# **FLUID MECHANICS** COLLEGE OF PETROLEUM AND MINING ENGINEERING

### Dr. Ibrahim Adil Ibrahim Al-Hafidh

Mining Engineering Department College of Petroleum and Mining Engineering University of Mosul



Email: iibrahim@uomosul.edu.iq

# LECTURE 1

Fluid Characteristics of Fluids Dimensions, Dimensional Homogeneity, and Units Systems of Units Analysis of Fluid Behavior



Email: iibrahim@uomosul.edu.iq



# Fluid

One of the first questions we need to explore is—what is a fluid? Or we might ask-what is the difference between a solid and a fluid? General we have a vague idea of the difference. A solid is "hard" and not easily deformed, whereas a fluid is "soft" and is easily deformed (we can readily move through air).

Although quite descriptive, these casual observations of the differences between solids and fluids are not very satisfactory from a scientific or engineering point of view.



# Fluid

A fluid is defined as a substance that deforms continuously under an external force, or when acted on by a shearing stress of any magnitude.

Fluids are a phase of matter and include liquids, gases and plasmas. They are substances with zero shear modulus, or, in simpler terms, substances which cannot resist any shear force applied to them.

Mechanics: the branch of applied mathematics that deals with the motion and equilibrium of bodies and the action of forces, and includes kinematics, dynamics, and statics.



- Fluid mechanics: Is the discipline within the broad field of applied mechanics that is concerned with the behavior of liquids and gases at rest or in motion.
- It covers a vast array of phenomena that occur in nature (with or without human intervention), in biology, and in numerous engineered, invented, or manufactured situations. There are few aspects of our lives that do not involve fluids, either directly or indirectly.
- In short description, it is a branch of science that studies the mechanics of those free moving particles.



# Fluid Mechanics

# Fluid Statics Study of fluid under rest

Fluid Kinematics Study of fluid under motion (velocities, acceleration) Fluid Dynamics Study of forces and their analysis causing the fluid motion/ deformation

# Some Characteristics of Tuids



# A closer look molecular structure of materials

1- Solid like (steel, concrete, etc.), has densely spaced molecular, with large intermolecular cohesive force. This allow the solid maintain its shape, and not be easily deformed.

2- Liquid like (water, oil, etc.), has farther apart spacing molecular, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement. Therefore, liquids can be easily deformed (but not easily compressed ) and can be poured into container or forced through a tube.

3- Gases like (air, oxygen, etc.) have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces, and as a consequence are easily deformed (and compressed ) and will completely fill the volume of any container in which they are placed. Both liquids and gases are fluids.

# The effect of a shear stress

A shearing stress (force per unit area) is created whenever a tangential force acts on a surface as shown by the figure.

A solids such as steel or other metals are acted on by a shearing stress, they will initially deform (usually a very small deformation), but they will not continuously deform.

A common fluids such as water, oil, and air satisfy the definition of a fluid that is, they will flow when acted on by a shearing stress.

Some materials, such as tar, putty, toothpaste, and so on, are not easily classified since they will behave as a solid if the applied shearing stress is small, but if the stress exceeds some critical value, the substance will flow.

Dr. Ibrahim Al-Hafidh / College of Petroleum and Mining Engineering







10/11/2021



# The most important parameters that describe fluid flow.

(1) The physical size of the flow, I.(2) The speed of the flow, V.(3) The pressure, p.

### Size (I),

Every flow has a characteristic length associated with it. For example, for flow of fluid within pipes, the pipe diameter is a characteristic length, the pipe length is also a characteristic length. Such examples include the flow of oil across Alaska through a 4-foot-diameter, 799-mile-long pipe.

Characteristic lengths of some other flows are shown in Fig. 1a.



(Photograph courtesy of CIRRUS Design Corporation.)



### Speed (V)

As we note from THE WEATHER CHANNEL, on a given day the wind speed may cover what we think of as a wide range, from a gentle 5-mph breeze to a 100-mph hurricane or a 250-mph tornado.

However, this speed range is large compared to that of the almost imperceptible flow of the fluid-like magma below the Earth's surface that drives the continental drift motion of the tectonic plates at a speed of about  $2 \times 10^{-8}$  m/s and small compered to the hypersonic airflow past a meteor as it streaks through the atmosphere at  $3 \times 10^4$  m/s.

