressure of this oil? Calculate the total formation volume factor and subsequently plot pressure versus $B_{o}$ and $B_{t}$ :

| Pressure (psia) | $R_{S}($ scf/STB $)$ | $B_{o}$ (res.bbl/STB) | Gas Z-factor |
| :---: | :---: | :---: | :---: |
| 4500 | 632 | 1.3474 |  |
| 4000 | 632 | 1.3575 |  |
| 3500 | 632 | 1.3686 |  |
| 3000 | 632 | 1.3811 |  |
| 2682 | 632 | 1.4040 |  |
| 2500 | 584 | 1.3782 | 0.8140 |
| 2200 | 509 | 1.3369 | 0.8165 |
| 2000 | 460 | 1.3109 | 0.8208 |
| 1800 | 414 | 1.2864 | 0.8269 |
| 1600 | 369 | 1.2634 | 0.8347 |
| 1400 | 326 | 1.2416 | 0.8440 |
| 1200 | 285 | 1.2208 | 0.8548 |
| 1000 | 245 | 1.2002 | 0.8670 |
| 800 | 205 | 1.1791 | 0.8808 |
| 600 | 163 | 1.1566 | 0.8964 |
| 400 | 119 | 1.1315 | 0.9140 |
| 200 | 70 | 1.1024 | 0.9339 |

## REFERENCES

Ahmed, T., 2019, "Reservoir Engineering Handbook", Gulf Professional Publishing.
Ahmed, T., 2016, "Equation of State and PVT Analysis Application for Improved Reservoir Modelling". Elsevier Inc, USA.

Amyx, J., Bass, D. and Whiting, R., 1960, "Petroleum Reservoir Engineering Physical Properties", McGraw-Hill Book Company. New York.

Burcik, E.J., 1997, "Properties of Petroleum Reservoir Fluids", John Wiley \& Sons, USA.
Bradley, H. B., 1987, "Petroleum Engineering Handbook", SPE, USA.
Clark, N.J., 1969, "Elements of Petroleum Reservoirs", Dallas, Texas.
McCain, William D., Jr.,1990, "The Properties of Petroleum Fluids", Tulsa, Oklahoma.
Nnaemeka Ezekwe, 2011, "Petroleum Reservoir Engineering Practice", Pearson Education, Inc., Boston.

Salah Abd., Waleed K., 1978, "Physical and thermodynamical properties of the reservoir oil of Buzurgan field", NIOC.

## Chapter Four

## Basic Reservoir Rock Properties

Rock properties are determined by performing laboratory analysis on cores from the reservoir. Basically, two main categories of core analysis tests are performed on core samples regarding physical properties of reservoir rocks. These are:

## Routine Core Analysis (RCAL):

- Porosity.
- Permeability.
- Saturation.


## Special Core Analysis (SCAL):

- Overburden pressure.
- Capillary pressure.
- Relative permeability.
- Wettability.
- Surface and interfacial tension.

These rock properties data are essential for reservoir engineering calculations as they directly affect both the quantity and the distribution of hydrocarbons and, when combined with fluid properties, control the flow of the existing phases (i.e., gas, oil, and water) within the reservoir.

### 4.1 Porosity:

The porosity of a rock is a measure of the storage capacity (pore volume) that is capable of holding fluids. Quantitatively, the porosity is the ratio of the pore volume to the total volume (bulk volume).

$$
\begin{equation*}
\phi=\frac{\text { pore volume }}{\text { bulk volume }}=\frac{V_{p}}{V_{b}}=\frac{V_{b}-V_{g}}{V_{b}} \tag{4.1}
\end{equation*}
$$

where:

$$
\begin{aligned}
& \phi=\text { fractional porosity } . \\
& V_{p}=\text { pore volume } . \\
& V_{b}=\text { bulk volume } . \\
& V_{g}=\text { grain volume } .
\end{aligned}
$$

Porosity may be classified as:

- Un-effective porosity
- Effective porosity $\qquad$

Un-effective porosity accounts for the isolated pores, whereas effective porosity only accounts for the interconnected pores. Total porosity accounts for all the pores in the rock (interconnected and isolated pores) Therefore, effective porosity will be less than or equal to total porosity depending on the amount of isolated pores in the rock, as shown in Fig. (4.1). From a reservoir engineering standpoint, it is the effective porosity that matters, not the total porosity.

Geologically porosity may also be classified as:

- Primary porosity.
- Secondary porosity.

Primary porosity is that which was formed at the time of deposition whereas secondary porosity was developed after deposition and burial of the formation. Sandstone porosity is practically all primary porosity whereas carbonate porosity tends to be secondary porosity.

Porosity can be measured directly from laboratory analysis of formation cores (see section 4.1.5) or indirectly from the analysis of well logs (as a sample see the example 4.1).


Fig.(4.1): effective, non-effective and total porosity.

### 4.1.1 Factors Affecting Sandstone Porosity:

These include packing, sorting and cementation. Packing describes the arrangement of the sand grains relative to one another. Fig. (4.3) shows three idealized types of packing for spherical sand grains and their theoretical porosities. The cubic packing has a porosity of $47.6 \%$; the hexagonal packing has a porosity of $39.5 \%$ and the rhombohedral packing has a porosity of $25.9 \%$.

Well sorted sandstone consists of grains having approximately the same size whereas poorly sorted sandstone consists of grains having a wide range of different grain sizes, Poor sorting reduce the porosity of the sandstone as may be seen in Fig. (4.4) in which the small grains fit into the pores created by the large grains, reducing the porosity.


Fig. (4.3): Effect of packing on porosity of uniform spheres.


A


B

Fig. (4.4): Effect of sorting on porosity. (A) Irregular grains, (B) Idealized spherical grains.

In consolidated rocks, the sand grains are cemented together usually by quartz or carbonates. Cementation reduce the porosity of the sand as shown in Fig.(4.5).


Fig. (4.5): Effect of cementation on porosity.

### 4.1.2 Factors Affecting Carbonate Porosity:

In carbonates, secondary porosity is usually more important than primary porosity. The major sources of secondary porosity are fracturing, solution and chemical replacement.

Fractures are cracks in the rock. Although fracture porosity is generally small, the fractures are very useful in allowing fluids to flow more easily through the rock. Therefore, they greatly enhance the flow capacity of the rock.

Solution is a chemical reaction in which water with dissolved carbon dioxide reacts with calcium carbonate to form calcium bicarbonate which is soluble. This reaction increases the porosity of the limestone.

$$
\begin{gather*}
\mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{~S}=\mathrm{H}_{2} \mathrm{CO}_{3}  \tag{4.2}\\
\mathrm{H}_{2} \mathrm{CO}_{3}+\mathrm{CaCO}_{3}=\mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2} \tag{4.3}
\end{gather*}
$$

Chemical Replacement is a chemical reaction in which one type of ion replaces another with a resulting shrinkage in the size of the new compound. An example is dolomitization in which some of the calcium ions in calcium carbonate are replaced by magnesium ions to form calcium magnesium carbonate (dolomite). This replacement causes a shrinkage of 12 to $13 \%$, with a corresponding increase in secondary porosity.

$$
\begin{equation*}
2 \mathrm{CaCO}_{3}+\mathrm{M}_{g} \mathrm{Cl}_{2}=\mathrm{CaM}_{g}\left(\mathrm{CO}_{3}\right)_{2}+\mathrm{CaCl}_{2} \tag{4.4}
\end{equation*}
$$

### 4.1.3 Typical Reservoir Porosity Values:

Sandstone have porosities that typically range from $8 \%$ to $38 \%$, with an average of $18 \%$. About $95 \%$ of sandstone porosity is effective porosity. Sandstone porosity is usually mostly intergranular porosity. Carbonate have porosities that typically range from $3 \%$ to $15 \%$, with an average of about $8 \%$. About $90 \%$ of carbonate porosity is effective porosity. Carbonate porosities are much more difficult to characterize and may consist of (1) intergranular, (2) intercrystalline, (3) fissures, and (4) vugular porosities.

### 4.1.4 Porosity Averaging:

The reservoir rock may generally show large variations in porosity vertically (vertical porosity) but does not show very great variations in porosity parallel to the bedding planes (horizontal porosity). In this case, the arithmetic average porosity or the thickness-weighted average porosity is used to describe the average reservoir porosity. A change in sedimentation or depositional conditions, however, can cause the porosity in one portion of the reservoir to be greatly different from that in another area. In such cases, the areal-weighted average or the volume-weighted average porosity is used to characterize the average rock porosity. These averaging techniques are expressed mathematically in the following forms:

$$
\begin{align*}
\text { Arithmetic average } & \phi=\frac{\sum \phi_{i}}{n}  \tag{4.5}\\
\text { Thickness - weighted average } & \phi=\frac{\sum \phi_{i} h_{i}}{\sum h_{i}}  \tag{4.6}\\
\text { Areal - weighted average } & \phi=\frac{\sum \phi_{i} A_{i}}{\sum A_{i}}  \tag{4.7}\\
\text { Volumetric - weighted average } & \phi=\frac{\sum \phi_{i} A_{i} h_{i}}{\sum A_{i} h_{i}} \tag{4.8}
\end{align*}
$$

where:
$\mathrm{n}=$ total number of core samples.
$h_{i}=$ thickness of core sample $i$ or reservoir area $i$.
$\phi_{i}=$ porosity of core sample $i$ or reservoir area $i$.
$A_{i}=$ reservoir area $i$.

## Example 4.1:

Calculate the arithmetic average and thickness-weighted average from the following measurements:

| Sample | Thickness, ft | Porosity, $\%$ |
| :---: | :---: | :---: |
| 1 | 1.0 | 10 |
| 2 | 1.5 | 12 |
| 3 | 1.0 | 11 |
| 4 | 2.0 | 13 |
| 5 | 2.1 | 14 |
| 6 | 1.1 | 10 |

## Solution:

- Arithmetic average:

$$
\phi=\frac{10+12+11+13+14+10}{6}=11.6 \%
$$

- Thickness-weighted average:

$$
\begin{aligned}
\phi & =\frac{(1)(10)+(1.5)(12)+(1)(11)+(2)(13)+(2.1)(14)+(1.1)(10)}{1+1.5+1+2+2.1+1.1} \\
& =12.11 \%
\end{aligned}
$$

### 4.1.5 Laboratory Measurement of Porosity:

The porosity of reservoir rocks can be determined essentially by two different methods: routine core analysis (laboratory measurements on core plugs drilled from whole core samples as shown in Figure (4.6)) and well logging techniques. In addition to the two methods of routine core analysis and well logging, other nonconventional techniques of porosity determination exist, such as x-ray computerized tomography (CT) scanning.

A given reservoir rock sample basically comprises three different volumes: bulk volume (BV), pore volume (PV), and grain volume (GV). These three volumes are related by the following simple relationship:

$$
\text { BulkVolume }=\text { PoreVolume }+ \text { Grain Volume }
$$

Therefore, in the laboratory measurement of porosity, it is necessary to determine only two of the three volumes: $\mathrm{BV}, \mathrm{PV}$, or GV.


Fig. (4.6): Core plugs drilled from a whole core sample.

### 4.1.5.1 Bulk Volume Measurement:

The most common types of samples used in routine core analysis are cylindrical core plugs which allow the determination of BV from the dimensions of the sample ( $L=$ length and $D=$ diameter; $\mathrm{BV}=(\pi / 4) D^{2} L$ ). This is the easiest and simplest method of determining the BV of a reservoir rock sample. For irregular core samples a method of displacing fluid (Archimedes principle) is used to find the core sample.

## Example 4.2:

An irregular sample of sandstone is 35.25 gm in mass, when coated with varnish, its mass increased to 36.55 gm . Compute the porosity if the coated sample displaced 15.7 ml of water when fully submerged if you have ( $\rho_{g}=$ $2.65 \mathrm{gm} / \mathrm{cm}^{3}, \rho_{w}=1 \mathrm{gm} / \mathrm{cm}^{3}, \rho_{v r}=1.8 \mathrm{gm} / \mathrm{cm}^{3}$ )

## Solution:

$$
\begin{gathered}
m_{v r}=\text { coated mass }- \text { sample mass } \\
m_{v r}=36.55-35.25=1.3 \mathrm{gm} \\
\rho_{v r}=\frac{m}{V} \rightarrow V_{v r}=\frac{m}{\rho}=\frac{1.3}{1.8}=0.72 \mathrm{cc}
\end{gathered}
$$

$$
\begin{gathered}
V_{\text {bulk }}=15.7-0.72=14.98 c c \\
V_{\text {grain }}=\frac{m}{\rho}=\frac{35.25}{2.65}=13.3 c c \\
V_{\text {pores }}=V_{\text {bulk }}-V_{\text {grain }}=14.98-13.3=1.68 c c \\
\phi=\frac{V_{\text {pores }}}{V_{\text {bulk }}}=\frac{1.68}{14.98}=0.112=11.2 \%
\end{gathered}
$$

### 4.1.5.2 Pore Volume Measurement:

All methods used for determining pore volume are based on either extraction of a fluid from the rock sample or reintroduction of a fluid in the pore spaces of the rock sample. When introducing fluids into the pore spaces of the rock sample, a number of methods are used for the determination of pore volume of reservoir rocks. These methods typically use three different types of fluids: helium, water or synthetic oil, and mercury. The porosity measured is, however, effective porosity because the saturating fluids only penetrate the interconnected and dead-end pore spaces. Although mercury has some distinct advantages, its use in laboratory testing is accompanied with the associated health hazards and, additionally, the rock sample is rendered unusable. The various methods that employ these saturating fluids are described in the following text.

## I. Helium Porosimeter:

The use of helium in the determination of porosity has certain obvious advantages over other gases and liquids: Helium is a clean inert gas and does not cause any unwanted rock-fluid interactions that may affect/change the original porosity; molecules are small that can rapidly penetrate the small pores, and it can be considered an ideal gas (compressibility factor $=1$ ) for pressures and temperatures usually employed in the procedure. Additionally, porosity measurements can be completed in a short amount of time. The use of helium in desktop-type porosimeters, commonly available in the market, is by far the most common technique for measuring porosities of plug size core samples. Fig. (4.7) shows a general arrangement of the helium porosimeter. In principle, the apparatus consists of two equal-volume chambers or cells called the reference chamber and the sample chamber. The reference chamber has a volume $V_{l}$ at initial pressure $P_{1}$ (usually 100 psig ), and the sample chamber has an unknown volume $V_{2}$ and initial pressure $P_{2}$ (normally atmospheric). The system is then
brought to equilibrium by opening the valve to the sample chamber, allowing the determination of the unknown volume $V_{2}$ by noting the resultant equilibrium pressure $P$. The application of Boyle's law allows the equalization of pressures (for isothermal conditions) before and after the opening of the valve to the sample chamber, as per the following equation:

$$
p_{1} V_{1}=p_{2} V_{2}
$$

where:
$V_{1}=$ reference chamber volume
$V_{2}=V_{1}+V_{\text {tube }}+V_{\text {pores }}$
$p_{1}=$ pressure at reference chamber
$p_{2}=$ final pressure after opening the valve to the sample chamber.


Figure (4.7): Schematic illustration of a helium porosimeter.

## Example 4.4:

Neglecting the tube volume in helium porosimeter method find $V_{p}$ and $\phi$ from the following data. $V_{1}=20 c c, \quad p_{1}=25 \mathrm{psia}, p_{2}=21 \mathrm{psia}$.

## Solution:

$$
\begin{aligned}
p_{1} V_{1} & =p_{2} V_{2} \\
25 \times 20 & =21 \times V_{2}
\end{aligned}
$$

$$
\begin{gathered}
V_{2}=23.81 c c \\
V_{2}=V_{1}+V_{\text {tube }}+V_{\text {pores }} \\
V_{\text {pores }}=23.81-20-0=3.81 c c \\
\phi=\frac{V_{p}}{V_{b}}=\frac{3.81}{20}=0.19=19 \%
\end{gathered}
$$

## II. Vacuum Saturation:

The vacuum saturation method is in fact one of the very basic methods of obtaining the pore volume of a rock sample. One of the advantages is the fact that pore volumes of multiple samples can be determined in one step. The method uses a large enough vacuum flask or a beaker, filled with a degassed liquid, normally water, in which dry rock samples are placed. Subsequently, as soon as the evacuation of the vacuum flask is initiated, air bubbles are seen in the saturating liquid as it replaces air from the pore spaces of the rock samples. The disappearance of the air bubbles gives an indication that the saturation is complete and at this point the evacuation is terminated, and porosity is calculated as follows:

$$
\begin{equation*}
\phi=\frac{(W W-D W)}{B V \rho_{w}} \times 100 \% \tag{4.9}
\end{equation*}
$$

Where:
WW: is the wet weight of the sample, after vacuum saturation.
DW: is the dry weight of the sample, before vacuum saturation.
$\rho_{\mathrm{w}}$ : is the saturating fluid (water) density.

## III. Liquid Saturation by Other Methods:

The other methods of introduction of a liquid into the pore spaces of a rock sample include forced saturation by either water or synthetic oil (liquids as chloroform and tetrachloroethane) also kerosene. The rock sample is held in a special device called a core holder, and a given liquid is injected through the sample by use of a pump. This method, however, requires advanced apparatus called a core flooding rig or a displacement apparatus, compared to the techniques discussed earlier.

### 4.1.5.3 Grain Volume Measurement:

All methods measuring grain volume usually yield total or absolute porosity, simply because the rock samples are normally crushed for grain volume measurements which actually destroy all pores, thus resulting in total porosity as grain volume is subtracted from the bulk volume. Although only the effective pore space has direct application in most reservoir engineering calculations, knowledge of the magnitude and distribution of the isolated pore spaces can reveal other characteristics of reservoir rocks.

### 4.1.5.4 Indirect Methods of Porosity Measurement (Logging Method):

These measurements are carried out in the well employing special instrument as part of well logging operations. Therefore, no core samples are needed and the porosity is estimated for the rock as it exists in the reservoir. Two of the most common well logging methods sonic and density logging. (for more details see (Bassiouni, 1994)).

## Example 4.5:

The bulk density of a clean, sandy interval saturated with water was measured by the density logging tool to be $\left(2.4 \mathrm{~g} / \mathrm{cm}^{3}\right)$. Assuming that the density of the formation water is $\left(1.04 \mathrm{~g} / \mathrm{cm}^{3}\right)$ and the density of the matrix is $\left(2.67 \mathrm{~g} / \mathrm{cm}^{3}\right)$, calculate the density porosity of the formation. Fig. (4.7) illustrates the solution.

## Solution:

$\rho_{b}=2.4 \mathrm{~g} / \mathrm{cm}^{3}, \quad \rho_{f w}=1.04 \mathrm{~g} / \mathrm{cm}^{3}$,
$\rho_{m a}=2.67 \mathrm{~g} / \mathrm{cm}^{3}$
$\Phi_{d}=$ ??


Fig. (4.7): illustrates the given data above.

$$
\Phi_{d}=\frac{\rho_{m a}-\rho_{b}}{\rho_{m a}-\rho_{f w}}
$$

$$
\Phi_{d}=\frac{2.67-2.4}{2.67-1.04}=0.166=16.6 \%
$$

## Example 4.6:

A sonic $\log$ measured travel time of $58 \mu s$ for a formation. If the formation is primarily limestone $(46 \mu s)$ and contains oil only $(190 \mu s)$ compute the rock porosity.

## Solution:

$$
\begin{gathered}
\Phi_{s}=\frac{\Delta t_{l o g}-\Delta t_{m a}}{\Delta t_{f}-\Delta t_{m a}} \\
\Phi_{s}=\frac{58-46}{190-46}=0.083=8.3 \%
\end{gathered}
$$

Note: This value is the average absolute porosity of the formation.

### 4.1.6 Pore Volume Compressibility:

Rock at reservoir conditions is subjected to overburden stress, while at the surface, recovered core has been relieved of the overburden stresses. It is not usual to perform routine porosity measurements under stress approaching reservoir conditions. Because of this, laboratory-measured porosities are generally expected to be higher than in situ values. Pore volume compressibility can be used to correct Laboratory-measured porosity to an in situ value and for other reservoir engineering calculations. Pore volume compressibility is defined as:

$$
\begin{equation*}
c_{f}=\frac{1}{\phi}\left[\frac{\partial \phi}{\partial p}\right]_{T=\text { constant }} \tag{4.10}
\end{equation*}
$$

Note that pore volume compressibility is defined without the negative sign because porosity decreases as pore pressure decreases.

Total system compressibility is a parameter of importance in pressure transient testing and material balance calculations. Total system compressibility is defined as the combined compressibility of pore volume and all fluids saturating the medium:

$$
\begin{equation*}
c_{t}=S_{o} c_{o}+S_{w} c_{w}+S_{g} c_{g}+c_{f} \tag{4.11}
\end{equation*}
$$

where:
$S_{o}, S_{w}, S_{g}=$ oil, water and gas saturations, fraction.
$c_{o}, c_{w}, c_{g}=$ oil, water and gas coefficients of compressibility, $\mathrm{psi}^{-1}$.
$c_{f}=$ pore volume coefficient of compressibility, $\mathrm{psi}^{-1}$.

### 4.2 Fluid Saturation:

Saturation is defined as that fraction, or percent, of the pore volume occupied by a particular fluid (oil, gas, or water). This property is expressed mathematically by the following relationship:

$$
\text { fluid saturation }=\frac{\text { total volume of fluid }}{\text { pore volume }}
$$

Applying this mathematical concept of saturation to each reservoir fluid gives

$$
\begin{gather*}
S_{o}=\frac{\text { volume of oil }}{\text { pore volume }}  \tag{4.12}\\
S_{g}=\frac{\text { volume of gas }}{\text { pore volume }}  \tag{4.13}\\
S_{w}=\frac{\text { volume of water }}{\text { pore volume }} \tag{4.14}
\end{gather*}
$$

where:

$$
\begin{aligned}
& S_{o}=\text { oil saturation } \\
& S_{g}=\text { gas saturation } \\
& S_{w}=\text { water saturation. }
\end{aligned}
$$

Thus, all saturation values are based on pore volume and not on gross reservoir volume.

The saturation of each individual phase ranges between 0 and $100 \%$. By definition, the sum of the saturations is $100 \%$; therefore,

$$
\begin{equation*}
S_{o}+S_{g}+S_{w}=1.0 \tag{2.15}
\end{equation*}
$$

In addition to the bottom (or edge) water, connate water will be distributed throughout the oil and gas zones. The water in these zones will have been reduced to some irreducible minimum. The forces retaining the water in the oil and gas zones are referred to as capillary forces because they are important only in pore spaces of capillary size.

Connate (interstitial) water saturation $S_{w c}$ is important primarily because it reduces the amount of space available between oil and gas. It is generally not uniformly distributed throughout the reservoir but varies with permeability, lithology, and height above the free water table.

Another particular phase saturation of interest is called the critical saturation and it is associated with each reservoir fluid. The definition and significance of the critical saturation for each phase is described next.

## Critical Oil Saturation, $\boldsymbol{S}_{o c}$

For the oil phase to flow, the saturation of the oil must exceed a certain value, which is termed critical oil saturation. At this particular saturation, the oil remains in the pores and, for all practical purposes, will not flow.

## Residual Oil Saturation, $\boldsymbol{S}_{\text {or }}$

During the displacing process of the crude oil system from the porous media by water or gas injection (or encroachment), some remaining oil will be left that is quantitatively characterized by a saturation value that is larger than the critical oil saturation. This saturation value is called the residual oil saturation, $S_{o r}$. The term residual saturation is usually associated with the nonwetting phase (oil or gas) when it is being displaced by a wetting phase (water), (see section 4.5).

## Movable Oil Saturation, $\boldsymbol{S}_{\text {om }}$

Movable oil saturation $S_{o m}$ is another saturation of interest and is defined as the fraction of pore volume occupied by movable oil as expressed by the following equation:

$$
\begin{equation*}
S_{o m}=1-S_{w c}-S_{o c} \tag{4.16}
\end{equation*}
$$

where:
$S_{w c}=$ connate water saturation.
$S_{o c}=$ critical oil saturation.

## Critical Gas Saturation, $\boldsymbol{S}_{\boldsymbol{g c}}$

As the reservoir pressure declines below the bubble-point pressure, gas evolves from the oil phase and consequently the saturation of the gas increases as the reservoir pressure declines. The gas phase remains immobile until its saturation exceeds a certain saturation, called critical gas saturation, above which gas
begins to move. So, $\boldsymbol{S}_{\boldsymbol{g}}$ is the minimum saturation at which gas flow through a porous medium. If $\boldsymbol{S}_{g c}=5 \%$ then gas doesn't flow until it's $\boldsymbol{S}_{g c}$ exceeds 5\%.

## Critical Water Saturation, $\boldsymbol{S}_{w c}$

The critical water saturation, connate water saturation, and irreducible water saturation are extensively used interchangeably to define the maximum water saturation at which the water phase will remain immobile.

### 4.2.1 Saturation Averaging:

Proper averaging of saturation data requires that the saturation values be weighted by both the interval thickness $h_{i}$ and interval porosity $\phi$. The average saturation of each reservoir fluid is calculated from the following equations:

$$
\begin{align*}
& S_{o}=\frac{\sum_{i=1}^{n} \phi_{i} h_{i} S_{o i}}{\sum_{i=1}^{n} \phi_{i} h_{i}}  \tag{4.17}\\
& S_{w}=\frac{\sum_{i=1}^{n} \phi_{i} h_{i} S_{w i}}{\sum_{i=1}^{n} \phi_{i} h_{i}}  \tag{4.18}\\
& S_{g}=\frac{\sum_{i=1}^{n} \phi_{i} h_{i} S_{g i}}{\sum_{i=1}^{n} \phi_{i} h_{i}} \tag{4.19}
\end{align*}
$$

where the subscript $i$ refers to any individual measurement and $h_{i}$ represents the depth interval to which $\phi_{i}, S_{o i}, S_{g i}$, and $S_{w i}$ apply.

## Example 4.7:

Calculate average oil and connate water saturation from the following measurements:

| Sample | $h_{i}, \mathrm{ft}$ | $\phi, \%$ | $S_{o}, \%$ | $S_{w c}, \%$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.0 | 10 | 75 | 25 |
| 2 | 1.5 | 12 | 77 | 23 |
| 3 | 1.0 | 11 | 79 | 21 |
| 4 | 2.0 | 13 | 74 | 26 |
| 5 | 2.1 | 14 | 78 | 22 |
| 6 | 1.1 | 10 | 75 | 25 |

## Solution:

Construct the following table and calculate the average saturation for the oil and water phase:

| Sample | $h_{i}, \mathrm{ft}$ | $\phi$ | $\phi h$ | $S_{o}$ | $S_{o} \phi h$ | $S_{w c}$ | $S_{w} \phi h$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.0 | 0.10 | 0.100 | 0.75 | 0.0750 | 0.25 | 0.0250 |
| 2 | 1.5 | 0.12 | 0.180 | 0.77 | 0.1386 | 0.23 | 0.0414 |
| 3 | 1.0 | 0.11 | 0.110 | 0.79 | 0.0869 | 0.21 | 0.0231 |
| 4 | 2.0 | 0.13 | 0.260 | 0.74 | 0.1924 | 0.26 | 0.0676 |
| 5 | 2.1 | 0.14 | 0.294 | 0.78 | 0.2293 | 0.22 | 0.0647 |
| 6 | 1.1 | 0.10 | 0.110 | 0.75 | 0.0825 | 0.25 | 0.0275 |
|  |  |  | 1.054 |  | 0.8047 |  | 0.2493 |

Calculate average oil saturation by applying Eq.(4.17):

$$
S_{o}=\frac{0.8047}{1.054}=0.7635
$$

Calculate average water saturation by applying Eq.(4.18):

$$
S_{w}=\frac{0.2493}{1.054}=0.2364
$$

### 4.2.2 Laboratory Measurement of Fluid Saturation:

Fluid saturation in reservoir rocks can be determined by essentially two different approaches: direct and indirect. The direct approach involves using preserved core plug samples or rather plug-end trims of the core plug, that is, a small section of the rock sample removed from a petroleum reservoir, for fluid saturation determination. The indirect method is further divided into two categories: (1) use of some other measurements on core plug samples such as capillary pressures based on which the fluid saturations are determined and (2) use based on traditional well-logging techniques where fluid saturations are not measured on core plugs but are measured in situ, for the entire formation itself at various depths.