Characteristic speeds of some other flows are shown in Fig. 1b.



### Pressure (p)

The pressure within fluids covers an extremely wide range of values. We are accustomed to the 35 psi (lb/in.<sup>2</sup>) pressure within our car's tires, the 120/70 mmHg typical blood pressure reading, or the standard 14.7 psi atmospheric pressure.

However, the large 10,000 psi pressure in the hydraulic ram of an earth mover or the tiny  $2 \times 10^{-6}$  psi pressure of a sound wave generated at ordinary talking levels are not easy to comprehend.

Characteristic pressures of some other flows are shown in Fig. 1c.

Fig 1, Characteristic values of some fluid flow parameters for a variety of flows: (a) object size.

(b) fluid speed.(c) fluid pressure.





# Dimensions, Dimensional Homogeneity, and Units

Since in our study of fluid mechanics we will be dealing with a variety of fluid characteristics, it is necessary to develop a system for describing these characteristics both qualitatively(نوعي) and quantitatively (كمي).

The qualitative aspect serves to identify the nature, or type, of the characteristics (such as length, time, stress, and velocity).

The quantitative aspect provides a numerical measure of the characteristics. The quantitative description requires both a number and a standard by which various quantities can be compared.

A standard for length might be a meter or foot, for time an hour or second, and for mass a slug or kilogram. Such standards are called **Units**.



The goaliantive description is conveniently given in terms of certain primary quantities,

length, L, time, T, mass, M, and temperature, O

These primary quantities can then be used to provide a qualitative description of any other secondary quantity: for example, Area  $\doteq L^2$ , velocity  $\doteq LT^{-1}$ , density  $\doteq ML^{-3}$ , where the symbol ( $\doteq$ ) is used to indicate the dimensions of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, V, we would write, Velocity = m/s

$$\mathbf{V} \doteq \mathbf{L}\mathbf{T}^{-1}$$

and say that "the dimensions of a velocity equal length divided by time." The primary quantities are also referred to as basic dimensions.



For a wide variety of problems involving fluid mechanics, only the three basic dimensions, [L, T, and M] are required. Alternatively, [L, T, and F] could be used, where F is the basic dimensions of force.

Since Newton's law states that force is equal to mass (Kg) (M) times acceleration  $(m/s^2)$  (LT<sup>-2</sup>), it follows that

$$\mathbf{F} \doteq \mathbf{M}\mathbf{L}\mathbf{T}^{-2}$$
 or  $\mathbf{M} \doteq \mathbf{F}\mathbf{L}^{-1}\mathbf{T}^2$ 



Thus, secondary quantities expressed in terms of M can be expressed in terms of F through the relationship above.

For example, stress,  $\sigma$  is a force per unit area, so that  $\sigma \doteq FL^{-2}$  but an equivalent dimensional equation is  $\sigma \doteq ML^{-1} T^{-2}$ .

The Table (1) in the next slide provides a list of dimensions for a number of common physical quantities. Tables (2 & 3) provide a list primary & secondary dimensions in fluid mechanics.

 $\sigma \doteq FL^{-2}$   $F \doteq MLT^{-2}$   $\sigma \doteq MLT^{-2} * L^{-2}$  $\sigma \doteq ML^{-1} T^{-2}$ 

## Table (1)



#### Dimensions Associated with Common Physical Quantities

	<i>FLT</i> System	MLT System
Acceleration	$LT^{-2}$	$LT^{-2}$
Angle	$F {}^{0}L^{0}T {}^{0}$	$M^{0}L^{0}T^{0}$
Angular acceleration	$T^{-2}$	$T^{-2}$
Angular velocity	$T^{-1}$	$T^{-1}$
Area	$L^2$	$L^2$
Density	$FL^{-4}T^2$	$ML^{-3}$
Energy	FL	$ML^{2}T^{-2}$
Force	F	$MLT^{-2}$
Frequency	$T^{-1}$	$T^{-1}$
Heat	FL	$ML^2T^{-2}$
Length	L	L
Mass	$FL^{-1}T^{2}$	M
Modulus of elasticity	$FL^{-2}$	$ML^{-1}T^{-2}$
Moment of a force	FL	$ML^{2}T^{-2}$
Moment of inertia (area)	$L^4$	$L^4$
Moment of inertia (mass)	FLT <sup>2</sup>	$ML^2$
Momentum	FT	$MLT^{-1}$

	FLT System	<i>MLT</i> System
Power	FLT <sup>-1</sup>	$ML^2T^{-3}$
Pressure	$FL^{-2}$	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	$FL^{-3}$	$ML^{-2}T^{-2}$
Strain	$F {}^{0}L^{0}T {}^{0}$	$M {}^{0}L^{0}T {}^{0}$
Stress	$FL^{-2}$	$ML^{-1}T^{-2}$
Surface tension	$FL^{-1}$	$MT^{-2}$
Temperature	θ	θ
Time	Т	Т
Torque	FL	$ML^{2}T^{-2}$
Velocity	$LT^{-1}$	$LT^{-1}$
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	$L^2 T^{-1}$	$L^2 T^{-1}$
Volume	$L^3$	$L^3$
Work	FL	$ML^2T^{-2}$



### Table (2) Primary Dimensions in SI and BG Systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass [M]	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length [L]	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time [T]	Second (s)	Second (s)	1 s = 1 s
Temperature [0]	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

### SI unite, International System BG unite, British Gravitational System





### Jable (3) Secondary Dimensions in Fluid Mechanics

Secondary dimension	SI unit	BG unit	<b>Conversion factor</b>
Area [ L <sup>2</sup> ]	m²	ft²	1 m <sup>2</sup> = 10.764 ft <sup>2</sup>
Volume [L <sup>3</sup> ]	m <sup>3</sup>	ft <sup>3</sup>	1 m <sup>3</sup> = 35.315 ft <sup>3</sup>
Velocity [LT <sup>-1</sup> ]	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration [LT <sup>-2</sup> ]	m/s²	ft/s²	$1 \text{ ft/s}^2 = 0.3048 \text{ m/s}^2$
Pressure or stress[ML <sup>-1</sup> T <sup>-2</sup> ]	$Pa = N/m^2$	lbf/ft <sup>2</sup>	1 lbf/ft² = 47.88 Pa
Energy, heat, work [ML <sup>2</sup> T <sup>-2</sup> ]	J = N . m	Ft.ibf	1 ft . lbf = 1.3558 J
Density [ML <sup>-3</sup> ]	Kg/m <sup>3</sup>	slugs/ft <sup>3</sup>	1 slug/ft <sup>3</sup> = 515.4 kg/m <sup>3</sup>
Viscosity (ML <sup>-1</sup> T <sup>-1</sup> ]	Kg/(m . s)	Slugs/(ft . s)	1 slug/(ft . s) =47.88 kg/(m . s)

Dr. Ibrahim Al-Hafidh / College of Petroleum and Mining Engineering



### **Dimensionally Homogeneous**

All theoretically derived equations are dimensionally homogeneous that is, the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions.

Example, the equation for the velocity, V, of a uniformly accelerated body is, V = V<sub>0</sub> + at Where, V<sub>0</sub> : is the initial velocity. a : the acceleration. t: the time interval. In terms of dimensions the equation is  $LT^{-1} \doteq LT^{-1} + LT^{-2}T$ 

#### **EXAMPLE 1.1** Restricted and General Homogeneous Equations

(1)

**GIVEN** A liquid flows through an orifice located in the side of a tank as shown in Fig. E1.1. A commonly used equation for determining the volume rate of flow, *Q*, through the orifice is

 $Q = 0.61 A \sqrt{2gh}$ 

where A is the area of the orifice, g is the acceleration of gravity, and h is the height of the liquid above the orifice.

FIND Investigate the dimensional homogeneity of this formula.

#### SOLUTION .

The dimensions of the various terms in the equation are  $Q = 1^{-1}$  volume/time  $\doteq L^3 T^{-1}$ ,  $A = \text{area} \doteq L^2$ ,  $g = \text{acceleration of gravity} \doteq LT^{-2}$ , and  $h = \text{height} \doteq L$ .