All the methods for measurement of original reservoir rock saturation are based on the principle of leaching that basically refers to the process of removal of liquids from a solid (rock sample in this case). Based on the principle of leaching, two methods are devised for the determination of fluid saturation. The first method involves using heat to extract the fluids present in the pore spaces of the rock and is termed retort distillation (see Figure (4.8)). The second method involves using both heat and an organic solvent to extract the pore fluids and is called Dean-Stark extraction (see Figure (4.9)).


Figure (4.8): Schematic of a retort distillation unit.


Figure (4.9): Schematic of the Dean-Stark distillation extraction unit.

### 4.3 Permeability:

Permeability is a property of the porous medium to transmit fluids through it. The rock permeability, $k$, is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by (Henry Darcy in 1856). In fact, the equation that defines permeability in terms of measurable quantities is called Darcy's Law.

Darcy developed a fluid flow equation that has since become one of the standard mathematical tools of the petroleum engineer. If a horizontal linear flow of an incompressible fluid is established through a core sample of length $L$ and a cross-section of area A (see Fig. (4.10)), then the governing fluid flow equation is defined as:

$$
\begin{equation*}
q=-\frac{k A}{\mu} \frac{d p}{d x} \tag{4.20}
\end{equation*}
$$

where:
$\mathrm{q}=$ volumetric flowrate, $\mathrm{cm}^{3} / \mathrm{sec}$.
$\mathrm{k}=$ absolute permeability of the rock, darcy.
$\mathrm{A}=$ cross sectional area in the flow direction, $\mathrm{cm}^{2}$.
$\mu=$ fluid viscosity, cp. ( $\mu=$ mass/length.time $=m / L . t=g m / c m . s e c$ (poise)).
$\frac{d p}{d x}=$ fluid pressure gradient, atm $/ \mathrm{cm}$.
One darcy is a relatively high permeability, as the permeabilities of most reservoir rocks are less than 1 darcy. In order to avoid the use of fractions in describing permeabilities, the term millidarcy is used.

$$
1 \text { darcy }=1000 \mathrm{md}
$$

The negative sign in equation (4.20) is necessary, as the pressure increases in one direction while the length increases in the opposite direction.
Equation (4.20) can be integrated when the geometry of the system through which fluid flows is known. For the simple linear system shown in Figure (4.10), the integration is performed as follows:

$$
q \int_{0}^{L} d L=-\frac{k A}{\mu} \int_{p_{1}}^{p_{2}} d p
$$

Integrating the above expression yields:

$$
q=-\frac{k A}{\mu L}\left(p_{2}-p_{1}\right)
$$

It should be pointed out that the volumetric flow rate, $q$, is constant for liquids because the density does not change significantly with pressure. Since $p_{l}$ is greater than $p_{2}$, the pressure terms can be rearranged, which will eliminate the negative term in the equation. The resulting equation is:

$$
\begin{equation*}
q=\frac{k A\left(p_{1}-p_{2}\right)}{\mu L} \tag{4.21}
\end{equation*}
$$

Equation (4.21) is the conventional linear flow equation used in fluid flow calculations.


Fig. (4.10): Linear flow model.

## Example 4.8:

A brine is used to measure the absolute permeability of a core plug. The rock sample is $(4 \mathrm{~cm})$ long and ( $3 \mathrm{~cm}^{2}$ ) in cross section. The brine has a viscosity of $(1.0 \mathrm{cp})$ and is flowing at a constant rate of $\left(0.5 \mathrm{~cm}^{3} / \mathrm{sec}\right)$ under a ( 2.0 atm ) pressure differential. Calculate the absolute permeability.

## Solution:

$$
\begin{gathered}
q=\frac{k A\left(p_{1}-p_{2}\right)}{\mu L} \\
0.5=\frac{k(3)(2)}{(1)(4)} \\
k=0.333 \text { darcy }
\end{gathered}
$$

For steady-state linear flow in oilfield units (see table (4.1) below), Eq. (4.21) becomes:

$$
\begin{equation*}
q=\frac{0.001127 k A}{\mu B_{o}} \frac{\left(p_{1}-p_{2}\right)}{L} \tag{4.22}
\end{equation*}
$$

where:
$q=$ volumetric flowrate, STB/Day.
$k=$ absolute permeability of the rock, millidarcy (md).
$A=$ cross sectional area in the flow direction, $\mathrm{ft}^{2}$.
$p_{1}=$ inlet pressure, psig.
$p_{2}=$ outlet pressure, psig.
$\mu=$ fluid viscosity, cp.
$L=$ length of the rock, ft.
$B_{o}=$ oil formation volume factor, $\mathrm{bbl} / \mathrm{STB}$.

## Note:

Field units and Lab. units of the above symbols are as shown in table below:
Table (4.1): Field and Lab. units of darcy's law.

| Parameter | Lab units | Field units | Conversion factor |
| :---: | :---: | :---: | :---: |
| $q$ | $c m^{3} / \mathrm{sec}$ | $b b l / d a y$ | $\frac{(30.48)^{3} f t^{3} .5 .615 \mathrm{bbl} / \mathrm{ft} t^{3}}{60 \times 60 \times 24 d a y}$ |
| $k$ | $d a r c y$ | $m d$ | $10^{-3}$ |
| $A$ | $c m^{2}$ | $f t^{2}$ | $(30.48)^{2}$ |
| $\Delta p$ | atm | $p s i$ | $1 / 14.7 \mathrm{psi}$ |
| $L$ | $c m$ | $f t$ | 30.48 |
| $\mu$ | $c p$ | $c p$ |  |

For radial flow of fluids into a wellbore (Fig. (4.11)), darcy's law may be expressed in radial coordinates in field units as:

$$
\begin{gather*}
q=\frac{0.001127 .2 \pi \cdot k h\left(p_{e}-p_{w}\right)}{\mu B_{o} \ln \left(\frac{r_{e}}{r_{w}}\right)} \\
q=\frac{0.00708 k h\left(p_{e}-p_{w}\right)}{\mu B_{o} \ln \left(\frac{r_{e}}{r_{w}}\right)} \tag{4.23}
\end{gather*}
$$

where:

```
\(\mathrm{q}=\) volumetric flowrate, STB/Day.
\(\mathrm{k}=\) absolute permeability of the rock, millidarcy (md).
\(\mathrm{h}=\) pay thickness, \(f t\).
\(p_{e}=\) pressure at external radius, psig.
\(p_{w}=\) pressure at wellbore, psig.
\(\mu=\) fluid viscosity, cp .
\(\mathrm{L}=\) length of the rock, \(f t\).
\(B_{o}=\) oil formation volume factor, \(\mathrm{bbl} / \mathrm{STB}\).
\(r_{e}=\) external drainage area, \(f t\).
\(r_{w}=\) wellbore radius, \(f t\).
\(\ln =\) natural logarithm.
```

Equation (4.23) can be deduced as follow:

$$
\begin{gathered}
q=\frac{k A}{\mu} \frac{d p}{d r} \\
q \int_{r_{w}}^{r_{e}} d r=\frac{k A}{\mu} \int_{p_{w}}^{p_{e}} d p \\
q \int_{r_{w}}^{r_{e}} \frac{d r}{A}=\frac{k}{\mu} \int_{p_{w}}^{p_{e}} d p \\
q \int_{r_{w}}^{r_{e}} \frac{d r}{2 \pi r h}=\frac{k}{\mu} \int_{p_{w}}^{p_{e}} d p \\
\frac{q}{2 \pi h} \int_{r_{w}}^{r_{e}} \frac{d r}{r}=\frac{k}{\mu} \int_{p_{w}}^{p_{e}} d p \\
\frac{q}{2 \pi h}\left[\ln \left(r_{e}\right)-\ln \left(r_{w}\right)\right]=\frac{k}{\mu}\left(p_{e}-p_{w}\right) \\
\frac{q}{2 \pi h}\left[\ln \frac{r_{e}}{r_{w}}\right]=\frac{k}{\mu}\left(p_{e}-p_{w}\right) \\
q=\frac{2 \pi k h\left(p_{e}-p_{w}\right)}{\mu\left[\ln \frac{r_{e}}{r_{w}}\right]}
\end{gathered}
$$



Fig. (4.11): Radial flow system.

Standard laboratory analysis procedures will generally provide reliable data on permeability of core samples. If the rock is not homogeneous, the whole core analysis technique will probably yield more accurate results than the analysis of core plugs (small pieces cut from the core). Procedures that have been used for improving the accuracy of the permeability determination include cutting the core with an oil-base mud, employing a pressure-core barrel, and conducting the permeability tests with reservoir oil.

Permeability is reduced by overburden pressure, and this factor should be considered in estimating permeability of the reservoir rock in deep wells because permeability is an isotropic property of porous rock in some defined regions of the system, that is, in other words, it is directional. Routine core analysis is generally concerned with plug samples drilled parallel to bedding planes and, hence, parallel to direction of flow in the reservoir. These yield horizontal permeabilities ( $\mathbf{k}_{\mathrm{h}}$ ).

The measured permeability on plugs that are drilled perpendicular to bedding planes is referred to as vertical permeability ( $\mathbf{k}_{\mathbf{v}}$ ). Figure (4.12) shows a
schematic illustration of the concept of the core plug and the associated permeability.


Fig. (4.12): Representative samples of porous media.

It is important to notify that there are three types of permeability which are:

1. Absolute permeability: is the permeability of a porous medium saturated with a single fluid.
2. Effective permeability: is a measure of the conductance of a porous medium for one fluid phase when the medium is saturated with more than one fluid.
3. Relative permeability: is the ratio of the effective permeability of a fluid at a given saturation to the absolute permeability. (see section 5.2).

## Example 4.9:

Calculate flow rate in $\mathrm{bbl} /$ day for oil flows in a linear porous media with the following properties. $\mathrm{L}=2000 \mathrm{ft}, \mathrm{h}=20 \mathrm{ft}, \mathrm{w}=300 \mathrm{ft}, \mathrm{k}=100 \mathrm{md}, \phi=15 \%, \mu=2$ cp, $p_{1}=2000$ psia, $p_{2}=1990$ psia.
solution:

$$
\begin{gathered}
A=h w=20 \times 300=6000 \mathrm{ft}^{2} \\
q=\frac{0.001127 \mathrm{kA}}{\mu} \frac{\left(p_{1}-p_{2}\right)}{L} \\
q=\frac{0.001127(100)(6000)}{(2)} \frac{(2000-1990)}{2000}=1.69 \mathrm{bbl} / \mathrm{day}
\end{gathered}
$$

### 4.3.1 Averaging Absolute Permeabilities:

The absolute permeability expression such as the one in Equation (4.21) is derived based on a fairly uniform or continuous value of permeability between the inflow and outflow faces. However, such uniformity and consistency is rarely seen in reservoir rocks. Most reservoir rocks have space variations of permeability. For example, reservoir rocks may contain distinct layers, blocks, or concentric rings of fixed permeability. In such cases, the permeability values are averaged according to the particular type of flow: parallel or series. The mathematical expressions for averaging permeability for these cases are developed in the following text.

### 4.3.1.1 Parallel Flow:

As shown in Figure (4.13), consider the case of fluid flow taking place in parallel through different layers of vertically stacked porous media. These individual layers of porous media that have varying permeability and thickness are separated from one another by infinitely thin impermeable barriers that preclude the possibility of cross flow or vertical flow. The average permeability for such a combination can be easily developed by applying Darcy's law to the individual layers. For layer 1,

$$
\begin{equation*}
q_{1}=\frac{k_{1} W h_{1} \Delta p}{\mu L} \tag{4.24}
\end{equation*}
$$

for layer 2,

$$
\begin{equation*}
q_{2}=\frac{k_{2} W h_{2} \Delta p}{\mu L} \tag{4.25}
\end{equation*}
$$

for layer 3,

$$
\begin{equation*}
q_{3}=\frac{k_{3} W h_{3} \Delta p}{\mu L} \tag{4.26}
\end{equation*}
$$

However, since flow is taking place in parallel, the total volumetric flow rate can be equated to the summation of the individual flow rates through the three layers:

$$
\begin{equation*}
q_{t}=q_{1}+q_{2}+q_{3} \tag{4.27}
\end{equation*}
$$

Similarly, the total height is given by:

$$
\begin{equation*}
h_{t}=h_{1}+h_{2}+h_{3} \tag{4.28}
\end{equation*}
$$



Fig. (4.13): Fluid flow through a parallel combination.
Based on Equations (4.27) and (4.28), Darcy's law can be written for the total flow rate for the entire systems using $k_{\text {avg }}$ as the average absolute permeability:

$$
\begin{equation*}
q_{t}=\frac{k_{\text {avg }} W h_{t} \Delta p}{\mu L}=\frac{k_{1} W h_{1} \Delta p}{\mu L}+\frac{k_{2} W h_{2} \Delta p}{\mu L}+\frac{k_{3} W h_{3} \Delta p}{\mu L} \tag{4.29}
\end{equation*}
$$

or,

$$
\begin{equation*}
k_{\text {avg }} h_{t}=k_{1} h_{1}+k_{2} h_{2}+k_{3} h_{3} \tag{4.30}
\end{equation*}
$$

that subsequently leads to the final generalized expression for calculating the average absolute permeability for a parallel system of $n$ layers:

$$
\begin{equation*}
k_{\text {avg }}=\frac{\sum_{i=1}^{n} k_{i} h_{i}}{\sum_{i=1}^{n} h_{i}} \tag{4.31}
\end{equation*}
$$

Equation (4.22) and the schematic used for deriving this equation demonstrate the practical significance from a petroleum reservoir point of view, that is, in horizontal flow situations, fluids travel through the reservoir strata to production wells and remain in the zone in which they originated. Or in other words, the
case of fluid flow in parallel is relevant to conventional wells (vertical) where fluid flow takes place parallel to the bedding planes (horizontal).

The average absolute permeability expressions for a more generalized case of the parallel flow can also be developed using an approach similar to the one described earlier. In such a case, the width of the layers is varied rather than being kept constant. For example, Figure (4.13) can be modified such that layer 1 has shortest width $\mathrm{W}_{1}$, layer 2 has medium width $\mathrm{W}_{2}$, and layer 3 has the largest width $\mathrm{W}_{3}$ as explained in figure (4.14) below:


Figure (4.14): Linear flow through layered beds with variable area.

Given this arrangement, the following equations can be set up for the three individual layers and the average for the entire system:

$$
\begin{gather*}
q_{1}=\frac{k_{1} W_{1} h_{1} \Delta p}{\mu L}  \tag{4.32}\\
q_{2}=\frac{k_{2} W_{2} h_{2} \Delta p}{\mu L}  \tag{4.33}\\
q_{3}=\frac{k_{3} W_{3} h_{3} \Delta p}{\mu L}  \tag{4.34}\\
q_{t}=\frac{k_{a v g}\left(W_{1} h_{1}+W_{2} h_{2}+W_{3} h_{3}\right) \Delta p}{\mu L} \tag{4.35}
\end{gather*}
$$

Using Equation (4.27),

$$
\begin{gather*}
q_{t}=q_{1}+q_{2}+q_{3} \\
\frac{k_{a v g}\left(W_{1} h_{1}+W_{2} h_{2}+W_{3} h_{3}\right) \Delta p}{\mu L}=\frac{k_{1} W_{1} h_{1} \Delta p}{\mu L}+\frac{k_{2} W_{2} h_{2} \Delta p}{\mu L}+\frac{k_{3} W_{3} h_{3} \Delta p}{\mu L}  \tag{4.36}\\
k_{a v g}=\frac{k_{1} W_{1} h_{1}+k_{2} W_{2} h_{2}+k_{3} W_{3} h_{3}}{W_{1} h_{1}+W_{2} h_{2}+W_{3} h_{3}} \tag{4.37}
\end{gather*}
$$

The earlier equation can be generalized for a system of $n$ layers to give the average permeability:

$$
\begin{equation*}
k_{\text {avg }}=\frac{\sum_{i=1}^{n} k_{i} A_{i}}{\sum_{i=1}^{n} A_{i}} \tag{4.38}
\end{equation*}
$$

where $A_{i}$ is the cross-sectional area of layer $i$, expressed as:

$$
\begin{equation*}
A_{i}=\sum_{i=1}^{n} W_{i} h_{i} \tag{4.39}
\end{equation*}
$$

## Example 4.10:

Given the following permeability data from a core analysis report, calculate the average permeability of the reservoir.

| Depth, ft | Permeability, md |
| :---: | :---: |
| $3998-4002$ | 200 |
| $4002-4004$ | 130 |
| $4004-4006$ | 170 |
| $4006-4008$ | 180 |
| $4008-4010$ | 140 |

## Solution:

| $h_{i}, f t$ | $k_{i}$ | $h_{i} k_{i}$ |
| :---: | :---: | :---: |
| 4 | 200 | 800 |
| 2 | 130 | 260 |
| 2 | 170 | 340 |
| 2 | 180 | 360 |
| 2 | 140 | 280 |
| $\mathrm{~h}_{\mathrm{t}}=12$ |  | $\sum h_{i} k_{i}=2040$ |

$$
k_{a v g}=\frac{2040}{12}=170 \mathrm{md}
$$

### 4.3.1.2 Series Flow (Harmonic Average Permeability):

Permeability variations can occur laterally in a reservoir as well as in the vicinity of a wellbore. Figure (4.15) illustrates a series flow taking place through a stack of porous media of varying absolute permeabilities and lengths. The mathematical expression for calculating the average absolute permeability for a flow system shown in Figure (4.15) is developed in the following text. Again writing Darcy's law for each of the layers or blocks of porous medium stacked in series, for layer 1,

$$
\begin{equation*}
q_{1}=\frac{k_{1} W h \Delta p_{1}}{\mu L_{1}} \tag{4.40}
\end{equation*}
$$

for layer 2,

$$
\begin{equation*}
q_{2}=\frac{k_{2} W h \Delta p_{2}}{\mu L_{2}} \tag{4.41}
\end{equation*}
$$

for layer 3,

$$
\begin{equation*}
q_{3}=\frac{k_{3} W h \Delta p_{3}}{\mu L_{3}} \tag{4.42}
\end{equation*}
$$

It should be noted that for series flow, each of these layers or blocks has a different differential pressure and the summation of these is equal to the total or overall differential pressure of the entire flow system. Additionally, the total flow rate is also equal to the individual flow rates:

$$
\begin{equation*}
\Delta p_{1}=p_{1}-p_{2}, \quad \Delta p_{2}=p_{2}-p_{3}, \quad \Delta p_{3}=p_{3}-p_{4} \tag{4.43}
\end{equation*}
$$

or,

$$
\begin{equation*}
\Delta \mathrm{p}=p_{1}-p_{4}=\Delta \mathrm{p}_{1}+\Delta \mathrm{p}_{2}+\Delta \mathrm{p}_{3} \tag{4.44}
\end{equation*}
$$

and,

$$
\begin{equation*}
q_{t}=q_{1}=q_{2}=q_{3} \tag{4.45}
\end{equation*}
$$

Now, Darcy's law can be written for the total flow rate as:

$$
\begin{equation*}
q_{t}=\frac{k_{a v g} W h \Delta p}{\mu L} \tag{4.46}
\end{equation*}
$$



Fig. (4.15): Fluid flow through a series combination.
Subsequently, the differential pressures can be separated from Equations (4.40) through (4.42) and (4.46) and substituted into Equation (4.44):

$$
\begin{gather*}
\Delta \mathrm{p}=\Delta \mathrm{p}_{1}+\Delta \mathrm{p}_{2}+\Delta \mathrm{p}_{3} \\
\frac{q_{t} \mu L}{k_{a v g} W h}=\frac{q_{1} \mu L_{1}}{k_{1} W h}+\frac{q_{2} \mu L_{2}}{k_{2} W h}+\frac{q_{3} \mu L_{3}}{k_{3} W h} \tag{4.47}
\end{gather*}
$$

or,

$$
\begin{align*}
& \frac{L}{k_{\text {avg }}}=\frac{L_{1}}{k_{1}}+\frac{L_{2}}{k_{2}}+\frac{L_{3}}{k_{3}}  \tag{4.48}\\
& k_{\text {avg }}=\frac{\sum_{i=1}^{n} L_{i}}{\sum_{i=1}^{n}\left(L_{i} / k_{i}\right)} \tag{4.49}
\end{align*}
$$

Equation (4.49) is used for calculating the average absolute permeability for the linear flow system.

In the radial system shown in Figure (4.16), the preceding averaging methodology can be applied to produce the following generalized expression:

$$
\begin{equation*}
k_{a v g}=\frac{\ln \left(r_{e} / r_{w}\right)}{\sum_{j=1}^{n}\left[\frac{\ln \left(r_{j} / r_{j-1}\right)}{k_{j}}\right]} \tag{4.50}
\end{equation*}
$$

Equation (4.50) comes from the following expressions:

$$
q_{t}=q_{1}=q_{2}=q_{3}
$$

$$
\begin{aligned}
& \Delta \mathrm{p}=\Delta \mathrm{p}_{1}+\Delta \mathrm{p}_{2}+\Delta \mathrm{p}_{3} \\
& \Delta p_{1}=\frac{q_{1} \mu \ln \frac{r_{1}}{r_{w}}}{k_{1} A} \\
& \Delta p_{2}=\frac{q_{2} \mu \ln \frac{r_{2}}{r_{1}}}{k_{2} A} \\
& \Delta p_{3}=\frac{q_{3} \mu \ln \frac{r_{3}}{r_{2}}}{k_{3} A} \\
& \Delta p_{t}=\frac{q_{t} \mu \ln \frac{r_{e}}{r_{w}}}{k_{a v g} A} \\
& \frac{q_{t} \mu \ln \frac{r_{e}}{r_{w}}}{k_{\text {avg }} A}=\frac{q_{1} \mu \ln \frac{r_{1}}{r_{w}}}{k_{1} A}+\frac{q_{2} \mu \ln \frac{r_{2}}{r_{1}}}{k_{2} A}+\frac{q_{3} \mu \ln \frac{r_{3}}{r_{2}}}{k_{3} A} \\
& \frac{\ln \frac{r_{e}}{r_{w}}}{k_{\text {avg }}}=\frac{\ln \frac{r_{1}}{r_{w}}}{k_{1}}+\frac{\ln \frac{r_{2}}{r_{1}}}{k_{2}}+\frac{\ln \frac{r_{3}}{r_{2}}}{k_{3}} \\
& k_{a v g}=\frac{\ln \frac{r_{e}}{r_{w}}}{\ln \frac{r_{1}}{r_{w}}}+\frac{\ln \frac{r_{2}}{r_{1}}}{k_{2}}+\frac{\ln \frac{r_{3}}{r_{2}}}{k_{3}}
\end{aligned}
$$

Fig. (4.16): Radial Flow through series beds.

The relationship in Equation (4.50) can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

## Example 4.11:

A hydrocarbon reservoir is characterized by five distinct formation segments that are connected in series. Each segment has the same formation thickness. The length and permeability of each section of the five-bed reservoir follow:

| Lenght, ft | Permeability, md |
| :---: | :---: |
| 150 | 80 |
| 200 | 50 |
| 300 | 30 |
| 500 | 20 |
| 200 | 10 |

Calculate the average permeability of the reservoir by assuming:
a. Linear flow system.
b. Radial flow system.

## Solution:

For a linear system:

| $\mathrm{L}_{\mathrm{i}}, \mathrm{ft}$ | $\mathrm{k}_{\mathrm{i}}, \mathrm{md}$ | $\mathrm{L}_{\mathrm{i}} / \mathrm{k}_{\mathrm{i}}$ |
| :---: | :---: | :---: |
| 150 | 80 | 1.8750 |
| 200 | 50 | 4.000 |
| 300 | 30 | 10.000 |
| 500 | 20 | 25.000 |
| 200 | 10 | 20.000 |
| 1350 |  | $\sum L_{i} / k_{i}=60.785$ |

Using Equation (4.49) gives:

$$
k_{\text {avg }}=\frac{1350}{60.875}=22.18 \mathrm{md}
$$

For a radial system:
The solution of the radial system can be conveniently expressed in the following tabulated form. The solution is based on Equation (4.50) and assumes a wellbore radius of 0.25 ft :

| Segment | $r_{j}, f t$ | $\ln \left(r_{j} / r_{j-1}\right)$ | $k_{j}$ | $\left[\ln \left(r_{j} / r_{j-1}\right)\right] / k_{j}$ |
| :---: | :---: | :---: | :---: | :---: |
| Wellbore | 0.25 | - | - | - |
| 1 | 150 | 6.397 | 80 | 0.080 |
| 2 | 350 | 0.847 | 50 | 0.017 |
| 3 | 650 | 0.619 | 30 | 10.021 |
| 4 | 1150 | 0.571 | 20 | 0.029 |
| 5 | 1350 | 0.160 | 10 | 0.016 |
|  |  |  |  | 0.163 |

From Equation (4.50),

$$
k_{a v g}=\frac{\ln (1350 / 0.25)}{0.163}=52.72 \mathrm{md}
$$

### 4.3.1.3 Geometric Average Permeability:

Warren and Price (1961) illustrated experimentally that the most probable behavior of a heterogeneous formation approaches that of a uniform system having a permeability that is equal to the geometric average. The geometric average is defined mathematically by the following relationship:

$$
\begin{equation*}
k_{\text {avg }}=\exp \left[\frac{\sum_{i=1}^{n}\left(h_{i} \ln \left(k_{i}\right)\right)}{\sum_{i=1}^{n}\left(h_{i}\right)}\right] \tag{4.51}
\end{equation*}
$$

where:
$k_{i}=$ permeability of core sample i.
$h_{i}=$ thickness of core sample i.
$n=$ total numbers of samples.