These terms, when substituted into the equation, yield the dimensional form:

$$L^{3}T^{-1}$$
 = (0.61) $(L^{2})(\sqrt{2})(LT^{-2})^{1/2}(L)^{1/2}$ 

or

 $(L^{3}T^{-1}) \doteq [0.61\sqrt{2}](L^{3}T^{-1})$ 

It is clear from this result that the equation is dimensionally homogeneous (both sides of the formula have the same dimensions of  $L^3T^{-1}$ ), and the number  $0.61\sqrt{2}$  is dimensionless.

If we were going to use this relationship repeatedly, we might be tempted to simplify it by replacing g with its standard value of 32.2 ft /s<sup>2</sup> and rewriting the formula as

 $Q = 4.90 A \sqrt{h}$ 

A quick check of the dimensions reveals that

 $L^{3}T^{-1} \doteq (4.90)(L^{5/2})$ 



(a) ■ Figure E1.1

and, therefore, the equation expressed as Eq. 1 can only be dimensionally correct if the number 4.90 has the dimensions of  $L^{1/2}T^{-1}$ . Whenever a number appearing in an equation or formula has dimensions, it means that the specific value of the number will depend on the system of units used. Thus, for the case being considered with feet and seconds used as units, the number 4.90 has units of  $ft^{1/2}/s$ . Equation 1 will only give the correct value for Q (in  $ft^3/s$ ) when A is expressed in square feet and h in feet. Thus, Eq. 1 is a *restricted* homogeneous equation, whereas the original equation is a *general* homogeneous equation that would be valid for any consistent system of units.

**COMMENT** A quick check of the dimensions of the various terms in an equation is a useful practice and will often be helpful in eliminating errors—that is, as noted previously, all physically meaningful equations must be dimensionally homogeneous. We have briefly alluded to units in this example, and this important topic will be considered in more detail in the next section.



1.1 The force, F, of the wind blowing against a building is given by  $F = C_D \rho V^2 A/2$ , where V is the wind speed,  $\rho$  the density of the air, A the cross-sectional area of the building, and  $C_D$  is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

 $F = C_0 \rho V^2 A/2$ or  $C_D = 2F/\rho V^2 A$ , where  $F = MLT^{-2}$  $\begin{array}{l} Q \doteq ML^{-3} \\ V \doteq LT^{-1} \end{array}$  $A \doteq L^2$ Thus,  $C_{\rm D} \doteq (M L T^{-2}) / \left[ (M L^{-3}) (L T^{-1})^2 (L^2) \right] = M^{\circ} L^{\circ} T^{\circ}$ Hence, Co is dimensionless.

1.2 Determine the dimensions, in both the *FLT* system and the *MLT* system, for (a) the product of mass times velocity, (b) the product of force times volume, and (c) kinetic energy divided by area.

- (a) mass x velocity =  $(M)(LT^{-1}) \doteq MLT^{-1}$ Since  $F \doteq MLT^{-2}$ mass x velocity =  $(FL^{-1}T^{-2})(LT^{-1}) \doteq FT$
- (b) force x volume  $\doteq \underline{FL}^{3}$  $\Rightarrow (MLT^{-2})(L^{3}) \doteq \underline{ML}^{4}T^{-2}$

(c) 
$$\frac{\text{Kinetic energy}}{\text{area}} \doteq \frac{FL}{L^2} \doteq \frac{FL^{-1}}{L^2}$$
  
 $\doteq \frac{(MLT^{-2})L}{L^2} \doteq \frac{MT^{-2}}{L^2}$ 

10/11/2021



### System of Units

In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a quantitative measure of any given quantity.

For example, if we measure the width of this page in the book and say that it is 10 units wide, the statement has no meaning until the unit of length is defined. If we indicate that the unit of length is a meter, and define the meter as some standard length, a unit system for length has been established and a numerical value can be given to the page width.

#### 24 Dr.Ibrahim Al Hafidh Fluid Mechanics

# The system of Units

# 1- International System (SI)

In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K).

Note: The kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale through the relationship:

$$K = {}^{\circ}C + 273.15$$

Although the Celsius scale is not in itself part of SI, it is common practice to specify temperatures in degrees Celsius when using SI units.