## Example 4.12:

Given the following core data, calculate the geometric average permeability:

| Sample | $\boldsymbol{h}_{\boldsymbol{i}}, \boldsymbol{f t}$ | $\boldsymbol{k}_{\boldsymbol{i}, \boldsymbol{m} \boldsymbol{d}}$ |
| :---: | :---: | :---: |
| 1 | 1 | 10 |
| 2 | 1 | 30 |
| 3 | 0.5 | 100 |
| 4 | 1.5 | 40 |
| 5 | 2 | 80 |
| 6 | 1.5 | 70 |


| 7 | 1 | 15 |
| :---: | :---: | :---: |
| 8 | 1 | 50 |
| 9 | 1.5 | 35 |
| 10 | 0.5 | 20 |

## Solution:

| Sample | $\boldsymbol{h}_{\boldsymbol{i}, \boldsymbol{f t}}$ | $\boldsymbol{k}_{\boldsymbol{i}, \boldsymbol{m} \boldsymbol{d}}$ | $\boldsymbol{h}_{\boldsymbol{i}} * \boldsymbol{\operatorname { l n } ( \boldsymbol { k } _ { \boldsymbol { i } } )}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 10 | 2.303 |
| 2 | 1 | 30 | 3.401 |
| 3 | 0.5 | 100 | 2.303 |
| 4 | 1.5 | 40 | 5.533 |
| 5 | 2 | 80 | 8.764 |
| 6 | 1.5 | 70 | 6.373 |
| 7 | 1 | 15 | 2.708 |
| 8 | 1.5 | 50 | 3.912 |
| 9 | 0.5 | 20 | 5.333 |
|  | $k_{\text {avg }}=\exp \left[\frac{\sum_{i=1}^{n}\left(h_{i} \ln \left(k_{i}\right)\right)}{\sum_{i=1}^{n}\left(h_{i}\right)}\right]$ |  |  |
|  | $k_{\text {avg }}=\exp \left[\frac{42.128}{11.5}\right]=39 m d$ |  |  |

### 4.3.2 Absolute Permeability Correlations:

Timur (1968) proposed the following expression for estimating the permeability from connate-water saturation and porosity:

$$
\begin{gather*}
k(m d)=\left(\frac{(93) \cdot \phi^{2.2}}{S_{w c}}\right)^{2}  \tag{4.52}\\
k(\text { darcy })=8.58102\left(\frac{\phi^{2.2}}{S_{w c}}\right)^{2} \tag{4.53}
\end{gather*}
$$

While Morris and Biggs (1967) presented the following two expressions for estimating the permeability if oil and gas reservoirs:

For an oil reservoir:

$$
\begin{equation*}
k(\text { darcy })=62.5\left(\frac{\phi^{3}}{S_{w c}}\right)^{2} \tag{4.54}
\end{equation*}
$$

For a gas reservoir:

$$
\begin{equation*}
k(\text { darcy })=2.5\left(\frac{\phi^{3}}{S_{w c}}\right)^{2} \tag{4.55}
\end{equation*}
$$

where:
$\mathrm{k}=$ absolute permeability.
$\phi=$ porosity, fraction.
$S_{w c}=$ connate water saturation, fraction.

## Example 4.13:

Estimate the absolute permeability of an oil zone with a connate-water saturation and average porosity of $25 \%$ and $19 \%$, respectively.

## Solution:

Applying the Timur equation:

$$
k=\left(\frac{(93) \cdot(0.19)^{2.2}}{0.25}\right)^{2}=92.8 m d
$$

From the Morris and Biggs correlation:

$$
k=62.5\left(\frac{0.19^{3}}{0.25}\right)^{2}=0.047 \mathrm{darcy}
$$

### 4.3.3 Permeability of Fractures and Channels:

The various aspects of absolute permeability looked at so far were confined to the matrix permeability. However, petroleum reservoir rocks such as sandstones and carbonates frequently contain solution channels and natural or artificial fractures. Therefore, the key to understanding the flow of fluids in petroleum reservoir rocks is to account for flow in all the three permeability elements: the matrix, channel, and fracture.

Matrix permeability refers to the flow in primary pore spaces in a reservoir rock. Fracture or channel permeability refers to the flow in cracks or breaks in
the rock or in the so-called secondary network. However, these channels and fractures do not change the permeability of the matrix but do change the effective permeability of the overall flow network, mainly because the latter permeability is saturation dependent and pore space saturation distribution varies in matrix or fracture/channel network.

### 4.3.4 Factors Affecting Absolute Permeability:

A number of factors affect the absolute permeability of a reservoir rock. The discussion of these factors clearly categorizes and subsequently reviews each factor which can be grouped as rock-related factors, fluid phase-related factors, thermodynamic factors, and mechanical factors:

- Rock-related factors are basic characteristics, structure, or indigenous properties of reservoir rocks, such as grain size and shape and clay cementing. These can in fact also be termed natural factors.
- The type of fluid medium (i.e., gas/brine/water) used for permeability measurement as well as the physical and chemical characteristics of these fluids are also major factors that affect the absolute permeability. These factors can be characterized as artificial or laboratory factors that may temporarily affect permeability.
- The thermodynamic factors affecting absolute permeability basically consist of temperature effects, based on some literature data, these fall under the category of fluid-rock interaction-induced laboratory artifacts that affect permeability.
- The mechanical factors are related to the effect of mechanical stresses or confining pressures on absolute permeability and also fall under the category of laboratory artifacts.


### 4.3.5 Laboratory Measurement of Absolute Permeability:

Laboratory measurement of absolute permeability usually involves the direct application of the Darcy equation, based on the measurement of individual variables such as flow rate, pressure drop, sample dimensions, and fluid properties. Most laboratory measurements are carried out on formation samples of well-defined geometry, such as cylindrical core plugs. These core plugs are generally 1 or 1.5 in . in diameter with lengths typically varying from 2 to 4 in .

Prior to using core plug samples for permeability measurements, the residual fluids or in situ formation fluids are removed so that the sample is $100 \%$
saturated by air. Considering the fact that Absolute permeability can only be measured by conducting a flow experiment in a porous media, gases or nonreactive liquids are commonly used as a fluid phase. Several commercial benchtop permeameters or minipermeameters that use gases or nonreactive liquids for permeability measurement are available.

The apparatus used for conducting flow experiments on core plug samples is a core flooding rig or a displacement apparatus. The schematic of a typical displacement apparatus capable of using both liquids and gases for permeability measurement is shown in figure (4.17).


Fig. (4.17): Schematic of a typical displacement apparatus for absolute permeability measurement using gases and liquids, (Dandekar, 2013).

The most common liquids used for the measurement of absolute permeability are formation waters (sometimes called brine) or degassed crude oil. Formation water or crude oil used is generally from the same formation for which the absolute permeability measurement is desired. In some cases, synthetic oil is also used. Absolute permeability measurements of core plug samples are also carried out using gases instead of liquids. Dry gases such as nitrogen, helium, or air are commonly used as the fluid medium in permeability measurements. Choosing a gas is simply convenient and practical because a gas is clean, nonreactive, and does not alter the pore network; in other words, absolute permeability measurements are not influenced by any rock-fluid interactions.

### 4.3.6 Klinkenberg Effect:

Klinkenberg (1941) discovered that permeability measurements made with air as the flowing fluid showed different results from permeability measurements made with a liquid as the flowing fluid. The permeability of a core sample measured by flowing air is always greater than the permeability obtained when a liquid is the flowing fluid. Klinkenberg postulated, on the basis of his laboratory experiments, that liquids had a zero velocity at the sand grain surface, while gases exhibited some finite velocity at the sand grain surface. In other words, the gases exhibited slippage at the sand grain surface. This slippage resulted in a higher flow rate for the gas at a given pressure differential. Klinkenberg also found that for a given porous medium, as the mean pressure increased the calculated permeability decreased.

Mean pressure is defined as upstream flowing plus downstream flowing pressure divided by $2,\left[p_{m}=\left(p_{1}+p_{2}\right) / 2\right]$. If a plot of measured permeability versus $1 / \mathrm{pm}$ were extrapolated to the point where $1 / \mathrm{p}_{\mathrm{m}}=0$, in other words, where $\mathrm{p}_{\mathrm{m}}=$ infinity, this permeability would be approximately equal to the liquid permeability. A graph of this nature is shown in Figure (4.18). The absolute permeability is determined by extrapolation as shown in Figure (4.18).

The magnitude of the Klinkenberg effect varies with the core permeability and the type of gas used in the experiment as shown in Figures (4.19) and (4.20). The resulting straight-line relationship can be expressed as:

$$
\begin{equation*}
k_{g}=k_{L}+c\left[\frac{1}{p_{m}}\right] \tag{4.56}
\end{equation*}
$$

where:
$k_{g}=$ measured gas permeability.
$p_{m}=$ mean pressure.
$k_{L}=$ equivalent liquid permeability, i.e., absolute permeability, k .
$c=$ slop of the line.


Fig. (4.18): The Klinkenberg effect in gas permeability measurements.


Fig.(4.19): Effect of permeability on the magnitude of the Klinkenberg effect. (After Cole, F., 1969.)


Fig. (4.20): Effect of gas pressure on measured permeability for various gases. (After Calhoun, J., 1976).

### 4.4 Surface and Interfacial Tensions:

In petroleum reservoirs, up to three fluid phases, gas, oil, and water, may coexist. All these fluid phases are immiscible at the pertinent reservoir conditions. When these immiscible fluid phases in a petroleum reservoir are in contact, these fluids are separated by a well-defined interface between gas-oil, gas-water, and oil-water pairs. This particular interface is extremely small in thickness and is typically of the order of about $10 \AA\left(1 \AA=1.0 \times 10^{-10} \mathrm{~m}\right)$; see (Douillard 2009) for reported values of interface thicknesses for some common organic liquids and water. In dealing with multiphase systems such as those encountered in petroleum reservoirs, it is necessary to consider the effect of the forces that exist at the interface when two immiscible fluids are in contact.

The term surface tension (ST) is normally used when characterizing the gasliquid surface forces, i.e. at the liquid surface. However, in the case of two immiscible liquids, the term interfacial tension (IFT) is used when describing the liquid-liquid interfacial forces (also between liquid and solid). Technically, in a petroleum reservoir that contains all the three phases- gas, oil, and water-three different IFT or ST values are of significance: gas-oil ST, gaswater ST, and oil-water IFT.

To understand the concept of interfacial tension or surface tension, consider a system of two immiscible fluids, oil and water, as shown in Figure (4.21). An oil or water molecule, remote from the interface, is surrounded by other oil or
water molecules, thus having a resulting net attractive force on the molecule of zero as it is pulled in all directions. However, a molecule at the interface has a force acting upon it from the oil lying immediately above the interface and water molecules lying below the interface. The resulting forces are not balanced because the magnitude of forces is different (i.e., forces from above and below) and gives rise to interfacial tension. and make the surface to concave to downward.


Fig. (4.21): The concept of interfacial tension (IFT) between two immiscible liquids.

Given the earlier definition of surface or interfacial tension, it has the dimensions of force per unit length usually expressed as $(\mathrm{mN} / \mathrm{m})$ or $\left(10^{-3} \mathrm{~N} / \mathrm{m}\right.$ (dyne/cm)) and commonly denoted by the Greek symbol $\sigma$, (see Fig. (4.22)).
Despite the fact that interfacial tension or surface tension is an entirely fluid- or interface-related property and not a petroleum reservoir rock property, it significantly influences other important rock properties such as wettability (see section 4.5 ), capillary pressure (see section 5.1 ), and relative permeabilities, all of which in turn affect the recovery of hydrocarbon fluids from petroleum reservoirs.

If a glass capillary tube is placed in a large open vessel containing water, the combination of surface tension and wettability of tube to water will cause water
to rise in the tube above the water level in the container outside the tube as shown in Figure (4.22).


Fig. (4.22): Pressure relations in capillary tubes.

The water will rise in the tube until the total force acting to pull the liquid upward is balanced by the weight of the column (due to gravity force) of liquid being supported in the tube. Assuming the radius of the capillary tube is $r$, the total upward force $F_{u p}$, which holds the liquid up, is equal to the force per unit length of surface times the total length of surface, or

$$
\begin{equation*}
F_{u p}=(2 \pi r)\left(\sigma_{g w}\right)(\cos \theta) \tag{4.57}
\end{equation*}
$$

where:
$\sigma_{g w}=$ surface tension between air (gas) and water (oil), dynes/cm.
$\theta=$ contact angle.
$r=$ radius, cm.

The upward force is counteracted by the weight of the water, which is equivalent to a downward force of mass times acceleration, or:

$$
\begin{equation*}
F_{\text {down }}=\pi r^{2} h\left(\rho_{w}-\rho_{\text {air }}\right) g \tag{4.58}
\end{equation*}
$$

where:
$\mathrm{h}=$ height to which the liquid is held, cm .
$\mathrm{g}=$ acceleration due to gravity, $\mathrm{cm} / \mathrm{sec}^{2}$.
$\rho_{w}=$ density of water, $\mathrm{gm} / \mathrm{cm}^{3}$.
$\rho_{\text {air }}=$ density of gas, $\mathrm{gm} / \mathrm{cm}^{3}$.
Because the density of air is negligible in comparison with the density of water, Equation (4.58) is reduced to:

$$
\begin{equation*}
F_{\text {down }}=\pi r^{2} h \rho_{w} g \tag{4.59}
\end{equation*}
$$

At points 1 and 2 in Fig. (4.22):

$$
\begin{gathered}
F_{u p}=F_{\text {down }} \\
(2 \pi r)\left(\sigma_{g w}\right)(\cos \theta)=\pi r^{2} h \rho_{w} g
\end{gathered}
$$

Equating Equation (4.57) with (4.59) and solving for the surface tension between water and gas gives:

$$
\begin{equation*}
\sigma_{g w}=\frac{r h p_{w} g}{2 \cos \theta} \tag{4.60}
\end{equation*}
$$

The generality of Equations (4.57) through (4.60) will not be lost by applying them to the behavior of two liquids, i.e., water and oil. Because the density of oil is not negligible, Equation (4.60) becomes:

$$
\begin{equation*}
\sigma_{o w}=\frac{r h g\left(\rho_{w}-\rho_{o}\right)}{2 \cos \theta} \tag{4.61}
\end{equation*}
$$

where:
$\rho_{o}=$ density of oil, $\mathrm{gm} / \mathrm{cm}^{3}$.
$\sigma_{o w}=$ interfacial tension between the oil and the water, dynes $/ \mathrm{cm}$.

### 4.4.1 Effect of Pressure and Temperature on Interfacial and Surface Tension:

Katz (1959) presented ST data as a function of temperature, on a number of pure hydrocarbon components showing the decline in ST values with an increase in the temperature. The effect of pressure on ST is also somewhat similar to the temperature effect. When considering the gas-liquid ST at high pressures, in most cases, the high-pressure vapor over the surface of a liquid would result in a low ST by bringing a fairly large number of gaseous molecules within reach of the surface. The attractions of these molecules to the surface molecules of the liquid would neutralize, to some extent, the inward attraction
on the surface molecules diminishing the ST, (Danesh, A., 1998). The highpressure gas phase tends to develop miscibility toward the companion liquid phase, thereby reducing ST as pressure increases. There are many studies in this subject like Kundt's (1881), Dandekar (1994) and Hjelmeland and Larrondo (1986). These studies stated that the effects of pressure and temperature on interfacial tension indicate that observed trends would depend on the type of systems studied. This phenomenon has not been well explained, and a theoretical explanation is still lacking.

### 4.4.2 Laboratory Measurement of Interfacial Tension:

The experimental techniques that are used for measuring interfacial tension or surface tension are essentially identical, that is, an apparatus or experimental setup used for measuring the interfacial tension between two liquids, generally, can also be employed for conducting surface tension measurements. A variety of experimental techniques are available for the measurement of IFT or ST values and are referred to as tensiometers. These tensiometers are instruments that simply measure force. Some of the commonly used tensiometers are the Wilhelmy plate method, du Noüy ring method, spinning drop method, and the maximum pull force method, (for more details see (Dandekar, 2013)).

### 4.5 Wettability:

Wettability is the tendency of one fluid to spread on a solid surface in the presence of other immiscible fluids. In dealing with petroleum reservoir fluids in a reservoir system, it is necessary to consider not only the surface and interfacial tension between immiscible fluid phases but also the forces that are active at the interface between the liquids and the solids (reservoir rock surface). The consideration of the interface between the liquids and the solid assumes a significant importance in reservoir engineering simply because the petroleum reservoir fluids are ubiquitously in contact with the solid (reservoir rocks) until they are brought to the surface as part of the production process. It is the combination of all the active forces that determines the wettability of reservoir rocks. The knowledge of reservoir wettability is critical because it influences various important reservoir properties such as the distribution of gas, oil, and water within a reservoir rock, capillary pressure and relative permeability characteristics, and consequently the production of hydrocarbons.

For example, in a system comprising of oil, water (brine), and rock, (sandstone or a carbonate), one of the phases (either the oil phase or the water phase) has a tendency to preferentially wet the rock. The concept of wettability, from the very basic definition, is simple to illustrate and is shown in Figure (4.23).


Fig. (4.23): Illustration of wettability

The spreading tendency of a fluid can be expressed more conveniently as adhesion tension, $\boldsymbol{A}_{\boldsymbol{T}}$, which was first introduced by (Benner et al. in 1938). For the system shown in Figure (4.24), (oil and water system), the adhesion tension is defined by:

$$
\begin{equation*}
A_{T}=\sigma_{s o}-\sigma_{s w} \tag{4.62}
\end{equation*}
$$

where:
$A_{T}=$ adhesion tension.
$\sigma_{s o}=$ the interfacial tension between the solid and the lighter fluid phase (oil in this case).
$\sigma_{s w}=$ the interfacial tension between the solid and the denser fluid phase (water in this case).

The angle of contact, $\theta$ (measured through the denser liquid phase, water in this case), at the liquid-solid surface is also shown in Figure (4.24). Obviously, the value of $\theta$ will range from $0^{\circ}$ to $180^{\circ}$. By definition, the cosine of the contact angle $\theta$ is:

$$
\begin{gather*}
\cos \theta_{o w}=\frac{\sigma_{s o}-\sigma_{s w}}{\sigma_{o w}} \\
\cos \theta_{o w} \sigma_{o w}=\sigma_{s o}-\sigma_{s w} \tag{4.63}
\end{gather*}
$$

Combining Equations (4.62) and (4.63):

$$
\begin{equation*}
A_{T}=\sigma_{o w} \cos \theta_{o w} \tag{4.64}
\end{equation*}
$$



Fig. (4.24): Schematic of a system of two immiscible liquids (oil and water) in contact with a mineral surface.

According to Equation (4.64), positive adhesion tension indicates that the denser phase (water in this case) preferentially wets the solid surface, whereas a negative value of adhesion tension indicates a wetting preference by the lighter phase (oil in this case). An adhesion tension of zero indicates that both phases (oil and water) have equal wettability or affinity for the solid surface. The wetting preferences indicated by the adhesion tension can also be expressed in terms of the contact angle; a $0^{\circ}$ contact angle indicates a completely water-wet system, whereas a contact angle of $180^{\circ}$ indicates an oil-wet system. A $90^{\circ}$ contact angle indicates a neutral-wet system, that is, both phases have equal affinity for the solid surface.

### 4.5.1 Classification or Types of Wettability:

A variety of wettability states exist for petroleum reservoirs, primarily depending on both reservoir fluid and rock characteristics. On a pore level, wettability in porous media has been classified as either homogeneous or heterogeneous. By definition, homogeneous means the entire rock surface has uniform wetting tendencies, whereas heterogeneous indicates distinct rock surface regions that exhibit different wetting tendencies. (Radke et al., 1992) have stated that strongly water wet, strongly oil wet, and intermediate wet systems fall under the category of homogeneous, whereas fractional and mixedwet systems fall under the category of heterogeneous.

### 4.5.1.1 Water Wet:

In this wettability state, all pore surfaces of the rock have preference for the water phase rather than the hydrocarbon phase, and as a result of this condition,
the gas and oil are contained in the centers of the pores. See Figure (4.25, A) for a simple pore level illustration.

### 4.5.1.2 Oil Wet:

This wettability state is exactly the opposite of the water-wet state, that is, the relative positions of the hydrocarbons and water are reversed. It is believed that surface active asphaltenic components of the oil phase cause this wetting state. See Figure (4.25, B) for a simple pore level illustration.


Fig. (4.25): Schematic representation of water-wet and oil-wet pore spaces.

### 4.5.1.3 Intermediate Wet:

The definition of intermediate wettability state from a pore level standpoint is somewhat vague in that there is some tendency for both phases (oil and water) to have preference for the rock surface; however, if that tendency is equal, then this may be termed as neutral-wetting state or considered as a special category of intermediate wettability.

### 4.5.1.4 Fractional Wettability:

Fractional wettability has been variously characterized as Dalmatian, speckled, or spotted (Radke, et al, 1992) because some of the pores are water wet, while others are oil wet, or, in other words, a portion of the rock is strongly water wet, while the rest is strongly oil wet. Dalmatian wettability is where there are oilwet and water-wet regions in the same pore.

### 4.5.1.5 Mixed Wettability:

Mixed wettability was proposed by (Salathiel in 1973), referring to a special type of fractional wettability in which the oil-wet surfaces form continuous paths through the larger pores. Salathiel, however, states that mixed wettability
he introduced should be distinguished from the fractional wettability. (Jerauld and Rathmell) in their paper have introduced Prudhoe Bay oil field in Alaska as a large and prolific mixed-wet reservoir. However, they conclude that the wettability of Prudhoe Bay differs from the mixed wettability proposed by Salathiel because aspects of the wettability they studied are closer to Dalmatian wettability.

### 4.5.2 Factors Affecting Wettability:

Reservoir wettability is almost entirely dependent on the characteristics of the fluids involved and the lithology of the rock in question. Obviously, these factors primarily affect the reservoir wettability. Additionally, reservoir pressure and temperature, locations (with respect to depths) of fluid contacts in the reservoir, and effect of drilling-mud filtrate invasion are also some of the factors that play a role in dictating the reservoir wettability. However, many uncertainties exist as to the relative importance of these various factors in affecting the wettability in reservoirs.

### 4.5.3 Measurement of Reservoir Rock Wettability:

Reservoir wettability can be evaluated by two different groups of methods: qualitative and quantitative. In qualitative methods, wettability is indirectly inferred from other measurements, such as capillary pressure curves or relative permeability curves. Quantitative methods are direct measurement methods, where the wettability is measured on actual rock samples using reservoir fluid samples and wettability is reported in terms of a certain wettability index, signifying the degree of water, oil wetness, or intermediate wetness. These direct quantitative methods include contact angle measurement, the Amott test, and the U.S. Bureau of Mines (USBM) wettability method. for more details see (Dandekar A., 2013).

## Problems

1. The following measurements on pay zone are available:

| Sample | Thickness, ft | $\boldsymbol{\phi}, \%$ | $\boldsymbol{S}_{\boldsymbol{o} \boldsymbol{i}}, \%$ |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 12 | 75 |
| 2 | 3 | 16 | 74 |
| 3 | 1 | 10 | 73 |
| 4 | 4 | 14 | 76 |
| 5 | 2 | 15 | 75 |
| 6 | 2 | 15 | 72 |

Calculate:
a. Average porosity.
b. Average oil and water saturation (assuming no gas).
2. Assuming a steady-state laminar flow, calculate the permeability from the following measurement made on a core sample by using air.
flow rate $=2 \mathrm{~cm}^{3} / \mathrm{sec}$
upstream pressure $=2 \mathrm{~atm}$
$\mathrm{T}=65^{\circ} \mathrm{F}$
$\mathrm{A}=2 \mathrm{~cm}^{2}$
$\mathrm{L}=3 \mathrm{~cm}$
downstream pressure $=1 \mathrm{~atm}$
$\mathrm{A}=2 \mathrm{~cm}^{2} \quad \mathrm{~L}=3 \mathrm{~cm}$
viscosity $=0.018 \mathrm{cp}$
3. Calculate average permeability from the following core analysis data:

| Depth, ft | $\mathbf{k ,} \mathbf{m d}$ |
| :---: | :---: |
| $4000-4002$ | 50 |
| $4002-4005$ | 20 |
| $4005-4006$ | 70 |
| $4006-4008$ | 100 |
| $4008-4010$ | 85 |

4. Calculate the average permeability of a formation that consists of four beds in series, assuming:
a. Liner System
b. Radial system with $r_{w}=0.3$ and $r_{e}=1450 \mathrm{ft}$

| Bed | Length of bed <br> Linear or radial | $\mathbf{k}, \mathbf{m d}$ |
| :---: | :---: | :---: |
| 1 | 400 | 70 |
| 2 | 250 | 400 |
| 3 | 300 | 100 |

$4 \quad 500 \quad 60$
5. Calculate the arithmetic average and thickness-weighted average porosity for the following core data:

| Sample | Depth, ft | Porosity, \% |
| :---: | :---: | :---: |
| 1 | 3705.5 | 40.1 |
| 2 | 3706.5 | 35.1 |
| 3 | 3707.5 | 39.3 |
| 4 | 3708.5 | 36.5 |
| 5 | 3709.5 | 29.1 |

6. Brine flood in a ( 1.9 in .) long and ( 1.5 in .) diameter core plug from the North Sea resulted in a stabilized pressure drop of ( 46.05 psi ). The flood was carried out at $(0.05 \mathrm{~mL} / \mathrm{min})$ with brine viscosity of $(0.443 \mathrm{cp})$. Determine the absolute permeability of this plug in millidarcies.
7. Three beds of equal cross section have permeabilities of ( 100,200 , and 300 mD ) and lengths of ( 50,15 , and 85 ft ), respectively. What is the average permeability of the beds placed in series?
8. Three beds of ( 50,110 , and 795 mD ) and (5, 7 , and 15 ft ) thick, respectively, are conducting fluid in parallel flow. If all are of equal length and width, what is the average permeability?
9. The daily production from a certain oil well is ( 718.5 bbls). The drainage area is ( 160 acre) where the average pressure is at ( 1850 psi ). If the oil viscosity $=2.2 \mathrm{cp}$, well-bore radius $=0.5 \mathrm{ft}$, thickness of pay zone $=16 \mathrm{ft}$, and formation permeability $=180 \mathrm{mD}$, what is the well flowing pressure? Note: 1 acre $=43560 \mathrm{ft}^{2}$.
10.A ( $1 \mathrm{ft} \times 1 \mathrm{ft} \times 1 \mathrm{ft}$ ) North Sea chalk gridblock contains three solution channels $(0.1 \mathrm{~mm}$ in diameter), traversing the entire length. If the absolute permeability of the sample matrix is ( 5 mD ), then what would be the flow rate of a ( 10 cp ) oil through this gridblock under a pressure drop of ( 5 atm )?
11.For the same sample as in Problem 10, if it now contains three fractures ( 0.1 mm wide), traversing the entire length, what would be the oil flow rate through this gridblock under a pressure drop of (5 atm)?
12.Answer the following questions:
a. Injection of gas in a gas cap to increase oil recovery is an imbibition process. True or False
b. Water injection in a water-wet system is a drainage process. True or False
c. Hydrocarbon gas phase is always a nonwetting phase. True or False

## REFERENCES

Ahmed, T., 2019, "Reservoir Engineering Handbook", Gulf Professional Publishing
Ahmed, T., 2006, "Working Guide to Reservoir Rock Properties and Fluid Flow, Elsevier Inc.

Anderson,W.G., 1986, Wettability literature survey-Part 2: Wettability measurement, Journal of Petroleum Technology, 38, 1246-1262.
Bassiouni, Z., 1994 "Theory, Measurement, and Interpretation of Well Logs". SPE. TX.
Benner, F.C., Riches, W.W., and Bartell, F.E., 1938, Nature and importance of surface forces in production of petroleum, API Drilling and Production Practice, 442-448.

Benner, F.C. and Bartell, F.E., 1941, The effect of polar impurities upon capillary and surface phenomena in petroleum production, API Drilling and Production Practice, 341-348.
Clark, N., 1969, "Elements of Petroleum Reservoirs". Dallas, TX: SPE.
Croft, H.O., 1938 "Thermodynamics, Fluid Flow and Heat Transmission", McGraw-Hill, New York, p. 129.

Dandekar, A.Y., 2013, "Petroleum Reservoir Rock and Fluid Properties", CRC Press.
Dandekar, A.Y., 1994, Interfacial tension and viscosity of reservoir fluids, PhD thesis, Heriot-Watt University, Edinburgh, U.K.
Danesh, A., 1998, "PVT and Phase Behavior of Petroleum Reservoir Fluids", Elsevier Science, Amsterdam, the Netherlands.
Darcy, H., 1856, Les Fontaines Publiques de la Ville de Dyon, Victor Dalmont.
Douillard, J.M., 2009, Experimental approach of the relation between surface tension and interfacial thickness of simple liquids, Journal of Colloid and Interface Science, 337, 307-310.
Hjelmeland, O.S. and Larrondo, L.E., 1986, Experimental investigation of the effects of temperature, pressure, and crude oil composition on interfacial properties, SPE Reservoir Engineering, 321-328.
Jerauld, G.R. and Rathmell, J.J., 1997, Wettability and relative permeability of Prudhoe Bay: A case study in mixed-wet reservoirs, Society of Petroleum Engineers SPE paper number 28576.

Katz, D.L., 1959, "Handbook of Natural Gas Engineering", McGraw-Hill, New York.
Klinkenberg, L.J., 1941, The permeability of porous media to liquids and gases, Drilling and Production Practices, American Petroleum Institute, p. 200.

Kundt, A., 1881, Annalen der Physik, 12, 538; International Critical Tables, 4, 475.
Peters, E.J., 2012, "Advanced Petrophysics", The University of Texas at Austin, Austin, Texas 78712, USA.

Radke, C.J., Kovscek, A.R., and Wong, H., 1992, A pore-level scenario for the development of mixed wettability in oil reservoirs, Society of Petroleum Engineers SPE paper number 24880.

Salathiel, R.A., 1973, Oil recovery by surface film drainage in mixed-wettability rocks, Journal of Petroleum Technology, 25, 1216-1224.
S.C. Jones, 1972, A Rapid Accurate Unsteady State Klinkenberg Parameter, SPEJ 12(5) (1972) 383-397.

Tiab, D. and Donaldson, E.C., 1996, "Theory and Practice of Measuring Reservoir Rocks and Fluid Transport Properties", Gulf Publishing Company, Houston, TX.

Wang, W. and Gupta, A., 1995, Investigation of the effect of temperature and pressure on wettability using modified pendant drop method, Society of Petroleum Engineers SPE paper number 30544.

## Chapter Five <br> Capillary Pressure and Relative Permeability

### 5.1 Capillary pressure:

When two immiscible fluids are in contact, a discontinuity in pressure exists between the two fluids, which depends upon the curvature of the interface separating the fluids. We call this pressure difference the capillary pressure and it is referred to by $p_{c}$ (as seen previously in section 4.4).

Capillary forces in petroleum reservoirs are an expressing of the combination of IFT, wetting characteristics, and pore sizes of a given system. The presence of capillary forces in a porous medium causes hydrocarbon entrapment. The classic example is the migration of hydrocarbons from a source rock to the water-saturated reservoir rock. Petroleum trapped in a reservoir represents an equilibrium state between gravity that wants to move the petroleum upward (due to its low density), which is resisted by capillary pressures of water (with its high density).

Capillary forces also play a major role in the dynamic problems, involving the flow of immiscible fluid phases in porous media under the influence of capillarity, gravity, and an impressed external pressure differential, (Leverett, M.C., 1941).

Thus, it is clear from the foregoing that it is advantageous to understand the nature of these capillary forces both from a static reservoir structure (in terms of fluid contacts, transition zones, and free water level [FWL]) and the dynamic actual hydrocarbon recovery standpoint.

Denoting the pressure in the wetting fluid by $p_{w}$ and that in the nonwetting fluid by $p_{n w}$, the capillary pressure can be expressed as:

Capillary pressure $=$ (pressure of the nonwetting phase) - (pressure of the wetting phase)

$$
\begin{equation*}
p_{c}=p_{n w}-p_{w} \tag{5.1}
\end{equation*}
$$

That is, the pressure excess in the nonwetting fluid is the capillary pressure, and this quantity is a function of saturation. This is the defining equation for capillary pressure in a porous medium.

There are three types of capillary pressure:

- Water-oil capillary pressure (denoted $P_{c w o}$ ).
- Gas-oil capillary pressure (denoted $P_{c g o}$ ).
- Gas-water capillary pressure (denoted $P_{c g w}$ ).

Applying the mathematical definition of the capillary pressure as expressed by Equation (5.1), the three types of capillary pressure can be written as:

$$
\begin{aligned}
p_{c w o} & =p_{o}-p_{w} \\
p_{c g o} & =p_{g}-p_{o} \\
p_{c g w} & =p_{g}-p_{w}
\end{aligned}
$$

Where $p_{g}, p_{o}$, and $p_{w}$ represent the pressure of gas, oil, and water, respectively.
The pressure difference across the interface between points $\mathbf{1}$ and 2 as shown in figure (5.1) is essentially the capillary pressure, i.e.:

$$
\begin{equation*}
p_{c}=p_{1}-p_{2} \tag{5.2}
\end{equation*}
$$

The pressure of the water phase at point 2 is equal to the pressure at point 4 minus the head of the water (cohesion force), or:

$$
\begin{equation*}
p_{2}=p_{4}-g h \rho_{w} \tag{5.3}
\end{equation*}
$$

The pressure just above the interface at point 1 represents the pressure of the air and is given by:

$$
\begin{equation*}
p_{1}=p_{3}-g h \rho_{a i r} \tag{5.4}
\end{equation*}
$$

It should be noted that the pressure at point 4 within the capillary tube is the same as that at point 3 outside the tube. Subtracting Equation (5.3) from (5.4) gives:

$$
\begin{equation*}
p_{c}=g h\left(\rho_{w}-\rho_{a i r}\right)=g h \Delta \rho \tag{5.5}
\end{equation*}
$$

Where $\Delta \rho$ is the density difference between the wetting and nonwetting phase. The density of the air (gas) is negligible in comparison with the water density.

In practical units, Equation (5.5) can be expressed as:

$$
\begin{equation*}
p_{c}=\left(\frac{h}{144}\right) \Delta \rho \tag{5.6}
\end{equation*}
$$

where:
$p_{c}=$ capillary pressure, psi $\left(l b / i n^{2}\right)=l b / 144 f t^{2} ; 1 f t=12 \mathrm{in}$.
$h=$ capillary rise, ft.
$\Delta \rho=$ density difference, $l b / f t^{3}$.


Fig. (5.1): Pressure relations in capillary tubes.

In the case of an oil-water system, Equation (5.6) can be written as:

$$
\begin{equation*}
p_{c}=\left(\frac{h}{144}\right)\left(\rho_{w}-\rho_{o}\right) \tag{5.7}
\end{equation*}
$$

The capillary pressure equation can be expressed in terms of the surface and interfacial tension by combining interfacial tension equations with Equation (5.5) to give:

## - Gas-Liquid System.

$$
p_{c}=g h\left(\rho_{w}-\rho_{g}\right)
$$

from equation (4.61):

$$
\begin{gather*}
\sigma=\frac{r h g(\Delta \rho)}{2 \cos \theta} \\
g h(\Delta \rho)=\frac{2 \sigma \cos \theta}{r} \\
p_{c}=\frac{2 \sigma_{g w}(\cos \theta)}{r} \tag{5.8}
\end{gather*}
$$

and,

$$
\begin{equation*}
h=\frac{2 \sigma_{g w}(\cos \theta)}{r g\left(\rho_{w}-\rho_{g}\right)} \tag{5.9}
\end{equation*}
$$

where:
$\rho_{w}=$ water density, $\mathrm{gm} / \mathrm{cm}^{3}$.
$\sigma_{g w}=$ gas-water surface tension, dynes $/ \mathrm{cm}$.
$\mathrm{r}=$ capillary radius, cm .
$\theta=$ contact angle .
$\mathrm{h}=$ capillary rise, cm .
$\mathrm{g}=$ acceleration due to gravity, $\mathrm{cm} / \mathrm{sec}^{2}$.
$p_{c}=$ capillary pressure, dynes $/ \mathrm{cm}^{2}$.

## - Oil-Water System.

$$
\begin{equation*}
p_{c}=\frac{2 \sigma_{o w}(\cos \theta)}{r} \tag{5.10}
\end{equation*}
$$

and,

$$
\begin{equation*}
h=\frac{2 \sigma_{o w}(\cos \theta)}{r g\left(\rho_{w}-\rho_{o}\right)} \tag{5.11}
\end{equation*}
$$

where: $\sigma_{o w}$ is the water-oil interfacial tension.

## Example 5.1:

Calculate the pressure difference, i.e., capillary pressure, and capillary rise in an oil-water system from the following data:
$\theta=30^{\circ}$

$$
\begin{array}{ll}
\rho_{w}=1.0 \mathrm{gm} / \mathrm{cm}^{3} & \rho_{o}=0.75 \mathrm{gm} / \mathrm{cm}^{3} \\
\sigma_{o w}=25 \text { dynes } / \mathrm{cm} &
\end{array}
$$

$r=10^{-4} \mathrm{~cm}$

## Solution:

Step 1. Apply Equation (5.10) to give:

$$
\begin{gathered}
p_{c}=\frac{2 \sigma_{g w}(\cos \theta)}{r} \\
p_{c}=\frac{(2)(25)\left(\cos 30^{\circ}\right)}{0.0001}=4.33 \times 10^{5} \text { dynes } / \mathrm{cm}^{2}
\end{gathered}
$$

Since 1 dyne $/ \mathrm{cm}^{2}=1.45 \times 10^{-5} \mathrm{psi}$

$$
p_{c}=6.28 p s i
$$

This result indicates that the oil-phase pressure is 6.28 psi higher than the waterphase pressure.

Step 2. Calculate the capillary rise by applying Equation (5.11):

$$
\begin{gathered}
h=\frac{2 \sigma_{o w}(\cos \theta)}{r g\left(\rho_{w}-\rho_{o}\right)} \\
h=\frac{(2)(25)\left(\cos 30^{\circ}\right)}{(0.0001)(980.7)(1.0-0.75)}=1766 \mathrm{~cm}=75.9 \mathrm{ft}
\end{gathered}
$$

### 5.1.1 Dependence of Capillary Pressure on Rock and Fluid Properties:

Equations (5.8) and (5.10) show the capillary pressure of an immiscible pair of fluids expressed in terms of surface or interface forces, wettability, and capillary size. Capillary pressure is a function of the adhesion tension ( $\sigma \cos \theta$ ) and inversely proportional to the radius of the capillary tube. Now we make a qualitative examination of the effect of pore size (capillary radius in this case) and the adhesion tension on capillary pressure. Figure (5.2) shows the effect of varying the wetting characteristics of the system and varying the radius of the capillary tube.
In Figure (5.2):
a. The wetting characteristics are the same, that is, the same IFT and contact angle, but the radius of the capillary tube is different. In this case, the capillary pressure is inversely proportional to the capillary tube radius, while the adhesion tension remains constant. By merely looking at the mathematical expression of capillary pressure, it is easily understood that the higher the capillary tube radius, the lower the capillary pressure, or vice versa.


Fig. (5.2): Dependence of capillary pressure on wetting characteristics and pore size (tube radius in this case). Case (a) $\theta_{1}=\theta_{2}, \mathrm{~h}_{1}>\mathrm{h}_{2}$ and $r_{1}<r_{2}$, case (b) $\theta_{1}<\theta_{2}, \mathrm{~h}_{1}>\mathrm{h}_{2}$ and $r_{1}=r_{2}$.
b. On the other hand, when the capillary tubes of same radius but different wetting characteristics are considered, the denominator in the capillary pressure equation will be a constant, and the value of capillary pressure will be directly proportional to the adhesion tension or the wetting characteristics of the system. Figure (5.2b) shows such a system of same radius and different contact angles. In this case, the smaller the contact angle, the greater the height of liquid rise and stronger the adhesion tension, leading to higher capillary pressure, whereas the opposite is evident from the other tube having weaker wetting characteristics or adhesion tension that obviously results in lower capillary pressure.

### 5.1.2 Laboratory Measurement of Capillary Pressure:

All laboratory experiments are basically designed to first mimic or simulate the saturation history of the reservoir, that is, migration of hydrocarbons from source rocks into water-filled reservoir rocks. Under the assumption that the rocks are water wet, at least prior to hydrocarbon migration, this is accomplished by the drainage process by displacing the wetting phase (water) by a nonwetting phase (oil or gas), which primarily establishes the fluid saturations that are found when the reservoir is discovered.

The data from such an experiment are shown in Figure (5.3). Two important phenomena can be observed in Figure (5.3). First, a finite capillary pressure at $100 \%$ water saturation is necessary to force the nonwetting phase into a capillary filled with the wetting phase. This minimum capillary pressure is known as the displacement pressure, $\boldsymbol{p}_{d}$.
As the wetting phase is displaced, the second phenomenon of any immiscible displacement process is encountered, that is, reaching some finite minimum irreducible saturation. This irreducible water saturation is referred to as connate water, $S_{w c}$.

The other principal flow process of interest involves reversing the drainage process by displacing the nonwetting phase (oil or gas) with the wetting phase, (e.g., water). This displacing process is termed the imbibition process and the resulting curve is termed the capillary pressure imbibition curve.
Following the two primary displacement processes in the porous medium (drainage and imbibition), laboratory measurement of capillary pressure is confined to imbibition and drainage. In all the laboratory measurement methods, wetting-phase saturation is increased in the imbibition process, while in the drainage process, wetting-phase saturation is decreased from a maximum value (typically $100 \%$ ) to the irreducible minimum by increasing the capillary pressure from 0 to a large positive value. The plots of capillary pressure data obtained in such a fashion are called imbibition and drainage capillary pressure curves, respectively. The process of saturating (see curve (1) in Fig. (5.4)) and
desaturating (see curve (2) in Fig. (5.4)) a core with the nonwetting phase is called capillary hysteresis. Figure (5.4) shows typical drainage and imbibition capillary pressure curves. The two capillary pressure-saturation curves are not the same.


Fig. (5.3): Capillary pressure curve.


Fig. (5.4): Capillary pressure drainage and imbibition curves.

On the other hand, oil production performance (oil as a nonwetting phase) is generally governed by the imbibition process (water flooding). Therefore, imbibition capillary pressure curves are generally used in reservoir studies, (Ertekin, T. et al, 2001).

In almost all laboratory measurement methods, capillary pressures are measured on cylindrical core plug samples of the representative formation. The laboratory measured imbibition and drainage capillary pressure-saturation relationships can be understood as somewhat equivalent to measuring these relationships for a bundle of several capillaries of varying tortuosities and sizes. The core sample is normally flooded with representative formation water or brine so that a maximum in the wetting-phase saturation is obtained (under the presumption that the sample is water wet). This saturation is then reduced in steps by injecting a nonwetting phase to mimic the original reservoir fluid distribution. Subsequently, the imbibition cycle can be completed by increasing the wettingphase saturation but without reaching $100 \%$ water saturation because of residual oil saturation. Some of the most commonly used laboratory techniques for measuring capillary pressures are listed below:

- Leverett's capillary pressure measurement.
- Porous diagram method.
- Centrifuge method.
- Mercury injection method (MICP).

For more details see (Dandekar, 2013).

## Example 5.1:

Plot capillary pressure with saturation curve of the given data and find the $p_{d}$ and $S_{w i}$ values.

Capillary pressure, (psi) Mercury saturation

| 0 | 0 |
| :---: | :---: |
| 5 | 0 |
| 10 | 0.1 |
| 20 | 0.3 |
| 30 | 0.6 |
| 40 | 0.8 |
| 100 | 0.9 |
| 200 | 0.9 |

## Solution:

Water saturation $=1-$ mercury saturation

| Capillary pressure, <br> (psi) | Mercury saturation | Water Saturation |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 5 | 0 | 1 |
| 10 | 0.1 | 0.9 |
| 20 | 0.3 | 0.7 |
| 30 | 0.6 | 0.4 |
| 40 | 0.8 | 0.2 |
| 100 | 0.9 | 0.1 |
| 200 | 0.9 | 0.1 |

Figure (5.5) shows the capillary pressure vs. water saturation plot


Fig. (5.5): Capillary pressure vs. water saturation.
From figure (5.5) the $p_{d}$ value is $5 p s i$ and $S_{w i}$ is $10 \%$.

### 5.1.3 Averaging Capillary Pressure Data: J-Function:

Capillary pressure data are obtained on small core samples that represent an extremely small part of the reservoir and, therefore, it is necessary to combine all capillary data to classify a particular reservoir. The fact that the capillary pressure-saturation curves of nearly all naturally porous materials have many features in common has led to attempts to devise some general equation describing all such curves. (Leverett, 1941) approached the problem from the standpoint of dimensional analysis.

Realizing that capillary pressure should depend on: 1- the porosity, 2- interfacial tension, 3- and mean pore radius, Leverett defined the dimensionless function of saturation, which he called the J -function, as:

$$
\begin{gather*}
J\left(S_{w}\right)=\frac{p_{c}}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} \\
J\left(S_{w}\right)=0.21645 \frac{p_{c}}{\sigma} \sqrt{\frac{k}{\phi}} \tag{5.12}
\end{gather*}
$$

where:

$$
\begin{aligned}
& J\left(S_{w}\right)=\text { Leverett J-function. } \\
& p_{c}=\text { capillary pressure, psi. } \\
& \sigma=\text { interfacial tension, dynes } / \mathrm{cm} . \\
& k=\text { permeability, md. } \\
& \phi=\text { fractional porosity. }
\end{aligned}
$$

In doing so, Leverett interpreted the ratio of permeability, $k$, to porosity, $\phi$, as being proportional to the square of a mean pore radius.

The J-function was originally proposed as a means of converting all capillarypressure data to a universal curve. There are significant differences in correlation of the J -function with water saturation from formation to formation, so that no universal curve can be obtained. For the same formation, however, this dimensionless capillary-pressure function serves quite well in many cases to remove discrepancies in the $p_{c}$ versus $S_{w}$ curves and reduce them to a common curve. This is shown for various unconsolidated sands in Figure (5.6).

## Example 5.4:

A laboratory capillary pressure test was conducted on a core sample taken from the Nameless Field. The core has a porosity and permeability of $16 \%$ and 80 md , respectively. The capillary pressure-saturation data are given as follows:

| $S_{w}$ | $p_{c}, p s i$ |
| :---: | :---: |
| 1.0 | 0.50 |
| 0.8 | 0.60 |
| 0.6 | 0.75 |
| 0.4 | 1.05 |
| 0.2 | 1.75 |

The interfacial tension ( $\sigma$ ) is measured at 50 dynes $/ \mathrm{cm}$. Further reservoir engineering analysis indicated that the reservoir is better described at a porosity
value of $19 \%$ and an absolute permeability of 120 md . Generate the capillary pressure data for the reservoir.


Fig. (5.6): The Leverett J-function for unconsolidated sands. (After Leverett, 1941).

## Solution:

Step 1. Calculate the J-function using the measured capillary pressure data.

$$
J\left(S_{w}\right)=0.21645 \frac{p_{c}}{\sigma} \sqrt{\frac{k}{\phi}}
$$

$$
J\left(S_{w}\right)=0.21645\left(\frac{p_{c}}{50}\right) \sqrt{\frac{80}{0.16}}=0.096799 p_{c}
$$

| $S_{w}$ | $p_{c}, p s i$ | $J\left(S_{w}\right)=0.096799\left(p_{c}\right)$ |
| :---: | :---: | :---: |
| 1.0 | 0.50 | 0.048 |
| 0.8 | 0.60 | 0.058 |
| 0.6 | 0.75 | 0.073 |
| 0.4 | 1.05 | 0.102 |
| 0.2 | 1.75 | 0.169 |

Step 2. Using the new porosity and permeability values, solve Equation (5.12) for the capillary pressure $p_{c}$.

$$
\begin{gathered}
\left.p_{c}=\frac{J\left(S_{w}\right) \sigma}{\left[0.21645 \sqrt{\frac{k}{\phi}}\right.}\right] \\
p_{c}=\frac{J\left(S_{w}\right) 50}{\left[0.21645 \sqrt{\frac{120}{0.19}}\right]} \\
p_{c}=9.192 J\left(S_{w}\right)
\end{gathered}
$$

Step 3. Reconstruct the capillary pressure-saturation table.

| $S_{w}$ | $J\left(S_{w}\right)$ | $p_{c}=9.192 J\left(S_{w}\right)$ |
| :---: | :---: | :---: |
| 1.0 | 0.048 | 0.441 |
| 0.8 | 0.058 | 0.533 |
| 0.6 | 0.073 | 0.671 |
| 0.4 | 0.102 | 0.938 |
| 0.2 | 0.169 | 1.553 |

### 5.1.4 Converting Laboratory Capillary Pressure Data to Reservoir Conditions:

For experimental convenience, it is common in the laboratory determination of capillary pressure to use air-mercury or air-brine systems, rather than the actual water-oil system characteristic of the reservoir. Since the laboratory fluid system does not have the same surface tension as the reservoir system, it
becomes necessary to convert laboratory capillary pressure to reservoir capillary pressure. By assuming that the Leverett J-function is a property of rock and does not change from the laboratory to the reservoir, we can calculate reservoir capillary pressure as follows:

$$
\begin{equation*}
\left(p_{c}\right)_{r e s}=\left(p_{c}\right)_{l a b} \frac{\sigma_{r e s}}{\sigma_{l a b}} \tag{5.13}
\end{equation*}
$$

Even after the laboratory capillary pressure has been corrected for surface tension, it may be necessary to make further corrections for permeability and porosity. The reason for this is that the core sample that was used in performing the laboratory capillary pressure test may not be representative of the average reservoir permeability and porosity. If we assume that the J-function will be invariant for a given rock type over a range of porosity and permeability values, then the reservoir capillary pressure can be expressed as:

$$
\begin{equation*}
\left(p_{c}\right)_{r e s}=\left(p_{c}\right)_{l a b} \frac{\sigma_{r e s}}{\sigma_{l a b}} \sqrt{\frac{\left(\phi_{\text {res }} k_{\text {core }}\right)}{\left(\phi_{\text {core }} k_{r e s}\right)}} \tag{5.14}
\end{equation*}
$$

where:
$\left(p_{c}\right)_{\text {res }}=$ reservoir capillary pressure.
$\sigma_{r e s}=$ reservoir surface or interfacial tension.
$k_{\text {res }}=$ reservoir permeability.
$\phi_{\text {res }}=$ reservoir porosity .
$\left(p_{c}\right)_{l a b}=$ laboratory-measured capillary pressure.
$\sigma_{l a b}=$ laboratory surface or interfacial tension.
$\phi_{\text {core }}=$ core porosity.
$k_{\text {core }}=$ core permeability.

### 5.1.5 Capillary Pressure and Permeability:

Figure (5.7) shows the air-mercury capillary pressure data as a function of mercury saturation for five different core samples varying in absolute permeability in a range of $k_{1}-k_{5} m d$. The data show decreases in permeability have corresponding increases in the capillary pressure at a constant value of mercury saturation. In other words, when iso-saturation data are compared, the sample having a permeability of $k_{1} m d$ has the lowest capillary pressure, whereas the one with permeability of $k_{5} m d$ has the highest capillary pressure. Although the general trend of capillary pressure-saturation curves remains unchanged, the magnitude of the capillary pressures does change when data are compared for samples of different permeabilities. Therefore, the capillary pressure-saturation-permeability relationship is a reflection of the influence of pore sizes and grain sorting. Smaller size pores and poorly sorted grains invariably have lower permeabilities and large capillary pressures.


Mercury saturation, \%
Fig. (5.7): Variation of capillary pressure with permeability $\left(k_{1}>k_{2}>k_{3}>k_{4}>k_{5}\right)$.

### 5.1.6 Zonation, Fluid Contacts, and Initial Saturation Distribution in a Reservoir:

The determination of initial fluid contacts in the reservoir, such as the gas-oil contact (GOC) and oil-water contact (OWC), through different methods are available to the engineer's disposal. These include:

1. A production test which involves the direct determination of GOC and OWC during the drilling of a well prior to setting the casing.
2. The other method includes application of geophysical logs such as electrical and radioactive.
3. In addition to these, SCAL (which includes capillary pressure) represents one of the most reliable methods of determining the fluid contacts in a reservoir.
A great deal of practical information pertaining to zonation, fluid contacts, and initial fluid saturation distribution in a hydrocarbon reservoir prior to its exploitation can be derived from capillary pressure data. However, this requires the conversion of $P_{c}-S_{w}$ data to height-saturation data, which can be achieved by considering the generalized form of Equation (5.11):

$$
\begin{equation*}
h=\frac{2 \sigma \cos \theta}{r \Delta \rho g} \tag{5.15}
\end{equation*}
$$

By using generalized form of Equation (5.10), Equation (5.15) can be expressed in terms of capillary pressure:

$$
p_{c}=g h \Delta \rho
$$

$$
\begin{equation*}
h=\frac{p_{c}}{\Delta \rho g} \tag{5.16}
\end{equation*}
$$

where:
$h=$ the height above the plane of 0 capillary pressure between the nonwetting and wetting fluids.
$P_{c}=$ the capillary pressure.
$\Delta \rho=$ the density difference between nonwetting and wetting phase at reservoir conditions.
$g=$ the gravitational constant.
In Equation (5.16), when a value of $g$ is $9.81 \mathrm{~m} / \mathrm{s}^{2}, P_{c}$ is in $\mathrm{N} / \mathrm{m}^{2}$, and the density difference is in $\mathrm{kg} / \mathrm{m}^{3}$,

$$
\begin{equation*}
h(m)=\frac{0.102 p_{c}\left(N / \mathrm{m}^{2}\right)}{\Delta \rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)} \tag{5.17}
\end{equation*}
$$

Whereas in oil-field units, when $g$ is $32.2 \mathrm{ft} / \mathrm{sec}^{2}\left(9.81 \mathrm{~m} / \mathrm{sec}^{2} \times 3.28 \mathrm{ft} / \mathrm{m}\right)$, $P_{c}$ is in $\mathrm{lb}_{\text {force }} / \mathrm{in}^{2}$ (multipled by $32.2 \mathrm{ft} / \mathrm{sec}^{2}$ to convert $l b_{\text {force }}$ to $l b_{\text {mass }}$ ), and the density difference is in $l b_{\text {mass }} f t^{3}$ :

$$
\begin{equation*}
h(f t)=\frac{p_{c}\left(l b_{\text {force }} / \text { in. }^{2}\right) \times 32.2\left(f t / s^{2}\right)}{32.2\left(f t / s^{2}\right) \Delta \rho\left(l b_{\text {mass }} / f t^{3}\right)}=\frac{144 p_{c}}{\Delta \rho} \tag{5.18}
\end{equation*}
$$

Equation (5.18) provides a relationship between capillary pressure and the height above the plane of 0 capillary pressure; that is, capillary pressure data are easily converted to height-saturation data based on which the zonation, fluid contacts, and fluid distribution in a hydrocarbon reservoir are determined. It should be noted, however, that prior to converting $P_{c}-S_{w}$ data to $h-S_{w}$ data, capillary pressures should be converted from laboratory conditions to representative reservoir conditions based on the methods described earlier.