## International System (SI)



The unit of force, called the newton (N), is defined from Newton's second law as:

1 N = (1Kg) (1 m/s<sup>2</sup>) Force = Mass x Acceleration

Thus, a 1-N force acting on a 1-kg mass will give the mass an acceleration of  $1m/s^2$ .



If you apply more force to an object, it accelerates at a higher rate.



Standard gravity (g) in SI is 9.807 m/s<sup>2</sup> (commonly approximated as 9.81 m/s<sup>2</sup>).

That mean a 1-kg mass, weighs 9.81 N under standard gravity. Note that weight and mass are different (The weight and mass are not the same thing but related), both qualitatively and quantitatively.

#### 26 Dr.Ibrahim Al Hafidh Fluid Mechanics

## International System (SI)

The unit of work in SI is the joule (J), which is the work done when the point of application of a 1-N force is displaced through a 1-m distance in the direction of a force. Thus,

1 J = N . m

The unit of power is the watt (W) defined as a joule per second. Thus, 1 W = 1 J / s = N . M / s

Prefixes for forming multiples and fractions of SI units are given in Table 4.

The point of application is the exact location at which a force is applied to a body. This point is usually described by a set of coordinates and is represented graphically by the tip of the arrowhead. The point of application is unique to each force.



### Table (4)

#### **Prefixes for SI Units**

Factor by Which Unit Is Multiplied	Prefix	Symbol	Factor by Which Unit Is Multiplied	Prefix	Symbol
10 <sup>15</sup>	peta	Р	10 <sup>-2</sup>	centi	с
$10^{12}$	tera	Т	10 <sup>-3</sup>	milli	m
10 <sup>9</sup>	giga	G	10 <sup>-6</sup>	micro	$\mu$
$10^{6}$	mega	Μ	10 <sup>-9</sup>	nano	n
$10^{3}$	kilo	k	10 <sup>-12</sup>	pico	р
$10^{2}$	hecto	h	10 <sup>-15</sup>	femto	f
10	deka	da	10 <sup>-18</sup>	atto	а
$10^{-1}$	deci	d			



# The system of Units

# 2- British Gravitational (BG) System

In the BG system the unit of length is the foot (ft), the time unit is the second (s), the force unit is the pound (lb), and the temperature unit is the degree Fahrenheit (°F) or the absolute temperature unit is the degree Rankine (°R), where:

°R = °F + 459.67

The mass unit, called the (slug), is defined from Newton's second law (force = mass x acceleration)as

 $1 \text{ lb} = (1 \text{ slug}) (1 \text{ ft/s}^2)$ 

This relationship indicates that a 1-lb force acting on a mass of 1 slug will give the mass an acceleration of 1 ft/s<sup>2</sup>.

Dr. Ibrahim AI-Hafidh / College of Petroleum and Mining Engineering



# British Gravitational (BG) System

The weight, W (which is the force due to gravity, g), of a mass, m, is given by the equation,

W = mg

And in BG units

W (Ib) = m (slug)  $g(ft/s^2)$ 

Since Earth's standard gravity is taken as (commonly approximated as 32.2 ft/s<sup>2</sup>), it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