In order to understand the application of height-saturation data to determine the fluid distribution, zonation, and fluid contacts in a reservoir, we consider the drainage capillary pressure curve such as the one in Figure (5.8), the data for which have been converted to reservoir conditions (for an oil-water system) and subsequently to height-saturation, as shown in Figure (5.9). Thus, the figure essentially shows the water saturation distribution in an oil-water system.


Fig. (5.8): A typical example of drainage and imbibition MICP data.


Fig. (5.9): Profile of fluid distribution, zonation, and fluid contacts based on the capillary pressure or height versus water saturation data.

### 5.1.6.1 Free Water Level, FWL:

From the capillary pressure standpoint, the FWL occurs at zero capillary pressure at which water saturation is $100 \%$. Consequently, the FWL is
represented by the base of the height-saturation curve, as schematically illustrated in Figure (5.9), below which a water zone or aquifer may exist. However, moving upward vertically from the base of the height-saturation curve in Figure (5.9), the water saturation is still $100 \%$ up to a certain finite value of " h " that extends from $\mathrm{h}=0$, which is an outcome of the capillary entry pressure or displacement pressure $\left(P_{d}\right)$ or the threshold pressure. In terms of depth, there is a difference between the FWL and the minimum depth at which $100 \%$ water saturation still exists, which is a manifestation of the largest pore size in the reservoir. On the other hand, if this pore size is so large that there is no capillary rise in this size pore, then the FWL and the $100 \%$ water saturation level will be the same, basically implying the absence or nonexistence of the displacement pressure. This is, however, not the case with most reservoirs because even the largest pore is small enough to result in some capillary rise. (Slider, 1976) states that in the case of very tight reservoirs, $P_{d}$ values can be large; similarly, low-permeability chalk reservoirs are also characterized by high capillary entry pressures, thus resulting in a difference of many feet between the two heights as outlined earlier. FWL can be expressed mathematically by the following relationship:

$$
\begin{equation*}
F W L=O W C+\frac{144 p_{d}}{\Delta \rho} \tag{5.19}
\end{equation*}
$$

where:

```
\(p_{d}=\) displacement pressure, psi (from capillary pressure curve).
\(\Delta \rho=\) density difference, \(l b_{\text {mass }} \delta f t^{3}\).
FWL = free-water level, ft.
OWC \(=\) Oil-Water contact, \(f t\).
```


### 5.1.6.2 Oil-Water Contact, OWC:

As shown in Figure (5.9), the OWC and $100 \%$ water saturation point on the height-saturation curve is represented by the coordinates $p_{c}=p_{d} ; S_{w}=100 \%$. In Equation (5.18), $P_{c}$ can be replaced by $p_{d}$ to express the OWC in terms of height above the FWL:

$$
\begin{equation*}
O W C=\frac{144 p_{d}}{\Delta p} \tag{5.20}
\end{equation*}
$$

where:
OWC = Oil-Water contact, ft.
$p_{d}=$ displacement pressure, psi (from capillary pressure curve).
$\Delta \rho=$ density difference, $l b_{\text {mass }} / f t^{3}$.

In terms of depth, the OWC is defined as the uppermost depth in the reservoir where a $100 \%$ water saturation exists, which can be mathematically expressed as:

$$
\begin{equation*}
O W C=F W L-\frac{144 p_{d}}{\Delta p} \tag{5.21}
\end{equation*}
$$

## Example 5.2:

The reservoir capillary pressure-saturation data of the Big Butte Oil Reservoir is shown graphically in Figure (5.10). Geophysical log interpretations and core analysis establish the WOC at (5023 ft). The following additional data are available:

- Oil density $=43.5 \mathrm{lb} / f t^{3}$.
- Water density $=64.1 \mathrm{lb} / f t^{3}$.
- Interfacial tension $=50$ dynes $/ \mathrm{cm}$.

Calculate:

- Connate-water saturation $\left(S_{w c}\right)$.
- Depth to FWL.
- Thickness of the transition zone.
- Depth to reach $50 \%$ water saturation.


Fig (5.10): Capillary pressure-saturation data.

## Solution:

a. From Figure (5.10), connate-water saturation is $20 \%$.
b. Applying Equation (5.19) with a displacement pressure of 1.5 psi gives:

$$
F W L=5023+\frac{(144)(1.5)}{(64.1-43.5)}=5033.5 \mathrm{ft}
$$

c. Thickness of transition zone:

$$
T Z \text { thickness }=\frac{144(6.0-1.5)}{(64.1-43.5)}=31.5 f t
$$

d. $\quad P_{c}$ at $50 \%$ water saturation $=3.5 \mathrm{psia}$.

### 5.1.6.3 Transition Zone:

The presence of transition zones in petroleum reservoirs is a manifestation of the capillary forces, the absence of which would essentially result in complete segregation of the fluid phases. The relationship as expressed in Equation (5.15), when applied for $h_{@ s w i}-h_{\text {owc }}$ suggests that the transition zone thickness is basically influenced by (1) the radius of the pore, $r$, and (2) the difference in density, $\Delta \rho$. Therefore, a reservoir rock system having small pore sizes has a large transition zone (typical characteristics of low-permeability chalk reservoirs) than a system comprised of large pore sizes. Additionally, the more uniform the pore sizes (well-sorted grains), the flatter the middle portion of the capillary pressure curve, thus resulting in thinner transition zones. Such uniform pore size systems will also have high permeabilities; thus, the thickness of the transition zone can also be indirectly related to permeability; a highpermeability reservoir rock system has shorter transition zones than lowpermeability reservoirs. As far as the influence of $\Delta \rho$ is concerned, from a practical standpoint, in a gas reservoir having a gas-water contact, the thickness of the transition zone will be small since the density difference is significantly large (gravity dominating over capillarity). Similarly, in the case of a light oil (smaller density), the density difference is large, resulting in a lower thickness of the transition zone. In the case of heavy oils (larger density), the density difference is small, thus increasing the thickness of the transition zone. However, a combined effect of $r$ and $\Delta \rho$ would normally dictate the thickness of the transition zone. (Ahmed, 2019) points out that the thickness of the transition zone may range from a few feet to several hundred feet in some reservoirs. Figure (5.11) shows the variations of transition zone with fluid gravity and permeability.


Fig. (5.11): Transition zone variation with (a) fluid gravity, (b) permeability.

## Example 5.3:

A four-layer oil reservoir is characterized by a set of reservoir capillary pressure-saturation curves as shown in Figure (5.12). The following data are also available:


Fig. (5.12): Variation of $p_{c}$ with $k$.
$\mathrm{WOC}=4060 \mathrm{ft}$.
Water density $=65.2 \mathrm{lb} / \mathrm{ft}^{3}$.
Oil density $=55.2 \mathrm{lb} / \mathrm{ft}^{3}$.
Calculate and plot water saturation versus depth for this reservoir.

## Solution:

Step 1. Establish the FWL by determining the displacement pressure $p_{d}$ for the bottom layer, i.e., Layer 4, and apply Equation (5.19):

- $P_{d}=0.75 \mathrm{psi}$

$$
F W L=4060+\frac{(144)(0.75)}{(65.2-55.2)}=4070.8 f t
$$

Step 2. The top of the bottom layer is located at a depth of $4,035 \mathrm{ft}$, which is 35.8 ft above the FWL. Using that height $h$ of 35.8 ft , calculate the capillary pressure at the top of the bottom layer:

$$
p_{c}=\left(\frac{h}{144}\right) \Delta \rho=\left(\frac{35.8}{144}\right)(65.2-55.2)=2.486 p s i
$$

- From the capillary pressure-saturation curve designated for layer 4, read the water saturation that corresponds to a $p_{c}$ of 2.486 to give $S_{w}=0.23$.
- Assume different values of water saturations and convert the corresponding capillary pressures into height above the FWL by applying Equation (5.18).

$$
h=\frac{144 p_{c}}{\rho_{w}-\rho_{o}}
$$

| $S_{w}$ | $p_{c}, p s i$ | $h, f t$ | Depth $=\mathrm{FWL}-\mathrm{h}$ |
| :---: | :---: | :---: | :---: |
| 0.23 | 2.486 | 35.8 | 4035 |
| 0.25 | 2.350 | 33.84 | 4037 |
| 0.30 | 2.150 | 30.96 | 4040 |
| 0.40 | 1.800 | 25.92 | 4045 |
| 0.50 | 1.530 | 22.03 | 4049 |
| 0.60 | 1.340 | 19.30 | 4052 |
| 0.70 | 1.200 | 17.28 | 4054 |
| 0.80 | 1.050 | 15.12 | 4056 |
| 0.90 | 0.900 | 12.96 | 4058 |

Step 3. The top of layer 3 is located at a distance of 50.8 ft from the FWL (i.e., h $=4,070.8-4,020=50.8 \mathrm{ft})$. Calculate the capillary pressure at the top of the third layer:

- $p_{c}=\left(\frac{50.8}{144}\right)(65.2-55.2)=3.53 p s i$
- The corresponding water saturation as read from the curve designated for layer 3 is 0.370 .
- Construct the following table for layer 3 .

| $S_{w}$ | $p_{c}, p s i$ | $h, f t$ | Depth $=$ FWL - h |
| :---: | :---: | :---: | :---: |
| 0.37 | 3.53 | 50.8 | 4020 |
| 0.40 | 3.35 | 48.2 | 4023 |
| 0.50 | 2.75 | 39.6 | 4031 |
| 0.60 | 2.50 | 36.0 | 4035 |

Step 4. Distance from the FWL to the top of layer 2 is:

- $h=4070.8-4010=60.8 \mathrm{ft}$
- $p_{c}=\left(\frac{60.8}{144}\right)(65.2-55.2)=4.22 p s i$
- $S_{w}$ at $p_{c}$ of 4.22 psi is 0.15
- The distance from the FWL to the bottom of the layer is 50.8 ft , which corresponds to a $p_{c}$ of 3.53 psi and an $S_{w}$ of 0.15 . This indicates that the second layer has a uniform water saturation of $15 \%$.

Step 5. For layer 1, the distance from the FWL to the top of the layer is

- $h=4070-4000=70.8 \mathrm{ft}$
- $p_{c}=\left(\frac{70.8}{144}\right)(10)=4.92 p s i$
- $S_{w}$ at the top layer $1=0.25$.
- The capillary pressure at the bottom of the layer is 3.53 psi with a corresponding water saturation of 0.27 .

Step 6. Figure (5.13) documents the calculated results graphically. The figure indicates that layer 2 will produce $100 \%$ oil while all remaining layers produce oil and water simultaneously.


Fig. (5.13): Water saturation profile.

### 5.2 Relative Permeability:

Numerous laboratory studies have concluded that the effective permeability of any reservoir fluid is a function of the reservoir fluid saturation and the wetting characteristics of the formation. It becomes necessary, therefore, to specify the fluid saturation when stating the effective permeability of any particular fluid in a given porous medium. Just as $k$ is the accepted universal symbol for the absolute permeability, $k_{o}, k_{g}$, and $k_{w}$ are the accepted symbols for the effective permeability to oil, gas, and water, respectively. The saturations, i.e., $S_{o}, S_{g}$, and $S_{w}$, must be specified to completely define the conditions at which a given effective permeability exists.

Effective permeabilities are normally measured directly in the laboratory on small core plugs. Owing to many possible combinations of saturation for a single medium, however, laboratory data are usually summarized and reported as relative permeability.

The absolute permeability is a property of the porous medium and is a measure of the capacity of the medium to transmit fluids. When two or more fluids flow at the same time, the relative permeability of each phase at a specific saturation is the ratio of the effective permeability of the phase to the absolute permeability, or:

$$
k_{r o}=\frac{k_{o}}{k}
$$

$$
\begin{aligned}
& k_{r g}=\frac{k_{g}}{k} \\
& k_{r w}=\frac{k_{w}}{k}
\end{aligned}
$$

where:
$k_{r o}=$ relative permeability to oil.
$k_{r g}=$ relative permeability to gas.
$k_{r w}=$ relative permeability to water.
$k=$ absolute permeability.
$k_{o}=$ effective permeability to oil for a given oil saturation.
$k_{g}=$ effective permeability to gas for a given gas saturation.
$k_{w}=$ effective permeability to water at some given water saturation.
For example, if the absolute permeability $k$ of a rock is 200 md and the effective permeability $k_{o}$ of the rock at an oil saturation of $80 \%$ is 60 md , the relative permeability $k_{r o}$ is 0.30 at $S_{o}=0.80$.
Since the effective permeabilities may range from zero to $k$, the relative permeabilities may have any value between zero and one, or:

$$
0 \leq k_{r w}, k_{r o}, k_{r g} \leq 1.0
$$

It should be pointed out that when three phases are present the sum of the relative permeabilities ( $k_{r o}+k_{r g}+k_{r w}$ ) is both variable and always less than or equal to unity. An appreciation of this observation and of its physical causes is a prerequisite to a more detailed discussion of two and three-phase relative permeability relationships.
It has become a common practice to refer to the relative permeability curve for the nonwetting phase as $k_{n w}$ and the relative permeability for the wetting phase as $k_{w}$.

### 5.2.1 Two-Phase Relative Permeability:

When a wetting (water for example) and a nonwetting phase (as oil) flow together in a reservoir rock, each phase follows separate and distinct paths. The distribution of the two phases according to their wetting characteristics results in characteristic wetting and nonwetting phase relative permeabilities. Since the wetting phase occupies the smaller pore openings at small saturations, and these pore openings do not contribute materially to flow, it follows that the presence of a small wetting phase saturation will affect the nonwetting phase permeability only to a limited extent. Since the nonwetting phase occupies the central or larger pore openings that contribute materially to fluid flow through
the reservoir, however, a small nonwetting phase saturation will drastically reduce the wetting phase permeability.

Figure (5.14) presents a typical set of relative permeability curves for a wateroil system with the water being considered the wetting phase. Figure (5.14) shows the following four distinct and significant observations:


Fig. (5.14): Typical two-phase flow behavior.

## Observation 1

The wetting phase (water in this case) relative permeability shows that a small saturation of the nonwetting phase (oil in this case) will drastically reduce the relative permeability of the wetting phase. For example see in the figure above at the water saturation of $70 \%$, relative permeability for the water will be
reduced from (1 to 0.37). The reason for this is that the nonwetting phase occupies the larger pore spaces, and it is in these large pore spaces that flow occurs with the least difficulty.

## Observation 2

The nonwetting phase relative permeability curve shows that the nonwetting phase begins to flow at the relatively low saturation of the nonwetting phase. The saturation of the oil at this point is called critical oil saturation $S_{o c}$, (see in the figure above at oil saturation of $16 \%$ ).

## Observation 3

The wetting phase relative permeability curve shows that the wetting phase will cease to flow at a relatively large saturation, (see in the figure above at water saturation of $20 \%$ ). This is because the wetting phase preferentially occupies the smaller pore spaces, where capillary forces are the greatest. The saturation of the water at this point is referred to as the irreducible water saturation $S_{\text {wir }}$ or connate-water saturation $S_{w c}$-both terms are used interchangeably.

## Observation 4

The nonwetting phase relative permeability curve shows that, at the lower saturations of the wetting phase, changes in the wetting phase saturation have only a small effect on the magnitude of the nonwetting phase relative permeability curve, (for example in the figure above at $30 \%$ and $40 \%$ of water saturation, $k_{r o}$ will be 0.94 and 0.72 respectively). The reason for this phenomenon is that at the low saturations, the wetting phase fluid occupies the small pore spaces that do not contribute materially to flow, and therefore changing the saturation, in these small pore spaces has a relatively small effect on the flow of the nonwetting phase.
In addition to the observations above note that the total permeability to both phases, $k_{r w}+k_{r o}$, is less than 1 , in regions $B$ and $C$.

Another important phenomenon associated with fluid flow through porous media is the concept of residual saturations. As when one immiscible fluid is displacing another, it is impossible to reduce the saturation of the displaced fluid to zero. At some small saturation, which is presumed to be the saturation at which the displaced phase ceases to be continuous, flow of the displaced phase will cease. This saturation is often referred to as the residual saturation. This is an important concept as it determines the maximum recovery from the reservoir. Conversely, a fluid must develop a certain minimum saturation before the phase will begin to flow. This is evident from an examination of the relative
permeability curves shown in Figure (5.14). The saturation at which a fluid will just begin to flow is called the critical saturation.

Theoretically, the critical saturation and the residual saturation should be exactly equal for any fluid; however, they are not identical. Critical saturation is measured in the direction of increasing saturation, while irreducible saturation is measured in the direction of reducing saturation. Thus, the saturation histories of the two measurements are different.

The above discussion may be also applied to gas-oil relative permeability data, as can be seen for a typical set of data in Figure (5.15). Note that this might be termed gas-liquid relative permeability since it is plotted versus the liquid saturation. This is typical of gas-oil relative permeability data in the presence of connate-water. Since the connate (irreducible) water normally occupies the smallest pores in the presence of oil and gas, it appears to make little difference whether water or oil that would also be immobile in these small pores occupies these pores. Consequently, in applying the gas-oil relative permeability data to a reservoir, the total liquid saturation is normally used as a basis for evaluating the relative permeability to the gas and oil.
Note that the relative permeability curve representing oil changes completely from the shape of the relative permeability curve for oil in the water-oil system. In the water-oil system, as noted previously, oil is normally the nonwetting phase, whereas in the presence of gas the oil is the wetting phase. Consequently, in the presence of water only, the oil relative permeability curve takes on an $S$ shape whereas in the presence of gas the oil relative permeability curve takes on the shape of the wetting phase, or is concave upward. Note further that the critical gas saturation $S_{g c}$ is generally very small, (it's value in Fig. (5.15) is $5 \%$ ).

As was discussed for capillary pressure data, there is also a saturation history effect for relative permeability. The effect of saturation history on relative permeability is illustrated in Figure (5.16). If the rock sample is initially saturated with the wetting phase (e.g., water) and relative permeability data are obtained by decreasing the wetting phase saturation while flowing nonwetting fluid (e.g., oil) in the core, this process is classified as drainage or desaturation.
while if the rock sample is resaturated by increasing the wetting phase, this process is termed imbibition or resaturation. The nomenclature is consistent with that used in connection with capillary pressure. This difference in permeability when changing the saturation history is called hysteresis. Since relative permeability measurements are subject to hysteresis, it is important to duplicate, in the laboratory, the saturation history of the reservoir.


Fig. (5.15): Gas-oil relative permeability curves.

### 5.2.1.1 Drainage Process:

It is generally agreed that the pore spaces of reservoir rocks were originally filled with water, after which oil moved into the reservoir, displacing some of the water, and reducing the water to some residual saturation. When discovered, the reservoir pore spaces are filled with a connate-water saturation and an oil saturation. If gas is the displacing agent, then gas moves into the reservoir, displacing the oil. So, if non-wetting phase occupied wetting phase this process is called drainage.

The same history must be duplicated in the laboratory to eliminate the effects of hysteresis. The laboratory procedure is to first saturate the core with water, then displace the water to a residual, or connate, water saturation with oil after which the oil in the core is displaced by gas. This flow process is called the gas drive, or drainage, depletion process. In the gas drive depletion process, the nonwetting phase fluid is continuously increased, and the wetting phase fluid is continuously decreased.

### 5.2.1.2 Imbibition Process:

If wetting phase occupied non-wetting phase this process is called imbibition. The imbibition process is performed in the laboratory by first saturating the core with the water (wetting phase), then displacing the water to its irreducible (connate) saturation by injection oil. This "drainage" procedure is designed to establish the original fluid saturations that are found when the reservoir is discovered. The wetting phase (water) is reintroduced into the core and the water (wetting phase) is continuously increased. This is the imbibition process and is intended to produce the relative permeability data needed for water drive or water flooding calculations.
Figure (5.16) schematically illustrates the difference in the drainage and imbibition processes of measuring relative permeability, (as shown in the shifting or difference between curves 1 and 2 , and curves 3 and 4 in the figure). It is noted that the imbibition technique causes the nonwetting phase (oil) to lose its mobility at higher values of water saturation than does the drainage process. The two processes have similar effects on the wetting phase (water) curve. The drainage method causes the wetting phase to lose its mobility at higher values of wetting phase saturation than does the imbibition method.
As shown in Fig. (5.16) there are several important differences between waterwet and oil-wet relative permeability curves that are generally observed; these are as follows:

1. The water saturation at which oil and water permeabilities are equal, that is, the intersection point of the two curves, will generally be greater than $50 \%$ for water-wet systems and less than $50 \%$ for oil-wet systems.
2. The relative permeability to water at maximum water saturation (i.e., $\left(1-S_{o r}\right)$ ), will be less than 0.3 for water-wet systems and is roughly greater than 0.5 for oil-wet systems.
3. The connate-water saturation for a water-wet system, $S_{w c}$, is generally greater than $25 \%$, whereas for oil-wet systems it is generally less than $15 \%$.

### 5.2.2 Laboratory Measurement of Tow-Phase Relative Permeability:

Laboratory measurement techniques for obtaining the two-phase relative permeability data based on the flow experiments are fairly well established. Essentially two different types of flow experiments can be conducted in reservoir rock samples from which relative permeability data are determined. These methods are called steady state (SS) and unsteady state (USS) flows, which are by far the most common. In addition to the SS and USS methods, petroleum engineering literature also reports the use of centrifuge technique to determine relative permeability data.


Fig. (5.16): Hysteresis effects in relative permeability.
The procedure for determining gas-oil and oil-water relative permeabilities from the SS and USS displacement tests requires very comprehensive testing programs composed of various steps beginning with the initial preparation of the reservoir rock sample and ending with the determination of final fluid saturations. Additionally, considering that reservoir rock samples, fluid samples, and pressure and temperature conditions are the key elements of a successful relative permeability testing program. The SS and USS displacement tests are normally carried out in displacement apparatus or relative permeability rigs. These rigs are basically set up by integrating various individual components, such as displacement pumps, core holder, high-pressure tubing, a set of valves, and a variety of electronic controls. For more details see (Honarpour, 1988).

### 5.2.3 Two-Phase Relative Permeability Correlations:

In many cases, relative permeability data on actual samples from the reservoir under study may not be available, in which case it is necessary to obtain the desired relative permeability data in some other manner. Several methods have been developed for calculating relative permeability relationships. Various parameters have been used to calculate the relative permeability relationships, including:

- Residual and initial saturations.
- Capillary pressure data.

In addition, most of the proposed correlations use the effective phase saturation as a correlating parameter. The effective phase saturation is defined by the following set of relationships:

$$
\begin{align*}
S_{o}^{*} & =\frac{S_{o}}{1-S_{w c}}  \tag{5.22}\\
S_{w}^{*} & =\frac{S_{w}-S_{w c}}{1-S_{w c}}  \tag{5.23}\\
S_{g}^{*} & =\frac{S_{g}}{1-S_{w c}} \tag{5.24}
\end{align*}
$$

where:
$S_{o}^{*}, S_{w}^{*}, S_{g}^{*}=$ effective oil, water, and gas saturation, respectively.
$S_{o}, S_{w}, S_{g}=$ oil, water, and gas saturation, respectively.
$S_{w c}=$ connate (irreducible) water saturation.
Various correlations have been proposed for two-phase relative permeability calculations, which are listed below:

1. Wyllie and Gardner Correlation.
2. Torcaso and Wyllie Correlation.
3. Pirson's Correlation.
4. Corey's Method.
5. Relative Permeability from Capillary Pressure Data.
6. Relative Permeability from Analytical Equations.

### 5.2.3.1 Wyllie and Gardner Correlation:

Wyllie and Gardner (1958) observed that, in some rocks, the relationship between the reciprocal capillary pressure squared $\left(1 / p_{c}^{2}\right)$ and the effective water saturation $S_{w}^{*}$ is linear over a wide range of saturation. Honapour et al. (1988) conveniently tabulated Wyllie and Gardner correlations as shown below:

## Drainage Oil-Water Relative Permeabilities

| Type of formation | $\boldsymbol{k}_{\boldsymbol{r} \boldsymbol{o}}$ | $\boldsymbol{k}_{\boldsymbol{r} \boldsymbol{w}}$ | Equation |
| :--- | :--- | :--- | :---: |
| Unconsolidated sand, well sorted | $\left(1-S_{w}^{*}\right)^{3}$ | $\left(S_{w}^{*}\right)^{3}$ | $(5.25)$ |
| Unconsolidated sand, poorly sorted | $\left(1-S_{w}^{*}\right)^{2}\left(1-S_{w}^{* 1.5}\right)$ | $\left(S_{o}^{*}\right)^{3.5}$ | $(5.26)$ |
| Cemented sandstone, oolitic limestone | $\left(1-S_{w}^{*}\right)^{2}\left(1-S_{w}^{* 2}\right)$ | $\left(S_{o}^{*}\right)^{4}$ | $(5.27)$ |


| Drainage Gas-Oil Relative Permeabilities |  |  |  |
| :--- | :---: | :---: | :---: |
| Type of formation | $\boldsymbol{k}_{\boldsymbol{r o}}$ | $\boldsymbol{k}_{\boldsymbol{r g}}$ | Equation |
| Unconsolidated sand, well sorted | $\left(S_{o}^{*}\right)^{3}$ | $\left(1-S_{o}^{*}\right)^{3}$ | $(5.28)$ |
| Unconsolidated sand, poorly sorted | $\left(S_{o}^{*}\right)^{3.5}$ | $\left(1-S_{o}^{*}\right)^{2}\left(1-S_{o}^{* 1.5}\right)$ | $(5.29)$ |
| Cemented sandstone, oolitic limestone, <br> rocks with vugular porosity | $\left(S_{o}^{*}\right)^{4}$ | $\left(1-S_{o}^{*}\right)^{2}\left(1-S_{o}^{* 2}\right)$ | $(5.30)$ | rocks with vugular porosity

Wyllie and Gardner have also suggested the following two expressions that can be used when one relative permeability is available:

- Oil-water system:

$$
\begin{equation*}
k_{r w}=\left(S_{w}^{*}\right)^{2}-k_{r o}\left[\frac{S_{w}^{*}}{1-S_{w}^{*}}\right] \tag{5.31}
\end{equation*}
$$

- Gas-oil system:

$$
\begin{equation*}
k_{r o}=\left(S_{o}^{*}\right)-k_{r g}\left[\frac{S_{o}^{*}}{1-S_{o}^{*}}\right] \tag{5.32}
\end{equation*}
$$

## Example 5.5:

Generate the drainage relative permeability data for an unconsolidated wellsorted sand by using the Wyllie and Gardner method. Assume the following critical saturation values:
$S_{o c}=0.3$,
$S_{w c}=0.25$,
$S_{g c}=0.05$

## Solution:

Generate the oil-water relative permeability data by applying Equation (5.25) in conjunction with Equation (5.23), to give:

| $\boldsymbol{S}_{\boldsymbol{w}}$ | $\boldsymbol{S}_{\boldsymbol{w}}^{*}=\frac{\boldsymbol{S}_{\boldsymbol{w}}-\boldsymbol{S}_{\boldsymbol{w c}}}{\mathbf{1}-\boldsymbol{S}_{\boldsymbol{w c}}}$ | $\boldsymbol{k}_{\boldsymbol{r o}}=\left(\mathbf{1}-\boldsymbol{S}_{\boldsymbol{w}}^{*}\right)^{\mathbf{3}}$ | $\boldsymbol{k}_{r \boldsymbol{w}}=\left(\boldsymbol{S}_{\boldsymbol{w}}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| 0.25 | 0.0000 | 1.000 | 0.0000 |
| 0.30 | 0.0667 | 0.813 | 0.0003 |
| 0.35 | 0.1333 | 0.651 | 0.0024 |
| 0.40 | 0.2000 | 0.512 | 0.0080 |
| 0.45 | 0.2667 | 0.394 | 0.0190 |
| 0.50 | 0.3333 | 0.296 | 0.0370 |
| 0.60 | 0.4667 | 0.152 | 0.1017 |
| 0.70 | 0.6000 | 0.064 | 0.2160 |

Figure (5.18) shows the calculated data above.


Fig. (5.18): Drainage Oil-Water Relative Permeabilities.
Apply Equation (5.28) in conjunction with Equation (5.22) to generate relative permeability data for the gas-oil system.

| $\boldsymbol{S}_{\boldsymbol{g}}$ | $\boldsymbol{S}_{\boldsymbol{o}}=\mathbf{1}-\boldsymbol{S}_{\boldsymbol{g}}-\boldsymbol{S}_{\boldsymbol{w c}}$ | $\boldsymbol{S}_{\boldsymbol{o}}^{*}=\frac{\boldsymbol{S}_{\boldsymbol{o}}}{\mathbf{1}-\boldsymbol{S}_{\boldsymbol{w} \boldsymbol{c}}}$ | $\boldsymbol{k}_{\boldsymbol{r} \boldsymbol{o}}=\left(\boldsymbol{S}_{\boldsymbol{o}}^{*}\right)^{\mathbf{3}}$ | $\boldsymbol{k}_{\boldsymbol{r g}}=\left(\mathbf{1}-\boldsymbol{S}_{\boldsymbol{o}}^{*}\right)^{\mathbf{3}}$ |
| :--- | :---: | :---: | :---: | :---: |
| 0.05 | 0.70 | 0.933 | 0.813 | - |
| 0.10 | 0.65 | 0.867 | 0.651 | 0.002 |
| 0.20 | 0.55 | 0.733 | 0.394 | 0.019 |
| 0.30 | 0.45 | 0.600 | 0.216 | 0.064 |
| 0.40 | 0.35 | 0.467 | 0.102 | 0.152 |
| 0.50 | 0.25 | 0.333 | 0.037 | 0.296 |
| 0.60 | 0.15 | 0.200 | 0.008 | 0.512 |
| 0.70 | 0.05 | 0.067 | 0.000 | 0.813 |

Figure (5.1) shows the calculated data above.


Fig. (5.19): Drainage Gas-Oil Relative Permeabilities.

### 5.2.3.2 Torcaso and Wyllie Correlation:

Torcaso and Wyllie (1958) developed a simple expression to determine the relative permeability of the oil phase in a gas-oil system. The expression permits the calculation of $k_{r o}$ from the measurements of $k_{r g}$. The equation has the following form:

$$
\begin{equation*}
k_{r o}=k_{r g}\left[\frac{\left(S_{o}^{*}\right)^{4}}{\left(1-S_{o}^{*}\right)^{2}\left(1-\left(S_{o}^{*}\right)^{2}\right)}\right] \tag{5.33}
\end{equation*}
$$

The above expression is very useful since $k_{r g}$ measurements are easily made and $k_{r o}$ measurements are usually made with difficulty.
For other correlations details see (Ahmed, 2019).

### 5.2.4 Three-Phase Relative Permeability:

The relative permeability to a fluid is defined as the ratio of effective permeability at a given saturation of that fluid to the absolute permeability at $100 \%$ saturation. Each porous system has unique relative permeability characteristics, which must be measured experimentally. Direct experimental determination of three-phase relative permeability properties is extremely difficult and involves rather complex techniques to determine the fluid saturation distribution along the length of the core. For this reason, the more easily measured two-phase relative permeability characteristics are experimentally determined.
In a three-phase system of this type, it is found that the relative permeability to water depends only upon the water saturation. Since the water can flow only through the smallest interconnect pores that are present in the rock and able to accommodate its volume, it is hardly surprising that the flow of water does not
depend upon the nature of the fluids occupying the other pores. Similarly, the gas relative permeability depends only upon the gas saturation. This fluid, like water, is restricted to a particular range of pore sizes and its flow is not influenced by the nature of the fluid or fluids that fill the remaining pores.

The pores available for flow of oil are those that, in size, are larger than pores passing only water, and smaller than pores passing only gas. The number of pores occupied by oil depends upon the particular size distribution of the pores in the rock in which the three phases coexist and upon the oil saturation itself.
In general, the relative permeability of each phase, i.e., water, gas, and oil, in a three-phase system is essentially related to the existing saturation by the following functions:

$$
\begin{gather*}
k_{r w}=f\left(S_{w}\right)  \tag{5.34}\\
k_{r g}=f\left(S_{g}\right)  \tag{5.35}\\
k_{r o}=f\left(S_{w}, S_{g}\right) \tag{5.36}
\end{gather*}
$$

Function (5.36) is rarely known and, therefore, several practical approaches are proposed and based on estimating the three-phase relative permeability from two sets of two-phase data:

## Set 1: Oil-Water System:

$$
\begin{aligned}
& k_{\text {row }}=f\left(S_{w}\right) \\
& k_{r w}=f\left(S_{w}\right)
\end{aligned}
$$

## Set 2: Oil-Gas System:

$$
\begin{aligned}
& k_{r o g}=f\left(S_{g}\right) \\
& k_{r g}=f\left(S_{g}\right)
\end{aligned}
$$

where $k_{r o w}$ and $k_{r o g}$ are defined as the relative permeability to oil in the wateroil two-phase system and similarly $k_{\text {rog }}$ is the relative permeability of oil in the gas-oil system. The symbol $k_{r o}$ is reserved for the oil relative permeability in the three-phase system.
The triangular graph paper is commonly used to illustrate the changes in the relative permeability values when three phases are flowing simultaneously, as illustrated in Figures (5.20) and (5.21). The relative permeability data are plotted as lines of constant percentage relative permeability (oil, water, and gas isoperms). Figures (5.20) and (5.21) show that the relative permeability data, expressed as isoperms, are dependent on the saturation values for all three phases in the rock.

There are several practical correlations that have developed over the years for three-phase relative permeability calculation, including:

- Wyllie's Correlations.
- Stone's Model I.
- Stone's Model II.
- The Hustad-Holt Correlation.

For more details see (Ahmed, 2019).


Fig. (4.22): Three-plate relative permeability imbibition. (After Honarpour et al., 1988).


Fig. (5.23): Three-phase relative permeability drainage. (After Honarpour et al., 1988).

## Problems

1. The capillary pressure data for a water-oil system are given below:

| $\boldsymbol{S}_{\boldsymbol{w}}$ | $\boldsymbol{p}_{\boldsymbol{c}}$ |
| :---: | :---: |
| 0.25 | 35 |
| 0.30 | 16 |
| 0.40 | 8.5 |
| 0.50 | 5 |
| 1.0 | 0 |

The core sample used in generalizing the capillary pressure data was taken from a layer that is characterized by an absolute permeability of ( 300 md ) and a porosity of ( $17 \%$ ). Generate the capillary pressure data for a different layer that is characterized by a porosity and permeability of (15\%), (200 md ), respectively. The interfacial tension is measured at ( 35 dynes $/ \mathrm{cm}$ ).
2. A five-layer oil reservoir is characterized by a set of capillary pressuresaturation curves as shown in Figure below. The following additional data are also available:


| Layer | Depth, ft | Permeability, md |
| :---: | :---: | :---: |
| 1 | $6000-6016$ | 10 |
| 2 | $6016-6025$ | 300 |
| 3 | $6025-6040$ | 100 |
| 4 | $6040-6055$ | 30 |
| 5 | $6055-6070$ | 3 |
| WOC $=6070 \mathrm{ft}$ | Water density $=65 \mathrm{lb} / \mathrm{ft}^{3}$ | Oil density $=32 \mathrm{lb} / \mathrm{ft}^{3}$ |

Calculate and plot the water and oil saturation profiles for this reservoir.
3. Calculate the reservoir height of water ( $100 \%$ water saturated zone) above the FWL, using the following capillary pressure data :

| $p_{c}$ lab. $=18 \mathrm{psi}$ | $S_{w c}=35 \%$ | $\sigma_{w o}=24 \mathrm{dyne} / \mathrm{cm}$ |
| :--- | :--- | :--- |
| $\rho_{w}=68 \mathrm{lb} / \mathrm{ft}^{3}$ | $\sigma_{g w}=72$ dyne $/ \mathrm{cm}$ | $\rho_{o}=53 \mathrm{lb} / \mathrm{ft}^{3}$ |

If Water-Gas system was used during laboratory test while the true reservoir fluid are Water-Oil. Also, schematically draw the height Vs. Saturation. Showing the level and depth of each zone if the transition zone capillary pressure equal ( 3 psi ), FWL depth $=1200 \mathrm{ft}$.
4. The capillary pressure data for a water-oil System are given:

| $\boldsymbol{S}_{\boldsymbol{w}}$ (fraction) | $\boldsymbol{p}_{\boldsymbol{c}}($ psia $)$ |
| :---: | :---: |
| 0.25 | 35 |
| 0.3 | 16 |
| 0.4 | 8.5 |
| 0.5 | 5 |
| 1 | 3 |
| 1 | 0 |

The core sample used in generalizing the capillary pressure data was taken from a layer that is characterized by an absolute permeability of ( 300 md ) and porosity of $(17 \%)$. Generate the capillary pressure data for a different layer that is characterized by a porosity \& permeability of ( $15 \%$ ) and ( 200 md ) respectively. The interfacial tension is measured at ( 35 dyne $/ \mathrm{cm}$ ). Also draw and determine the connate water saturation, and FWL if you know that the WOC depth $=3120 \mathrm{ft}$.
5. Given the following Hg injection data:

| $\boldsymbol{S}_{\boldsymbol{H} \boldsymbol{g}}($ fraction $)$ | $\boldsymbol{p}_{\boldsymbol{c}}($ psia $)$ |
| :---: | :---: |
| 0 | 0 |


| 0 | 5 |
| :---: | :---: |
| 0.1 | 10 |
| 0.3 | 20 |
| 0.6 | 30 |
| 0.8 | 40 |
| 0.9 | 100 |
| 0.9 | 200 |

Also:

$$
\begin{array}{ll}
\sigma_{H g}=400 \mathrm{dyne} / \mathrm{cm} & \sigma_{w o}=40 \mathrm{dyne} / \mathrm{cm} \\
\theta_{H g}=120^{\circ} & \theta_{w o}=20^{\circ} \\
\rho_{w}=1.09 \mathrm{gm} / \mathrm{cm}^{3} & \rho_{o}=0.2 \mathrm{gm} / \mathrm{cm}^{3}
\end{array}
$$

Calculate:
a. The height of water saturation (0.5) above free water level.
b. Displacement pressure.
c. The thickness of transition zone.
6. A Laboratory capillary pressure test was conducted on a core sample taken from XX field. The core has a porosity and permeability of (16\%) and (80 md ), respectively. The capillary-pressure saturation data are given as follows:

| $\boldsymbol{S}_{\boldsymbol{w}}$, (fraction) | $\boldsymbol{p}_{\boldsymbol{c}}($ psia $)$ |
| :---: | :---: |
| 1.0 | 0.50 |
| 0.8 | 0.60 |
| 0.6 | 0.75 |
| 0.4 | 1.05 |
| 0.2 | 1.75 |

The interfacial tension is measured at ( 50 dyne/cm). Further - reservoir Engineering analysis indicates that the reservoir is better described at a porosity value of (19\%) and an absolute permeability of (120 md). Generate the capillary pressure data for the reservoir.
7. The average Leverett J -function $\mathrm{J}(\mathrm{Sw})$ was determined from a set of capillary pressure curves measured on a number of cores from a reservoir interval. Some values are tabulated below:

| $\boldsymbol{S}_{\boldsymbol{w}}$ <br> (fraction) | $\boldsymbol{J}\left(\boldsymbol{S}_{\boldsymbol{w}}\right)$ <br> (dimensionless) |
| :---: | :---: |
| 0 | 0 |
| 0 | 5 |


| 0.1 | 10 |
| :---: | :---: |
| 0.3 | 20 |
| 0.6 | 30 |
| 0.8 | 40 |
| 0.9 | 100 |
| 0.9 | 200 |

Calculate the thickness of the capillary transition zones between water and oil, and between oil and gas, using the following additional data on the reservoir:
average porosity

$$
\begin{aligned}
& \bar{\phi}=0.20 \\
& k=200 \mathrm{md} \\
& \sigma_{w o}=30 \mathrm{dyne} / \mathrm{cm} \\
& \theta_{w o}=35^{\circ} \\
& \sigma_{g o}=5 \mathrm{dyne} / \mathrm{cm} \\
& \theta_{g o}=10^{\circ} \\
& \rho_{o}=850 \mathrm{~kg} / \mathrm{m}^{3} \\
& \rho_{w}=1050 \mathrm{~kg} / \mathrm{m}^{3} \\
& \rho_{g}=120 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

average permeability
water/oil interfacial tension
water/oil angle of contact
gas/oil interfacial tension
gas/oil angle of contact
oil density
water density
gas density
If the reservoir contained only gas directly in contact with the aquifer, what would the thickness of the gas/water Capillary transition be, given that $\sigma_{w g}=45$ dyne/cm and $\theta=0^{\circ}$ ?
8. Given:

- $S_{w c}=0.30$
$S_{g c}=0.06$
$S_{o c}=0.35$
- unconsolidated well-sorted sand.

Generate the drainage relative permeability data by using Wyllie-Gardner correlation.

## REFERENCES

Ahmed, T., 2019, "Reservoir Engineering Handbook", Gulf Professional Publishing.
Ahmed, T., 2010, "Working guide Reservoir Rock Properties and Fluid Flow", Burlington, MA.

Bech, N., Frykman, P., and Vejbæk, O.V., Determination of free water levels in lowpermeability chalk reservoirs from logged saturations, Society of Petroleum Engineers SPE paper number 105622.
Buckley, S. E. and Leverett, M. C., 1942, Mechanism of fluid displacement in sands, Trans. AIME, 146,107.

Dandekar, A.Y., 2013 "Petroleum Reservoir Rock and Fluid Properties", CRC Press.
Ertekin, T., Abou-Kassem, J.H., and King, G.R., 2001, "Basic Applied Reservoir Simulation", Society of Petroleum Engineers, Richardson, TX, p. 406.
Honarpour, M. M., Koederitz, L. F., and Harvey, A. H., 1988, "Relative Permeability of Petroleum Reservoirs". CRC Press, Inc., 1988.
Leverett, M.C., 1939, Flow of oil-water mixtures through unconsolidated sands, Transactions of the AIME, 132, 381-401.
Leverett, M.C., 1941, Capillary behavior in porous solids, Transactions of the AIME, 142, 152.

Slider, H.C., 1976, "Worldwide Practical Petroleum Reservoir Engineering Methods", Petroleum Publishing Company, Tulsa, OK.
Special Core Analysis, Core Laboratories,Inc., Dallas, 1976.
Tiab, D. and Donaldson, E.C., 1996, "Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties", Gulf Publishing Company, Houston, TX.
Torcaso, M. A., and Wyllie, M. R. J., 1958, "A Comparison of Calculated krg/kro Ratios with Field Data," JPT, pp. 6, 57.
Welge, H.J., 1952, A simplified method for computing recovery by gas or water drive, Trans .AIME19, 5,91,1952.

Wyllie, M. R. J., and Gardner, G. H. F., 1958, "The Generalized Kozeny-Carmen EquationIts Application to Problems of Multi-Phase Flow in Porous Media," World Oil, pp. 121, 146.

# Chapter Six Introduction to Reserve Estimation 

### 6.1 Introduction:

The term "reserves" means different things to different subjects. To the oil and gas industry, reserves are the amount of crude oil, natural gas, and associated substances that can be produced profitably in the future from subsurface reservoirs.

Estimates of oil and/or gas reserves are inherently uncertain. The degree of uncertainty in estimates of reserves depends mainly on:

- The degree of geologic complexity.
- Maturity of the property.
- The quality and quantity of geologic and engineering data.
- The operating environment.
- The skill, experience and integrity of the estimators.

Over the years, there have been numerous classifications of oil and/or gas reserves published by various individuals and organizations. The principle purposes of a classification system are: (1) to promote uniformity and (2) to qualify the degree of risk for each class of reserve.

Procedures to estimate and classify reserves (ECR) have been described as deterministic or probabilistic. These procedures to estimate reserves, or a part of such reserves, may be classified as:

1. Proved reserves (with certainty of $90 \%$ ).
2. Or unproved reserves: this type of reserves divided into two categories: probable reserves (with certainty of $50 \%$ ) or possible reserves (with certainty at least of $10 \%$ ) based on the engineer's judgment and relevant guidelines regarding the probability of actually producing such reserves.

While it has been recognized that there is always some degree of uncertainty in estimating reserves, most of the published literature on reserve estimating has focused on deterministic methods. Probabilistic calculations, however, have received continuing attention in the geologic literature in the evaluation of exploration "plays".

### 6.1.1 Reserve Status Categories:

Reserve status categories define the development and producing status of wells and reservoirs.

Developed: Developed reserves are expected to be recovered from existing wells including reserves behind pipe. Improved recovery reserves are considered developed only after the necessary equipment has been installed, or when the costs to do so are relatively minor. Developed reserves may be subcategorized as producing or non-producing.

Producing: Reserves subcategorized as producing are expected to be recovered from completion intervals which are open and producing at the time of the estimate. Improved recovery reserves are considered producing only after the improved recovery project is in operation.

Non-producing: Reserves subcategorized as non-producing include shut-in and behind-pipe reserves. Shut-in reserves are expected to be recovered from (1) completion intervals which are open at the time of the estimate but which have not started producing, (2) wells which were shut-in for market conditions or pipeline connections, or (3) wells not capable of production for mechanical reasons. Behind-pipe reserves are expected to be recovered from zones in existing wells, which will require additional completion work or future recompletion prior to the start of production.

Undeveloped Reserves: Undeveloped reserves are expected to be recovered: (1) from new wells on undrilled acreage, (2) from deepening existing wells to a different reservoir, or (3) where a relatively large expenditure is required to (a) recomplete an existing well or (b) install production or transportation facilities for primary or improved recovery projects.

Terminology for oil and gas resources classification is illustrated by Fig. (6.1).

### 6.1.2 Reserve Estimation Methods:

Reserve estimation methods is discussed briefly in this chapter. Which may be classified as:

- Analogical Methods.
- Volumetric Methods.
- Performance Methods.
- Material Balance.
- Computer Simulation.
- Performance/Decline Trend Analysis.

a. In reporting to EIA. "other' reserves include only "indicated additional reserves that do not meet the criteria necessary to be classified as proved.
b. Nomproducing reserves include those that did not produce during the prior reporting year.

Fig. (6.1): Terminology for oil and gas resources.

### 6.2 Oil Recovery Methods:

Oil recovery methods are illustrated in Table (6.1) below. Primary recovery, production depends on the natural energy of the reservoir itself. Secondary recovery, when natural drive energy is depleted or too small for economic oil recovery, energy must be added to the reservoir to permit additional oil recovery. That additional energy is usually in the form of injected water or gas. Tertiary recovery, when secondary recovery is no longer economic, supplemental energy of a different kind permits additional oil recovery. Enhanced fluid flow conditions within the reservoir are usually induced by addition of heat, chemical interaction between the injected fluid and the reservoir oil, mass transfer, and/or changing of oil properties in such a way that the process facilitates oil movement through the reservoir. Tertiary recovery processes generally include thermal, chemical, gas miscible and microbial. They are also often referred to as enhanced oil recovery (EOR) processes.

Improved oil recovery (IOR) refers to any practice used to increase oil recovery. This can include EOR and secondary recovery processes such as waterflooding and gas pressure maintenance, as well as practices to increase
sweep such as infill drilling, horizontal wells and polymers for mobility control or improved conformance.

Table 6.1: oil recovery methods.

| Primary Recovery <br> (oil recovery less than 30\%) |  |  | Natural flow | - Rock and liquid expansion drive. <br> - Depletion drive. <br> - Gas-cap drive. <br> - Water drive. <br> - Gravity drainage drive. <br> - Combination drive. |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Artificial lift | - Pump <br> - Gas lift <br> - Other |
| O | Secondary Oil Recovery (oil recovery: 30-50\%) |  | Water flood |  |
|  |  |  | Pressure <br> Maintenance | Water/Gas Reinjection |
|  |  | Tertiary Oil Recovery (oil recovery more than $50 \%$ and up to 80\%) | Thermal | - Combustion <br> - Steam soak/cyclic <br> - Huff-and-puff <br> - Steam drive/flood <br> - Hot water drive <br> - Electromagnetic |
|  |  |  | Gas miscible/ immiscible | $-\mathrm{CO}_{2}$ <br> - Nitrogen <br> - Flue gas <br> - Hydrocarbon |
|  |  |  | Chemical \& other | - Alkaline <br> - Micellar-Polymer <br> - Microbial/foam |

### 6.3 Reservoir Primary Recovery Mechanisms:

The recovery of oil by any of the natural drive mechanisms is called primary recovery. The term refers to the production of hydrocarbons from a reservoir without the use of any process (such as fluid injection) to supplement the natural energy of the reservoir.

For a proper understanding of reservoir behavior and predicting future performance, it is necessary to have knowledge of the driving mechanisms that control the behavior of fluids within reservoirs. The overall performance of oil reservoirs is largely determined by the nature of the energy, i.e., driving mechanism, available for moving the oil to the wellbore. There are basically six driving mechanisms that provide the natural energy necessary for oil recovery:

- Rock and liquid expansion drive.
- Depletion drive.
- Gas-cap drive.
- Water drive.
- Gravity drainage drive.
- Combination drive.


### 6.3.1 Rock and Liquid Expansion:

When an oil reservoir initially exists at a pressure higher than its bubble-point pressure, the reservoir is called an undersaturated-oil reservoir. At pressures above the bubble-point pressure, crude oil, connate- water, and rock are the only materials present. As the reservoir pressure declines, the rock and fluids expand due to their individual compressibilities. The reservoir rock compressibility is the result of two factors:

- Expansion of the individual rock grains.
- Formation compaction.

As the expansion of the fluids and reduction in the pore volume occur with decreasing reservoir pressure, the crude oil and water will be forced out of the pore space to the wellbore. Because liquids and rocks are only slightly compressible, the reservoir will experience a rapid pressure decline. The oil reservoir under this driving mechanism is characterized by a constant gas-oil ratio that is equal to the gas solubility at the bubble point pressure.

This driving mechanism is considered the least efficient driving force and usually results in the recovery of only a small percentage of the total oil-inplace.

### 6.3.2 The Depletion-Drive Mechanism:

This driving form may also be referred to by the following various terms:

- Solution gas drive.
- Dissolved gas drive.
- Internal gas drive.

In this type of reservoir, the principal source of energy is a result of gas liberation from the crude oil and the subsequent expansion of the solution gas as the reservoir pressure is reduced. As pressure falls below the bubble-point pressure, gas bubbles are liberated within the microscopic pore spaces. These bubbles expand and force the crude oil out of the pore space, see figure (6.2).


Fig. (6.2): Dissolved gas drive reservoir, (Clark, 1960).

### 6.3.3 Gas-Cap Drive:

Gas-cap-drive reservoirs can be identified by the presence of a gas cap with little or no water drive. Due to the ability of the gas cap to expand, these reservoirs are characterized by a slow decline in the reservoir pressure. The
natural energy available to produce the crude oil comes from the following two sources:

- Expansion of the gas-cap gas.
- Expansion of the solution gas as it is liberated, see figure (6.3).


### 6.3.4 The Water-Drive Mechanism:

Many reservoirs are bounded on a portion or all of their peripheries by water bearing rocks called aquifers. Although water is considered incompressible, the total compressed volume is quite large when such great quantities of total water volume are involved. Even the great volume of rock in which the water exist is influenced by water pressure. As oil is produced, pressure declines at the point where oil is withdrawn from the reservoir. Water then moves in to replace the oil as it is produced because of expansion of the minutely compressed water; a reservoir producing in this manner is termed a water drive reservoir (Fig. 6.4). Many of the most important reservoir in the world are producing by energy supplied by water drives. Notable examples are the east Texas field; Alberta in Canada; Burgan field in Kuwait.


Fig. (6.3): Gas cap drive reservoir, (Clark, 1960).


A CROSS SECTION


## B MAP VEW

Fig. (6.4): Water drive reservoir, (Clark, 1960).

### 6.3.5 The Gravity-Drainage-Drive Mechanism:

The mechanism of gravity drainage occurs in petroleum reservoirs as a result of differences in densities of the reservoir fluids. The effects of gravitational forces can be simply illustrated by placing a quantity of crude oil and a quantity of water in a jar and agitating the contents. After agitation, the jar is placed at rest, and the denser fluid (normally water) will settle to the bottom of the jar, while the less dense fluid (normally oil) will rest on top of the denser fluid. The fluids have separated as a result of the gravitational forces acting on them.
The fluids in petroleum reservoirs have all been subjected to the forces of gravity, as evidenced by the relative positions of the fluids, i.e., gas on top, oil underlying the gas, and water underlying oil. Due to the long periods of time involved in the petroleum accumulation-and-migration process, it is generally
assumed that the reservoir fluids are in equilibrium. If the reservoir fluids are in equilibrium, then the gas-oil and oil-water contacts should be essentially horizontal. Although it is difficult to determine precisely the reservoir fluid contacts, best available data indicate that, in most reservoirs, the fluid contacts actually are essentially horizontal.

Gravity segregation of fluids is probably present to some degree in all petroleum reservoirs, but it may contribute substantially to oil production in some reservoirs, see Fig. (6.5).


Fig. (6.5): Gravitational segregation showing relative movement of gas and oil along the structure at various distance from a producing well, (Clark, 1960).

### 6.3.6 The Combination-Drive Mechanism:

Reservoirs are seldom found to fit exactly one type of drive classification. The most common type of drive encountered, is a combination drive see Fig. (6.6). Production problems are exceedingly complicated because of the infinite number of combinations characterizing the various reservoirs occurring naturally.


Fig. (6.6): Combination drive reservoir, (Clark, 1960).

### 6.4 Volumetric Methods:

The volumetric method involves calculating: (a) the amount of oil and gas initially in place by a combination of volumetric (geologic) mapping, petrophysical analysis, and reservoir engineering and (b) the fractions of oil, gas, and associated products initially in place that are expected to be recovered commercially-i.e., the recovery efficiencies-using analytical methods and/or analogy.

Procedures to estimate oil and gas initially in place and recovery efficiency are discussed in the following sections.

### 6.4.1 Oil Reservoirs:

For an oil reservoir, or for the oil column of an oil reservoir with a gas cap, oil initially in place (OIIP) may be calculated as:

$$
\begin{equation*}
O I I P=N_{i}=\frac{7758 \phi_{o}\left(1-S_{w o}\right) A_{o} h_{n o}}{B_{o i}} \tag{6.1}
\end{equation*}
$$

where:
$N_{i}=$ oil initially in place, STB.
$7758=$ (unit conversion constant) barrels in an acre foot (1
acre $=4046.85642 \mathrm{~m}^{2} \cong 4047 \mathrm{~m}^{2}$ ).
$\phi_{o}=$ average porosity in the oil zone, fraction.
$S_{w o}=$ average water saturation in the oil zone, fraction.
$A_{o}=$ area of the oil zone, acres.
$h_{n o}=$ average net oil pay, feet.
$B_{o i}=$ average initial formation volume factor, $\mathrm{bbl} / \mathrm{STB}$.
If the units for area and net pay are hectares and meters, respectively, the constant in Eq. (6.1) becomes 1.0, and the units for OIP are $\left(10^{4} \mathrm{~m}^{3}\right)$.

Porosity, water saturation, and oil formation volume factor should be volumeweighted averages in the oil zone.

Solution gas dissolved in the oil at initial reservoir conditions may be calculated as:

$$
\begin{equation*}
G_{S i}=N_{i} R_{S i} \tag{6.2}
\end{equation*}
$$

where:
$G_{S i}=$ solution gas initially in place, scf.
$R_{s i}=$ average initial solution gas/oil ratio, scf/STB.
In some areas, GOR units are cubic meters of gas per cubic meter of oil $\left(\mathrm{m}^{3} / \mathrm{m}^{3}\right)$. The conversion factor is $(5.614 \mathrm{scf} / \mathrm{STB})$.

## Example 6.1:

An oil reservoir exists at its bubble-point pressure of (3,000 psia) and temperature of $\left(160^{\circ} \mathrm{F}\right)$. The oil has an API gravity of $\left(42^{\circ}\right)$ and gas-oil ratio of ( $600 \mathrm{scf} / \mathrm{STB}$ ). The specific gravity of the solution gas is $(0.65)$. The following additional data are also available:

- Reservoir area $=640$ acres.
- Average thickness $=10 \mathrm{ft}$.
- Connate water saturation $=0.25$
- Effective porosity $=15 \%$.

Calculate the initial oil in place in STB.

## Solution:

Step 1. Determine the specific gravity of the stock-tank oil from Equation (2.42):

$$
\begin{gathered}
A P I=\frac{141.5}{\gamma_{o}}-131.5 \\
\gamma_{o}=\frac{141.5}{42+131.5}=0.8156
\end{gathered}
$$

Step 2. Calculate the initial oil formation volume factor by applying Standing's equation, i.e., Equation (2.50), to give:

$$
B_{o}=0.9759+0.000120\left[600\left(\frac{0.65}{0.8156}\right)^{0.5}+1.25(160)\right]^{1.2}=1.396 \mathrm{bbl} / \text { STB }
$$

Step 3. Calculate the initial oil in place by using equation (6.1):

$$
\text { OIIP }=N_{i}=\frac{7758(0.15)(1-0.25)(640)(10)}{1.396}=4001260.74 S T B
$$

### 6.4.2 Gas Reservoirs:

For a nonassociated gas reservoir, or for a gas cap, free gas initially in place may be calculated as:

$$
\begin{equation*}
G_{F i}=\frac{43560 \phi_{g}\left(1-S_{w g}\right) A_{g} h_{n g}}{B_{g i}} \tag{6.3}
\end{equation*}
$$

where:
$G_{F i}=$ free gas initially in place, scf.
$43560=$ (unit conversion constant) cubic feet in an acre foot.
$\phi_{g}=$ average porosity in the free gas zone, fraction.
$S_{w g}=$ average water saturation in the free gas zone, fraction.
$A_{g}=$ area of gas cap or gas reservoir, acres.
$h_{n g}=$ average net thickness of gas cap or gas reservoir, feet.
$B_{g i}=$ average initial formation volume factor, $f t^{3} / s c f$.
Condensate (also called distillate) in the vapor phase at initial reservoir conditions (but measured as a liquid at surface conditions) may be calculated as:

$$
\begin{equation*}
C_{i}=C_{F i} R_{c i} \tag{6.4}
\end{equation*}
$$

where:
$C_{i}=$ condensate (distillate) initially in place, STB.
$R_{c i}=$ initial condensate/gas ratio (CGR), STB condensate/MMscf.
$C_{F i}=$ free gas initially in place, MMscf.

## Example 6.2:

The following data are given for the Bell Gas Field:

- Area $=160$ acres.
- Net productive thickness $=40 \mathrm{ft}$.
- Porosity $=22 \%$.
- Connate water $=23 \%$.
- Initial gas $\mathrm{FVF}=0.00533 \mathrm{ft}^{3} / S C F$.

Find the gas initially in place.

## Solution:

Calculating initially gas in place by using Eq. (6.3):

$$
G_{F i}=\frac{43560(0.22)(1-0.23)(160)(40)}{0.00533}=8860 \mathrm{MMSCF}
$$

Note: Each " M " is equal $10^{3}$, So "MM" equals $10^{6}$.

### 6.5 Material Balance Equation:

The material balance equation (MBE) has long been recognized as one of the basic tools of reservoir engineers for interpreting and predicting reservoir performance. The MBE, when properly applied, can be used to:

- Estimate initial hydrocarbon volumes in place.
- Predict future reservoir performance.
- Predict ultimate hydrocarbon recovery under various types of primary driving mechanisms.

The equation is structured to simply keep inventory of all materials entering, leaving, and accumulating in the reservoir. The concept of the material balance equation was presented by Schilthuis in 1941. In its simplest form, the equation can be written on a volumetric basis as:

$$
\text { Initial volume }=\text { volume remaining }+ \text { volume removed }
$$

Since oil, gas, and water are present in petroleum reservoirs, the material balance equation can be expressed for the total fluids or for any one of the fluids present.

Before deriving the material balance equation, it is convenient to denote certain terms by symbols for brevity. The symbols used conform where possible to the standard nomenclature adopted by the Society of Petroleum Engineers.

| $p_{i}$ | Initial reservoir pressure, psi |
| :---: | :--- |
| $p$ | Volumetric average reservoir pressure |
| $\Delta p$ | Change in reservoir pressure $=p_{i}-\mathrm{p}, \mathrm{psi}$ |
| $P_{p}$ | Bubble point pressure, psi |
| N | Initial (original) oil-in-place, STB |
| $N_{p}$ | Cumulative oil produced, STB |
| $G_{p}$ | Cumulative gas produced, scf |
| $W_{p}$ | Cumulative water produced, bbl |
| $R_{p}$ | Cumulative gas-oil ratio, scf/STB |
| $\mathrm{GOR}^{\prime}$ | Instantaneous gas-oil ratio, scf/STB |
| $R_{s i}$ | Initial gas solubility, scf/STB |
| $R_{s}$ | Gas solubility, scf/STB |
| $B_{o i}$ | Initial oil formation volume factor, bbl/STB |
| $B_{o}$ | Oil formation volume factor, bbl/STB |
| $B_{g i}$ | Initial gas formation volume factor, bbl/scf |
| $B_{g}$ | Gas formation volume factor, bbl/scf |
| $W_{i n j}$ | Cumulative water injected, STB |
| $G_{i n j}$ | Cumulative gas injected, scf |
| $W_{e}$ | Cumulative water influx, bbl |
| m | Ratio of initial gas-cap gas reservoir volume to initial reservoir oil |
| G | volume, bbl/bbl |
| Initial gas-cap gas, scf |  |
| $\mathrm{P} . \mathrm{V}$ | Pore volume, bbl |
| $c_{w}$ | Water compressibility, psi ${ }^{-1}$ |
| $c_{f}$ | Formation (rock) compressibility, psi ${ }^{-1}$ |

Several of the material balance calculations require the total pore volume (P.V) as expressed in terms of the initial oil volume $N$ and the volume of the gas cap. The expression for the total pore volume can be derived by conveniently introducing the parameter $m$ into the relationship as follows:
Defining the ratio $m$ as:

$$
\begin{equation*}
m=\frac{\text { Initial volume of gas cap }}{\text { Volume of oil initially in place }}=\frac{G B_{g i}}{N B_{o i}} \tag{6.5}
\end{equation*}
$$

Solving for the volume of the gas cap gives:

$$
\text { Initial volume of the gas cap }=G B_{g i}=m N B_{o i}
$$

The total volume of the hydrocarbon system is then given by:

Initial oil volume + initial gas cap volume

$$
=(P . V)\left(1-S_{w i}\right) N B_{o i}+m N B_{o i}=(P . V)\left(1-S_{w i}\right)
$$

or,

$$
\begin{equation*}
P . V=\frac{N B_{o i}(1+m)}{1-S_{w i}} \tag{6.6}
\end{equation*}
$$

where:
$S_{w i}=$ initial water saturation.
$N=$ initial oil-in-place, STB.
$P . V=$ total pore volume, bbl.
$m=$ ratio of initial gas-cap gas reservoir volume to initial reservoir oil volume, bbl/bbl.

Treating the reservoir pore as an idealized container as illustrated in Figure (6.2), volumetric balance expressions can be derived to account for all volumetric changes which occur during the natural productive life of the reservoir.


Fig. (6.2): Tank-model concept.

The MBE can be written in a generalized form as follows:

๔

## (8)

$]+\left[\begin{array}{c}\text { Pore } \\ \text { volume } \\ \text { occupied } \\ \text { by the } \\ \text { net } \\ \text { water } \\ \text { influx } \\ \text { at } p\end{array}\right]+\left[\begin{array}{c}\text { Change } \\ \text { in pore } \\ \text { volume } \\ \text { due to } \\ \text { connat } \\ \text { water } \\ \text { expansion } \\ \text { and rock } \\ \text { expansion }\end{array}\right]+\left[\begin{array}{c}\text { Pore } \\ \text { volume } \\ \text { occupied } \\ \text { by the } \\ \text { injected } \\ \text { gas } \\ \text { at } p\end{array}\right]+\left[\begin{array}{c}\text { Pore } \\ \text { voleum } \\ \text { occupied } \\ \text { by the } \\ \text { injected } \\ \text { water } \\ \text { at } p\end{array}\right]$
$(L \cdot 9) \cdot b$ g

[^0]The above nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties, as follows:

## (1) Pore Volume Occupied by the Oil Initially in Place:

$$
\begin{equation*}
\text { Volume occupied by initial oil }- \text { in }- \text { place }=N B_{o i} \tag{6.8}
\end{equation*}
$$

where:
$N=$ oil initially in place, STB.
$B_{o i}=$ oil formation volume factor at initial reservoir pressure $p_{i}, \mathrm{bbl} / \mathrm{STB}$.
(2) Pore Volume Occupied by the Gas in the Gas Cap:

$$
\begin{equation*}
\text { Volume of gas cap }=m N B_{o i}=G B_{g i} \tag{6.9}
\end{equation*}
$$

where $m$ is a dimensionless parameter and defined as the ratio of gas-cap volume to the oil zone volume.

## (3) Pore Volume Occupied by the Remaining Oil:

$$
\begin{equation*}
\text { Volume of the remaining oil }=\left(N-N_{p}\right) B_{o} \tag{6.10}
\end{equation*}
$$ where:

$N_{p}=$ cumulative oil production, STB.
$B_{o}=$ oil formation volume factor at reservoir pressure $p, \mathrm{bbl} / \mathrm{STB}$.

## (4) Pore Volume Occupied by the Gas Cap at Reservoir Pressure p:

As the reservoir pressure drops to a new level $p$, the gas in the gas cap expands and occupies a larger volume. Assuming no gas is produced from the gas cap during the pressure decline, the new volume of the gas cap can be determined as:

$$
\begin{equation*}
\text { Volume of the gas cap at } p=\left[\frac{m N B_{o i}}{B_{g i}}\right] B_{g} \tag{6.11}
\end{equation*}
$$

where:
$B_{g i}=$ gas formation volume factor at initial reservoir pressure, $\mathrm{bbl} / \mathrm{scf}$. $B_{g}=$ current gas formation volume factor, $\mathrm{bbl} / \mathrm{scf}$.

## (5) Pore Volume Occupied by the Evolved Solution Gas:

This volumetric term can be determined by applying the following material balance on the solution gas:

$$
\left[\begin{array}{c}
\text { volume } \\
\text { of the } \\
\text { evolved } \\
\text { solution } \\
\text { gas }
\end{array}\right]=\left[\begin{array}{c}
\text { volume } \\
\text { of gas } \\
\text { initially } \\
\text { in } \\
\text { solution }
\end{array}\right]-\left[\begin{array}{c}
\text { volume } \\
\text { of gas } \\
\text { produced }
\end{array}\right]-\left[\begin{array}{c}
\text { volume } \\
\text { of gas } \\
\text { remaining } \\
\text { in } \\
\text { solution }
\end{array}\right]
$$

or,

$$
\left[\begin{array}{c}
\text { volume }  \tag{6.12}\\
\text { of the } \\
\text { evolved } \\
\text { solution } \\
\text { gas }
\end{array}\right]=\left[N R_{s i}-N_{p} R_{p}-\left(N-N_{p}\right) R_{s}\right] B_{g}
$$

where:
$N_{p}=$ cumulative oil produced, STB.
$R_{p}=$ net cumulative produced gas-oil ratio, scf/STB.
$R_{S}=$ current gas solubility factor, scf/STB.
$B_{g}=$ current gas formation volume factor, $\mathrm{bbl} / \mathrm{scf}$.
$R_{s i}=$ gas solubility at initial reservoir pressure, scf/STB.

## (6) Pore Volume Occupied by the Net Water Influx:

$$
\begin{equation*}
\text { net water influx }=W_{e}-W_{p} B_{w} \tag{6.13}
\end{equation*}
$$

where:
$W_{e}=$ cumulative water influx, bbl.
$W_{p}=$ cumulative water produced, STB.
$B_{w}=$ water formation volume factor, $\mathrm{bbl} / \mathrm{STB}$.

## (7) Change in Pore Volume Due to Initial Water and Rock Expansion:

The component describing the reduction in the hydrocarbon pore volume due to the expansion of initial (connate) water and the reservoir rock cannot be neglected for an undersaturated-oil reservoir. The water compressibility $c_{w}$ and rock compressibility $c_{f}$ are generally of the same order of magnitude as the compressibility of the oil. The effect of these two components, however, can be generally neglected for the gas-cap-drive reservoir or when the reservoir pressure drops below the bubble-point pressure.

The compressibility coefficient $c$, which describes the changes in the volume (expansion) of the fluid or material with changing pressure, is given by:

$$
c=\frac{-1}{V} \frac{\partial V}{\partial p}
$$

or,

$$
\Delta V=V c \Delta p
$$

where $\Delta \mathrm{V}$ represents the net changes or expansion of the material as a result of changes in the pressure. Therefore, the reduction in the pore volume due to the expansion of the connate-water in the oil zone and the gas cap is given by:

$$
\text { Connate }- \text { water expansion }=\left[(\text { pore volume }) S_{w i}\right] c_{w} \Delta p
$$

Substituting for the pore volume (P.V) with Equation (6.6) gives:

$$
\begin{equation*}
\text { Exapantion of connate water }=\frac{N B_{o i}(1+m)}{1-S_{w i}} S_{w i} c_{w} \Delta p \tag{6.14}
\end{equation*}
$$

where:
$\Delta p=$ change in reservoir pressure, $(\mathrm{pi}-\mathrm{p})$.
$c_{w}=$ water compressibility coefficient, $\mathrm{psi}^{-1}$.
$m=$ ratio of the volume of the gas-cap gas to the reservoir oil volume, bbl/bbl

Similarly, the reduction in the pore volume due to the expansion of the reservoir rock is given by:

$$
\begin{equation*}
\text { Change in pore volume }=\frac{N B_{o i}(1+m)}{1-S_{w i}} c_{f} \Delta p \tag{6.15}
\end{equation*}
$$

Combining the expansions of the connate-water and formation as represented by Equations (6.14) and (6.15) gives:

$$
\left[\begin{array}{c}
\text { Total changes in }  \tag{6.16}\\
\text { the pore volume }
\end{array}\right]=N B_{o i}(1+m)\left(\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right) \Delta p
$$

## (8) \& (9) Pore Volume Occupied by the Injection Gas and Water:

Assuming that $G_{i n j}$ volumes of gas and $W_{i n j}$ volumes of water have been injected for pressure maintenance, the total pore volume occupied by the two injected fluids is given by:

$$
\begin{equation*}
\text { Total volume }=G_{i n j} B_{g i n j}+W_{i n j} B_{w} \tag{6.17}
\end{equation*}
$$

where:
$G_{i n j}=$ cumulative gas injected, scf.
$B_{g i n j}=$ injected gas formation volume factor, $\mathrm{bbl} / \mathrm{scf}$.
$W_{i n j}=$ cumulative water injected, STB.
$B_{w}=$ water formation volume factor, $\mathrm{bbl} / \mathrm{STB}$.
Combining Equations (6.8) through (6.17) with Equation (6.7) and rearranging gives:

$$
\begin{equation*}
N=\frac{N_{p} B_{o}+\left(G_{p}-N_{p} R_{s}\right) B_{g}-\left(W_{e}-W_{p} B_{w}\right)-G_{i n j} B_{g i n j}-W_{i n j} B_{w}}{\left(B_{o}-B_{o i}\right)+\left(R_{s i}-R_{s}\right) B_{g}+m B_{o i}\left[\frac{B_{g}}{B_{g i}}-1\right]+B_{o i}(1+m)\left[\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right] \Delta p} \tag{6.18}
\end{equation*}
$$

where:
$N=$ initial oil-in-place, STB.
$G_{p}=$ cumulative gas produced, scf.
$N_{p}=$ cumulative oil produced, STB.
$R_{s i}=$ gas solubility at initial pressure, scf/STB.
$m=$ ratio of gas-cap gas volume to oil volume, $\mathrm{bbl} / \mathrm{bbl}$.
$B_{g i}=$ gas formation volume factor at $p_{i}, \mathrm{bbl} / \mathrm{scf}$.
$B_{g i n j}=$ gas formation volume factor of the injected gas, bbl/scf.
The cumulative gas produced $G_{p}$ can be expressed in terms of the cumulative gas-oil ratio $R_{p}$ and cumulative oil produced $N_{p}$ by:

$$
\begin{equation*}
G_{p}=R_{p} N_{p} \tag{5.19}
\end{equation*}
$$

Combining Equation (5.19) with Equation (5.18) gives:

$$
\begin{equation*}
N=\frac{N_{p}\left[B_{o}+\left(R_{p}-R_{s}\right) B_{g}\right]-\left(W_{e}-W_{p} B_{w}\right)-G_{i n j} B_{g i n j}-W_{i n j} B_{w}}{\left(B_{o i}-B_{o}\right)+\left(R_{s i}-R_{s}\right) B_{g}+m B_{o i}\left[\frac{B_{g}}{B_{g i}}-1\right]+B_{o i}(1+m)\left[\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right] \Delta p} \tag{5.20}
\end{equation*}
$$

The above relationship is referred to as the material balance equation (MBE). A more convenient form of the MBE can be determined by introducing the concept of the total (two-phase) formation volume factor $B_{t}$ into the equation. This oil PVT property is defined as:

$$
\begin{equation*}
B_{t}=B_{o}+\left(R_{s i}-R_{s}\right) B_{g} \tag{6.21}
\end{equation*}
$$

Introducing $B_{t}$ into Equation (6.20) and assuming, for the sake of simplicity, no water or gas injection gives:

$$
\begin{equation*}
N=\frac{N_{p}\left[B_{t}+\left(R_{p}-R_{s i}\right) B_{g}\right]-\left(W_{e}-W_{p} B_{w}\right)}{\left(B_{t}-B_{t i}\right)+m B_{t i}\left[\frac{B_{g}}{B_{g i}}-1\right]+B_{t i}(1+m)\left[\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right] \Delta p} \tag{6.22}
\end{equation*}
$$

## Example 6.3:

The Big Butte field is a combination-drive reservoir. The current reservoir pressure is estimated at ( $2,500 \mathrm{psi}$ ). The reservoir production data and PVT information are given below:

|  | Initial Reservoir <br> Conditions | Current Reservoir <br> Conditions |
| :--- | :---: | :---: |
| $\mathrm{p}, \mathrm{psi}$ | 3000 | 2500 |
| $B_{o}$, bbl/STB | 1.35 | 1.33 |
| $R_{s}$, sc//STB | 600 | 500 |
| $N_{p}$, MMSTB | 0 | 5 |
| $G_{p}$, MMMscf | 0 | 5.5 |
| $B_{w}$, bbl/STB | 1.00 | 1.00 |
| $W_{e}$, MMbbl | 0 | 3 |
| $W_{p}$, MMbbl | 0 | 0.2 |
| $B_{g}$, bbl/scf | 0.0011 | 0.0015 |
| $c_{w}, c_{f}$ | 0 | 0 |

The following additional information is available:
Volume of bulk oil zone $=100,000 \mathrm{ac}-\mathrm{ft}$.
Volume of bulk gas zone $=20,000 \mathrm{ac}-\mathrm{ft}$.
Calculate the initial oil-in-place.

## Solution:

Step 1. Assuming the same porosity and connate-water for the oil and gas zones, calculate $m$ :

$$
m=\frac{\text { Initial volume of gas cap }}{\text { Volume of oil initially in place }}=\frac{20,000}{100,000}=0.2
$$

Step 2. Calculate the cumulative gas-oil ratio $R_{p}$ :

$$
R_{p}=\frac{G_{p}}{N_{p}}=\frac{5.5 \times 10^{9}}{5 \times 10^{6}}=1100 \mathrm{scf} / \mathrm{STB}
$$

Step 3. Solve for the initial oil-in-place by applying Equation (6.20):

$$
N=\frac{N_{p}\left[B_{o}+\left(R_{p}-R_{s}\right) B_{g}\right]-\left(W_{e}-W_{p} B_{w}\right)}{\left(B_{o i}-B_{o}\right)+\left(R_{s i}-R_{s}\right) B_{g}+m B_{o i}\left[\frac{B_{g}}{B_{g i}}-1\right]}
$$

$$
\begin{gathered}
N=\frac{5 \times 10^{6}[1.33+(1100-500) 0.0015]-\left(3 \times 10^{6}-0.2 \times 10^{6}\right)}{(1.35-1.33)+(600-500) 0.0015+(0.2)(1.35)\left[\frac{0.0015}{0.0011}-1\right]} \\
N=31.14 M M S T B
\end{gathered}
$$

## Example 6.4:

A combination-drive reservoir contains 10 MMSTB of oil initially in place. The ratio of the original gas-cap volume to the original oil volume, i.e., $m$, is estimated as 0.25 . The initial reservoir pressure is 3,000 psia at $150^{\circ} \mathrm{F}$. The reservoir produced 1 MMSTB of oil, $1,100 \mathrm{MMscf}$ of 0.8 specific gravity gas, and $50,000 \mathrm{STB}$ of water by the time the reservoir pressure dropped to 2,800 psi. The following PVT is available:

|  | $\mathbf{3 0 0 0} \mathbf{~ p s i}$ | $\mathbf{2 8 0 0} \mathbf{~ p s i}$ |
| :--- | :---: | :---: |
| $B_{o}, \mathrm{bbl} / \mathrm{STB}$ | 1.58 | 1.48 |
| $R_{s}, \mathrm{scf} / \mathrm{STB}$ | 1040 | 850 |
| $B_{g}, \mathrm{bbl} / \mathrm{scf}$ | 0.00080 | 0.00092 |
| $B_{t}, \mathrm{bbl} / \mathrm{STB}$ | 1.58 | 1.655 |
| $B_{w}, \mathrm{bbl} / \mathrm{STB}$ | 1.000 | 1.000 |

The following data are also available:
$S_{w i}=0.20$
$c_{w}=1.5 \times 10^{-6} p s i^{-1}$
$c_{f}=1 \times 10^{-6} p s i^{-1}$

Calculate:
a. Cumulative water influx.
b. Net water influx.

## Solution:

Because the reservoir contains a gas cap, the rock and fluid expansion can be neglected, i.e., set $c_{f}$ and $c_{w}=0$. For illustration purposes, however, the rock and fluid expansion term will be included in the calculations.

Part A. Cumulative water influx.
Step 1. Calculate cumulative gas-oil ratio $R_{p}$ :

$$
R_{p}=\frac{G_{p}}{N_{p}}=\frac{1100 \times 10^{6}}{1 \times 10^{6}}=1100 \mathrm{scf} / \mathrm{STB}
$$

Step 2. Arrange Equation (6.22) to solve for We:

$$
\begin{aligned}
& \begin{array}{l}
W_{e}=N_{p}\left[B_{t}+\left(R_{p}-R_{s i}\right) B_{g}\right] \\
\\
\quad-N\left[\left(B_{g}-B_{t i}\right)+m B_{t i}\left(\frac{B_{g}}{B_{g i}}-1\right)+B_{t i}(1+m)\left(\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right) \Delta p\right]+W_{p} B_{w p}
\end{array} \\
& \begin{aligned}
W_{e}= & 10^{6}[1.655+(1100-1040) 0.00092] \\
& \quad-10^{7}\left[(1.655-1.58)+0.25(1.58)\left(\frac{0.00092}{0.00080}-1\right)\right. \\
& \left.\quad+1.58(1+0.25)\left(\frac{0.2\left(1.5 \times 10^{-6}\right)}{1-0.2}\right)(3000-2800)\right]+50000
\end{aligned} \\
& \begin{array}{l}
W_{e}=411281 b b l
\end{array}
\end{aligned}
$$

Neglecting the rock and fluid expansion term, the cumulative water influx is 417,700 bbl.

Part B. Net water influx:

$$
\text { Net water influx }=W_{e}-W_{p} B_{w}=411281-50000=361281 \mathrm{bbl}
$$

### 6.6 Decline Curve analysis and History Matching:

Decline curve analysis is a means of predicting future oil well or gas well production behavior and life based on past production history with time. While history matching is the process of building one or more sets of numerical models (representing a reservoir which account for observed and measured data).

### 6.7 Computer Simulation:

Computer simulation may be considered a form of material balance analysis in which the spatial distribution of rock, fluid, and rock/fluid properties in the reservoir are represented in a computer model by a grid system, or a set of interconnected "tanks". The computer model may be used to calculate oil and/or gas initially in place, to match observed pressure performance history, and to forecast future production rates. The grid system may be configured to simulate the well or reservoir (or sections of the reservoir) under study. For some applications, the reservoir (or well) model may be coupled with models to simulate production facilities.

In high-permeability reservoirs, where reservoir pressure does not exhibit large areal variations, "zero dimensional" or "tank" model material balance calculations usually will be acceptable. In low-permeability reservoirs, where
reservoir pressure may exhibit large areal variations, or in geologically complex reservoirs, it may be necessary to use one or more multidimensional reservoir simulation models. As an example, computer grid system of porosity distribution of Gullfaks oil field is shown in Figure (6.3).


Fig. (6.3): Porosity distribution of Gullfaks oil field, (prepared by Al-Nasir, 2016).

## Problems

1. A petroleum reservoir has an areal extent of $\left(55,000 \mathrm{ft}^{2}\right)$ and an oil pay zone thickness of $(100 \mathrm{ft})$. The reservoir rock has a uniform porosity of $(25 \%)$ and the connate water saturation is $(30 \%)$. Calculate the initial oil in place.
2. Determine the OOIP (in STB and $\mathrm{m}^{3}$ ) for Oil Reservoirs 1 through 5 with the average parameters given in the table below:

| Reservoir | $\boldsymbol{h}$ | $\boldsymbol{\phi}$ | $\boldsymbol{S}_{\boldsymbol{w}}$ | $\boldsymbol{A}$ | $\boldsymbol{B}_{\boldsymbol{o}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 25 ft | 0.16 | 0.25 | 950 acres | $1.440 \mathrm{bbl} / \mathrm{STB}$ |
| 2 | 35 ft | 0.196 | 0.20 | $2 \times 10^{6} f t^{2}$ | $1.250 \mathrm{bbl} / \mathrm{STB}$ |
| 3 | 250 ft | 0.21 | 0.15 | 3750 acres | $1.530 \mathrm{bbl} / \mathrm{STB}$ |
| 4 | 28.5 ft | 0.177 | 0.275 | 15 sq miles | $1.045 \mathrm{bbl} / \mathrm{STB}$ |
| 5 | 25 m | 0.22 | 0.215 | 762 ha | $1.244 \mathrm{res} \mathrm{m}^{3} / \mathrm{std} \mathrm{m}^{3}$ |

3. A gas reservoir has the average parameters and gas properties listed in the table below. Use these to determine the OGIP for Oil Reservoirs 1 through 5 in the table.

| Reservoir | $\boldsymbol{h}$ | $\boldsymbol{\phi}$ | $\boldsymbol{S}_{\boldsymbol{w}}$ | $\boldsymbol{A}$ | $\boldsymbol{B}_{\boldsymbol{g}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 25 ft | 0.16 | 0.25 | 950 acres | $0.00300 \mathrm{ft}^{3} / \mathrm{scf}$ |
| 2 | 35 ft | 0.196 | 0.20 | $2 \times 10^{6} \mathrm{ft}^{2}$ | $0.00550 \mathrm{ft}^{3} / \mathrm{scf}$ |
| 3 | 250 ft | 0.21 | 0.15 | 3750 acres | $0.00463 \mathrm{ft}^{3} / \mathrm{scf}$ |
| 4 | 28.5 ft | 0.177 | 0.275 | 15 sq miles | $0.00235 \mathrm{ft}^{3} / \mathrm{scf}$ |
| 5 | 25 m | 0.22 | 0.215 | 762 ha | $0.00610 \mathrm{res} \mathrm{m}^{3} / \mathrm{std} \mathrm{m}^{3}$ |

4. A gas reservoir under partial water drive produced (12.0 MMM SCF) when the average reservoir pressure had dropped from (3000 psia to 2200 psia). During the same interval, an estimated ( 5.20 MM bbl ) of water entered the reservoir based on the volume of the invaded area. If the gas deviation factor at ( 3000 psia ) and bottom-hole temperature of $\left(170^{\circ} \mathrm{F}\right)$ is ( 0.88 ) and at ( 2200 psia ) is ( 0.78 ), what is the initial volume of gas in place measured at $\left(14.7\right.$ psia and $\left.60^{\circ} \mathrm{F}\right)$ ?
5. Using the letter symbols for reservoir engineering, write expressions for the following terms for a volumetric, undersaturated reservoir:
a. The initial reservoir oil in place in stock tank barrels.
b. The volume occupied by the remaining oil (liquid) after producing $\mathrm{N}_{\mathrm{p}}$ STB.
c. The SCF of gas produced.
d. The SCF of initial gas.
e. The SCF of gas in solution in the remaining oil.
f. The volume occupied by the escaped, or free, gas.
6. If (1 million STB) of oil have been produced from the XX reservoir at a cumulative produced GOR of ( $2700 \mathrm{SCF} / \mathrm{STB}$ ), causing the reservoir pressure to drop from the initial reservoir pressure of (4400 psia to 2800 psia), what is the initial stock tank oil in place?
7. Calculating Initial Oil in Place in a Volumetric, Undersaturated Reservoir given:

$$
\begin{aligned}
& \mathrm{B}_{\mathrm{ti}}=1.35469 \mathrm{bbl} / \mathrm{STB} \\
& \mathrm{~B}_{\mathrm{t}} \text { at } 3600 \mathrm{psig}=1.37500 \mathrm{bbl} / \mathrm{STB} \\
& \text { Connate water }=0.20 \\
& \mathrm{c}_{\mathrm{w}}=3.6(10)^{-6} \mathrm{psi}^{-1} \\
& \mathrm{~B}_{\mathrm{w}} \text { at } 3600 \mathrm{psig}^{-1} .04 \mathrm{bbl} / \mathrm{STB} \\
& \mathrm{c}_{\mathrm{f}}=5.0(10)^{-6} \mathrm{psi}^{-1} \\
& \mathrm{p}_{\mathrm{i}}=5000 \mathrm{psig} \\
& \mathrm{~N}_{\mathrm{p}}=1.25 \mathrm{MM} \mathrm{STB} \\
& \Delta \mathrm{p} \text { at } 3600 \mathrm{psig}=1400 \mathrm{psi} \\
& \mathrm{~W}_{\mathrm{p}}=32,000 \mathrm{STB} \\
& \mathrm{~W}_{\mathrm{e}}=0
\end{aligned}
$$

8. Calculating the Stock-Tank Barrels of Oil Initially-in-Place in a Combination Drive Reservoir, Given:

Volume of bulk oil zone $=112,000 \mathrm{ac}-\mathrm{ft}$
Volume of bulk gas zone $=19,600 \mathrm{ac}-\mathrm{ft}$
Initial reservoir pressure $=2710 \mathrm{psia}$
Initial formation volume factor $=1.340 \mathrm{bbl} / \mathrm{STB}$
Initial gas volume factor $=0.006266 \mathrm{ft}^{3} / \mathrm{SCF}$
Initial dissolved GOR $=562 \mathrm{SCF} / \mathrm{STB}$
Oil produced during the interval $=20 \mathrm{MM}$ STB
Reservoir pressure at the end of the interval $=2000 \mathrm{psia}$
Average produced GOR $=700 \mathrm{SCF} / \mathrm{STB}$
Two-phase formation volume factor at $2000 \mathrm{psia}=1.4954 \mathrm{bbl} / \mathrm{STB}$
Volume of water encroached $=11.58 \mathrm{MM} \mathrm{bbl}$
Volume of water produced $=1.05 \mathrm{MM}$ STB
Formation volume factor of the water $=1.028 \mathrm{bbl} / \mathrm{STB}$
Gas volume factor at $2000 \mathrm{psia}=0.008479 \mathrm{ft}^{3} / \mathrm{SCF}$

## REFERENCES

Ahmed, T., 2019, "Reservoir Engineering Handbook", Gulf Professional Publishing.
Clark, N., 1969, "Elements of Petroleum Reservoirs". Dallas, TX: SPE.
Cole, F., 1969, "Reservoir Engineering Manual". Houston, TX: Gulf Publishing Co.
Craft, B. C., and Hawkins, M. (Ronald E. Terry, J. Brandon Rogers.), 2015, "Applied Petroleum Reservoir Engineering, Third edition. Englewood Cliffs, NJ: Prentice Hall.
Dake, L. P., 1978, "Fundamentals of Reservoir Engineering". Amsterdam: Elsevier.
Dake, L., 1994, "The Practice of Reservoir Engineering". Amsterdam: Elsevier.
DeSorcy GJ., et al, 1994, "Determination of Oil and Gas Reserve", society of petroleum engineers.

Economides, M., and Hill, D., 1993, "Petroleum Production System". Prentice Hall.
George J. Stosur, et al, 2003, The Alphabet Soup of IOR, EOR, AOR: Effective Communication Requires a Definition of Terms, Kuala Lumpur, Malaysia, SPE-84908-MS.
Henry L. Doherty, 2001, "Estimation and Classification of Reserves of Crude Oil, Natural Gas, and Condensate", Society of petroleum engineers.
Schilthuis, R., 1936, "Active Oil and Reservoir Energy," Trans. AIME, Vol. 118, p. 33.

## A

absolute permeability $\cdot 147,148,149,150,152,153$,
$154,155,157,158,163,164,165,166,167,178,192$,
194, 204, 205, 215, 219, 220, 221
Abu-Kassem • 50, 85
Abu-Khamsin $\cdot 77$
acidizing $\cdot 160$
adhesion tension $\cdot 173,174,186,187$
Ahmed • 60, 73, 85, 129, 180, 200, 215, 217, 223, 250
Alani-Kennedy • 65
Analogical • 225
API • III, 62, 63, 66, 67, 68, 69, 73, 75, 76, 77, 83, 84, 86, $96,126,180,234$
apparent molecular weight $\cdot 19,21,61$
Apparent molecular weight $\cdot 18,79$
Argon 20
artificial fractures $\cdot 163$
atmospheric pressure $\cdot 60,63,70,76,109$
attractive forces $\cdot 12$
average permeability $\cdot 153,156,160,161,177,178,222$
Avogadro's law • 3, 7, 9

## B

Beal's • 76, 78
Beggs • 67, 68, 70, 73, 76, 77, 78, 86
Benner • 173, 180
Berthelot • 14
Biggs • 162, 163
binary system $\cdot 28,29$
binodal curve $\cdot 31$
black oil • 96, 115, 126, 127
boiling point curve $\cdot 25$
boiling point temperature $\cdot 25$
bottom-hole pressure • 99, 100
Boyle's law • 3, 4, 138
bubble point curve $\cdot 23,24,28,31,89$
bubble point pressure $\cdot 23,28,66,67,68,69,70,72,73$
$74,76,77,92,104,105,109,114,116,117,228$
bubble point pressure, $\cdot 28,67,72,73,74,92,105,114$
bulk density 140
bulk volume • 130, 135, 140
butanes • 18

## C

capillary forces $\cdot 142,182,200,207$
capillary hysteresis $\cdot 188$
capillary pressure • $169,172,176,182,183,184,185$,
186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197,
198, 199, 200, 201, 202, 203, 208, 212, 219, 220, 221
capillary tube $\cdot 169,170,183,186$
carbon $\cdot 1,18,45,82,133$
carbonate porosity $\cdot 131,134$
Carr-Kobayashi-Burrows • 46, 48, 57, 83
CCE • 68, 106, 127
Celsius • 2
Centrifuge • 189
channel permeability $\cdot 163$
Charles's law • 3
Chew-Connally • 77
chromatographic test • 102
chromatography • 102
Clausius • 15
Combination drive $\cdot 227,228,233$
compositional analysis $\cdot 47,65$
Compositional analysis • 102, 127
compressibility factor $\cdot 11,12,36,37,42,44,45,47,49$, 50, 51, 55, 110, 137
computerized tomography • 135
concave upward. $\cdot 208$
connate water • 142, 143, 144, 163, 187, 220, 248
Constant composition expansion $\cdot 102,104$
contact angle $\cdot 170,173,174,176,185,186,187$
core analysis • 130, 135, 136, 151, 156, 177, 199
core flooding • 139, 165
core permeability $\cdot 166,194$
core plug $\cdot 135,136,145,148,151,152,164,165,166$, 178, 189, 204
core sample • 130, 134, 135, 136, 137, 140, 151, 190, 194
correction factor $\cdot 11,12,46$
corresponding state $\cdot 51$
corresponding states $\cdot 32,33,36$
cricondenbar • 88
cricondentherm $\cdot 88,89,92,94,126$
critical point $\cdot 24,25,29,31,88,89,93$
critical pressure $\cdot 24,29,32,36,46,47,89$
critical saturation $\cdot 143,208,213$
critical state $\cdot 25,32$
critical temperature $\cdot 24,32,36,37,46,47,89,90,92$, 93
critical volume $\cdot 24,32$
crude oil system $\cdot 63,67,68,72,83,84,143$

## D

Dalton's Law • II, 10
Dandekar • 165, 172, 176, 180, 189, 223
Decline curve • 246
density logging • 140
depletion • 92, 102, 109, 110, 209
Depletion drive • 227, 228
deposits • 1
desaturating $\cdot 188$
desaturation $\cdot 208$
deterministic $\cdot 224$
Developed • 225
development project $\cdot 1$
deviation factor • 12, 84, 113, 127, 248
dew point curve $\cdot 23,24,29,31,89$
dew point pressure $\cdot 23,24,28,92$
differential liberation • 109, 117, 120, 122
Differential liberation • 102
displacement pressure • 187, 198, 200, 202
drainage process $\cdot 179,187,210$
Dranchuk 50, 85
drawdown 99
drilling fluids • 99
drive mechanisms • 228
Dry Gas • 95

## E

effective permeability • 152, 164, 204, 205, 215
Effective permeability $\cdot 152$
effective porosity $\cdot 131,134,137$
enhanced oil recovery $\cdot 226$
EOR • 226, 227, 250
EoS • 3, 13, 14
Equation of State • II, 3, 85, 129
equilibrium test $\cdot 102,103,105,106,110$
Ertekin • 189, 223
Ethane • 1, 11, 26, 33, 79, 80, 81, 82
evolved gas $\cdot 105$

## F

Fahrenheit $\cdot 2$
Farshad • 67, 68, 70, 73, 77, 85, 86
flash expansion $\cdot 103$
flash liberation • 103, 121
flash vaporization $\cdot 103$
fluid contacts • 176, 182, 195, 196, 197, 232
fluid injection • 228
Formation Testing 100
formation volume factor $\cdot 54,55,56,62,66,70,82,84$, $96,97,120,121,127,149,150,234,235,237,240$,
241, 242, 243, 249
fractional porosity • 130, 191
free water level • 182, 221
fusion curve - 25
FVF • 66, 69, 70, 71, 74, 103, 110, 113, 114, 118, 236
FWL • V, 182, 197, 198, 199, 202, 203, 220

## G

Gardner • V, 212, 213, 222, 223
gas composition $\cdot 22,42,44,79,83$
gas compressibility $\cdot 12,18,42,44,49,50,52,53,55,83$
gas density $\cdot 21,22,48,49,60,61,222$
Gas density $\cdot 18,79$
Gas expansion factor • 18

Gas formation volume factor • 18, 79, 237
gas injection • 102, 143, 243
gas phase $\cdot 28,57,58,87,111,121,143,172,179$
gas reservoir • $18,42,44,56,58,82,84,90,92,95,126$,
163, 200, 235, 237, 238, 248
Gas reservoirs 90
gas saturation • 142, 143, 205, 208, 212, 216
Gas solubility • 62, 237
Gas-cap drive - 227, 228
gas-cap reservoir $\cdot 92$
gas-oil contact • 195
gasoline plant • 124, 125
Gay-Lussac's law • 3
geophysical logs • 195
Glaso $\cdot 67,68,70,75,76,77,85$
Glaso's • 67, 70, 75, 76, 77
GOR • 64, 65, 96, 97, 99, 101, 110, 113, 115, 116, 118,
126, 234, 237, 249
grain volume • 130, 135, 140
gravitational forces • 231
Gravity drainage • 227, 228
gravity force • 170

## H

Hall - 50, 85, 250
heavy components $\cdot 87,92,111,122$
heavy molecules • 93, 95
helium $\cdot 62,137,138,166$
Henry Darcy 147
heptane $\cdot 18,28,30$
heptanes-plus - 49, 97
heterogeneous • 161, 174
hexanes $\cdot 18,97$
history matching $\cdot 246$
Hjelmeland • 172, 180
homogeneous • 18, 87, 151, 174
Honarpour • 211, 217, 218, 223
horizontal permeability 151
Hustad • 217
hydrocarbon entrapment • 182
hydrogen $\cdot 1,18,45,82$
hysteresis • 208, 209

## I

ideal gas • $3,6,7,8,10,11,13,16,20,21,22,23,32,36$, $50,53,79,80,81,137$
Ideal gas law • 3
imbibition process $\cdot 179,187,189,210$
immiscible • 168, 169, 172, 174, 182, 186, 187, 207, 227
intensive properties $\cdot 19,25,57,88$
interconnected pores $\cdot 131$
interfacial tension $\cdot 130,168,169,171,172,173,184$,
$185,190,191,194,219,220,221,222$
intermediate gases • 18

IOR • 226, 227, 250
isolated pores • 131
isoperms • 216
isothermal compressibility $\cdot 50,51,52,71,72,106$
isotherms • 24,28

## J

Jerauld • 176, 180
J-function • 190, 191, 192, 194, 221

## $K$

Katz $\cdot 33,34,35,37,38,39,40,41,43,46,49,65,85$, 171, 180
Kelvin • 2
kerosene • 139
Khan's • 78
Klinkenberg • IV, 166, 167, 180, 181
known composition $\cdot 32,36$

## L

Larrondo • 172, 180
Leverett • 182, 189, 190, 191, 192, 194, 221, 223
liberated gas $\cdot 74,109,110,121$
light gases • 18
linear flow • 147, 148, 149, 158
liquid phase $\cdot 24,25,28,87,89,93,95,105,172,173$
lithology • 143, 176
Low-shrinkage oil • 96

## M

Marhoun • 67, 68, 70, 75, 77
Marhoun's • 67, 70, 75
material balance equation • 70, 236, 243
Matrix permeability 163
MBE • 236, 239, 240, 243
mercury • VII, 81, 104, 137, 189, 193, 194
Mercury injection - 189
methane • $9,11,16,18,33,36,37,80,81,95,97,102$
micropoises 57
millidarcy $\cdot 147,149,150$
Molar Mass • II, 8, 26
mole fraction $\cdot 10,11,18,19,37,46,48,79,81$
Morris • 163
multicomponent $\cdot 25,28,31,87,88$

## $N$

natural energy • 226, 228, 230
natural gas $\cdot 18,19,22,26,43,45,47,48,49,57,58,59$, $60,66,82,93,127,224$
Near-critical -90, 97
nitrogen • 18, 45, 48, 62, 166
Nitrogen • 16, 20, 27, 81, 82, 227
nonhydrocarbon gases $\cdot 18,57$
non-hydrocarbon gases • 45
non-producing • 225
nonwetting fluid $\cdot 182,208$
nonwetting phase $\cdot 143,179,182,183,187,188,189$, 205, 206, 207, 208, 209, 210
numerical simulator • 101

## 0

octane • 18
oil density • 62, 65, 66, 73, 77, 222
oil gravity • 62, 65, 68, 71, 75
oil phase • $72,74,92,143,173,175,215$
oil production $\cdot 120,121,189,232,240$
oil properties $\cdot 65,74,84,226$
oil recovery • 122, 123, 179, 226, 227, 228
Oil reservoirs $\cdot 89$
oil saturation • $142,143,145,189,205,207,209,216$, 220
oil viscosity $\cdot 62,75,76,77,178$
oil zone • 163, 234, 240, 242, 244, 249
oil-water contact - 195
optimum separation $\cdot 124,125$
overburden stress $\cdot 141$
Oxygen • 20, 27

## $P$

partial pressure $\cdot 10,11,80,81$
partial volumes $\cdot 11$
pentanes 18
Performance • 225
petrophysical $\cdot 233$
Petrosky • 67, 68, 70, 73, 85
Petrosky-Farshad's • 67, 73
phase behavior • 28, 30, 31, 87, 88, 101
phase diagram $\cdot 23,31,89,92,93,94,95,96,97$
Phase diagram $\cdot 87,95,96$
phase envelope $\cdot 24,29,31,89,90,91,126$
pore volume $\cdot 130,135,137,139,141,142,143,228$, 237, 238, 241, 242
porosimeter 137
porous media $\cdot 75,143,152,153,157,165,174,180$, 182, 207
primary porosity $\cdot 131$
Primary recovery $\cdot 226$
probabilistic $\cdot 224$
producing $\cdot 1,21,22,56,58,68,96,100,101,125,126$, 224, 225, 230, 232, 248
producing zone $\cdot 100$
production rate $\cdot 99$
Propane • 1, 11, 26, 33, 79, 80, 81, 82
Pseudocritical • 36
pseudocritical pressure $\cdot 36,37,43,48,57$
pseudocritical temperature $\cdot 36,43,46,48,57$
pseudoreduced compressibility • 52, 53
pseudoreduced conditions $\cdot 37$
$p-T$ diagram $\cdot 25,26,87,88$
pure component $\cdot 20,23,24,25,26,28,42$
Purvis • 50, 85
PVT • III, 18, 23, 65, 75, 84, 85, 86, 96, 99, 100, 101, 102, 103, 109, 120, 127, 129, 180, 240, 243, 244, 245

## R

radioactive 195
Radke • 174, 175, 180
Rankine • VIII, 2
Rathmell - 176, 180
Real Gas • II, 11
Redlich-Kwong • 14, 17
reduced pressure • 32, 42
reduced temperature $\cdot 32,38$
reduced volume 32
reference point $\cdot 32$
relative permeability $\cdot 172,176,180,204,205,206,207$,
208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 222
Relative permeability $\cdot 130,152$
repulsive forces $\cdot 3,11,12$
resaturation $\cdot 208$
reserve estimation 1
reserves • 124, 125, 224, 225
reservoir conditions • VIII, 22, 42, 44, 54, 80, 82, 92, 121, 141, 168, 196, 234, 235
reservoir engineer $\cdot 1$
reservoir engineering $\cdot I, 1,18,62,72,130,131,140$, 141, 172, 191, 233, 248
reservoir pressure $\cdot 21,42,44,61,65,67,69,70,72,82$, $83,84,89,92,99,104,108,126,127,143,176,228$, 229, 237, 240, 241, 242, 244, 245, 246, 248, 249
reservoir temperature $\cdot 58,60,61,69,71,75,76,77,89$, 90, 92, 93, 94, 103, 109, 120, 126
residual saturation $\cdot 143,207,208,209$
retort distillation $\cdot 145,146$
Retrograde 92
Robinson • 17, 50, 76, 77, 85

## $S$

Salathiel - 175, 181
sampling chamber $\cdot 100$
Sandstone porosity • 131, 134
saturated oil $\cdot 65,77,92,126$
saturation pressure $\cdot 66,77,99,100,104,105,106,107$, 108, 109, 120

Schilthuis • 236, 250
secondary porosity $\cdot 131,133$
Secondary recovery $\cdot 226$
segregation - 200, 232
separator conditions $\cdot 97$
separator system • 120, 124
separator tests $\cdot 120$
Separator tests • 64, 102, 119
series flow • 157
shrinkage factor • 121
shrinkage oil • 102, 111, 112, 124
Simulation • VI, 223, 225, 246
single-component $\cdot 24,25,26,28,32$
Slim-tube test $\cdot 102$
solid phase $\cdot 25,87$
sonic • 140, 141
sorted grains • 194, 200
sour gases • 45, 61, 86
SPE • 7, 17, 85, 86, 129, 180, 181, 223, 250
specific gravity • VIII, IX, 21, 22, 43, 45, 47, 48, 53, 56, $57,58,62,63,64,66,67,68,71,80,81,82,83,84$, 111, 120, 234, 245
Specific gravity $\cdot 18,62,79,82$
specific volume $\cdot 22,23,30$
Specific volume • II, 18
standard conditions $\cdot$ VIII, IX, 7, 12, 54, 55, 63, 120
standard pressure $\cdot 21,55$
standard temperature $\cdot 7,21$
Standing • 33, 37, 38, 43, 45, 46, 49, 60, 65, 66, 67, 68, $69,70,71,73,75,83,84,85,86,235$
steady state $\cdot 149,177,210$
stimulation • 99
stock tank • IX, 111, 119, 121, 122, 123, 124, 125, 126, 248, 249
stock-tank • VII, VIII, IX, 64, 66, 71, 74, 96, 113, 118, 120, 126, 234
Stone • 217
subsurface sampling • 99
sulfide $\cdot 18,27,45,81,82$
Surface sampling 101
surface tension $\cdot 168,169,170,171,172,180,185,193$, 194
Swelling test $\cdot 102$
synthetic oil • 137, 139, 166

## $T$

ternary diagram - 30, 97
ternary system 30
Tertiary recovery $\cdot 226$
tight formation 99
Timur • 162, 163
Torcaso • V, 212, 215, 223
total volume • 3, 11, 19, 28, 74, 104, 108, 109, 119, 121, 130, 237
transient testing - 141
transition zone • 182, 200, 222

Trube • 52, 53, 73, 86
two-phase region • 24, 31, 89, 95

## $u$

uncertainty • 224
unconsolidated • 191, 192, 213, 222, 223
uniformity • 125, 153, 224
universal constant $\cdot 8$
unsteady state $\cdot 210$

## v

vacuum saturation 139
Van der Waals • II, 13, 14, 17
vapor pressure $\cdot$ VIII, 25,28
vapor pressure curve • 25
vaporization process - 93
Vasquez • 67, 68, 70, 73, 78, 86
Vasquez-Beggs's • 67, 73
vertical permeability • 151
Viscosity • III, 18, 56, 60, 75, 76, 77, 85
viscous oil • 99
Volatile crude oil • 96
volatile oil • 65
volume fraction $\cdot 19$
Volumetric • VI, 85, 86, 225, 233, 237, 249
volumetric flowrate • 147, 149, 150

## $w$

water drive • 210, 223, 227, 229, 230, 248
water phase • 144, 173, 174, 183
water saturation $\cdot 142,143,144,145,162,163,177,187$, 189, 190, 191, 196, 197, 198, 199, 200, 202, 203, 205, 206, 207, 209, 210, 212, 215, 220, 221, 234, 235, 238, 248
water-oil system $\cdot 193,206,208,219$
weight fraction - 19, 20, 79
well conditioning • 99
well logging • 135, 140
wellhead pressure $\cdot 99$
Wet Gas • 94
wettability • 169, 172, 173, 174, 175, 176, 180, 181, 186
wetting fluid $\cdot 182$
wetting phase $\cdot 143,182,187,196,205,206,207,208$,

$$
209,210
$$

While Morris • 162
Wichert-Aziz • 46
Wyllie • V, 212, 213, 215, 217, 222, 223

## $X$

x-ray 135

## $\gamma$

Yarborough • 50, 85
Y-function • 103, 108

## Z

z-factor $\cdot 12,43,45,47,50,81,82,84$
Z-factor $\cdot \mathrm{II}, 12,13,32,46,48,49,55,56,85,116,117$, 118, 119, 128


[^0]:    $\left[N B_{o i}\right]+\left[m N B_{o i}\right]=\left[\left(N-N_{p}\right) B_{o}\right]+\left[\left(\frac{m N B_{o i}}{B_{g i}}\right) B_{g}\right]+\left[N R_{s i} B_{g}-N_{p} R_{p} B_{g}-\left(N R_{s} B_{g}-N_{p} R_{s} B_{g}\right)\right]+\left[W_{e}-W_{p} B_{w}\right]+\left[N B_{o i}(1+m)\left(\frac{S_{w i} c_{w}+c_{f}}{1-S_{w i}}\right) \Delta p\right]+\left[G_{i n j} B_{g i n j}\right]+\left[W_{i n j} B_{w}\right]$