# 3- English Engineering (EE) System



### (5) and (6) conversion factors for a large variety of unit systems

Conversion Factors from BG and EE Units to SI Units <sup>a</sup>			
	To Convert from	to	Multiply by
Acceleration	ft/s <sup>2</sup>	m/s <sup>2</sup>	3.048 E - 1
Area	ft <sup>2</sup>	m <sup>2</sup>	9.290 E - 2
Density	lbm/ft <sup>3</sup>	kg/m <sup>3</sup>	1.602 E + 1
	slugs/ft <sup>3</sup>	kg/m <sup>3</sup>	5.154 E + 2
Energy	Btu	J	1.055 E + 3
	ft • lb	J	1.356
Force	lb	Ν	4.448
Length	ft	m	3.048 E - 1
	in.	m	2.540 E - 2
	mile	m	1.609 E + 3
Mass	lbm	kg	4.536 E - 1
	slug	kg	1.459 E + 1
Power	ft · lb/s	W	1.356
	hp	W	7.457 E + 2
Pressure	in. Hg (60 °F)	$N/m^2$	3.377 E + 3
	lb/ft <sup>2</sup> (psf)	$N/m^2$	4.788 E + 1
	lb/in. <sup>2</sup> (psi)	$N/m^2$	6.895 E + 3
Specific weight	lb/ft <sup>3</sup>	N/m <sup>3</sup>	1.571 E + 2
Temperature	°F	°C	$T_C = (5/9)(T_F - 32^\circ)$
	°R	Κ	5.556 E - 1
Velocity	ft/s	m/s	3.048 E - 1
	mi/hr (mph)	m/s	4.470 E − 1
Viscosity (dynamic)	$lb \cdot s/ft^2$	$N \cdot s/m^2$	4.788 E + 1
Viscosity (kinematic)	ft <sup>2</sup> /s	m <sup>2</sup> /s	9.290 E - 2
Volume flowrate	ft <sup>3</sup> /s	m <sup>3</sup> /s	2.832 E - 2
	gal/min (gpm)	m <sup>3</sup> /s	6.309 E - 5

<sup>a</sup> If more than four-place acc	aracy is desired, refer to Appendix E.
--	--

	To Convert from	to	Multiply by
Acceleration	m/s <sup>2</sup>	ft/s <sup>2</sup>	3.281
Area	m <sup>2</sup>	ft <sup>2</sup>	1.076 E + 1
Density	kg/m <sup>3</sup>	lbm/ft <sup>3</sup>	6.243 E - 2
	kg/m <sup>3</sup>	slugs/ft <sup>3</sup>	1.940 E - 3
Energy	J	Btu	9.478 E - 4
	J	ft • lb	7.376 E - 1
Force	Ν	lb	2.248 E - 1
Length	m	ft	3.281
	m	in.	3.937 E + 1
	m	mile	6.214 E - 4
Mass	kg	lbm	2.205
	kg	slug	6.852 E - 2
Power	W	ft • lb/s	7.376 E - 1
	W	hp	1.341 E - 3
Pressure	$N/m^2$	in. Hg (60 °F)	2.961 E - 4
	N/m <sup>2</sup>	lb/ft <sup>2</sup> (psf)	2.089 E - 2
	$N/m^2$	lb/in. <sup>2</sup> (psi)	1.450 E - 4
Specific weight	N/m <sup>3</sup>	lb/ft <sup>3</sup>	6.366 E - 3
Temperature	°C	°F	$T_F = 1.8 T_C + 32^{\circ}$
	K	°R	1.800
Velocity	m/s	ft/s	3.281
	m/s	mi/hr (mph)	2.237
Viscosity (dynamic)	$N \cdot s/m^2$	$lb \cdot s/ft^2$	2.089 E - 2
Viscosity (kinematic)	m <sup>2</sup> /s	ft <sup>2</sup> /s	1.076 E + 1
Volume flowrate	m <sup>3</sup> /s	ft <sup>3</sup> /s	3.531 E + 1
	m <sup>3</sup> /s	gal/min (gpm)	1.585 E + 4

<sup>a</sup>If more than four-place accuracy is desired, refer to Appendix E.

Conversion Factors from SI Units to BG and EE Units<sup>a</sup>

Dr. Ibrahim Al-Hafidh / College of Petroleum and Mining Engineering



#### **EXAMPLE 1.2** BG and SI Units

**GIVEN** A tank of liquid having a total mass of 36 kg rests on a support in the equipment bay of the Space Shuttle.

**FIND** Determine the force (in newtons) that the tank exerts on the support shortly after lift off when the shuttle is accelerating upward as shown in Fig. E1.2*a* at 15 ft /s<sup>2</sup>.

#### SOLUTION

A free-body diagram of the tank is shown in Fig. E1.2*b*, where W is the weight of the tank and liquid, and  $F_f$  is the reaction of the floor on the tank. Application of Newton's second law of motion to this body gives

$$\sum \mathbf{F} = m \mathbf{a}$$

or

$$F_f - \mathcal{W} = ma$$

where we have taken upward as the positive direction. Since  $\mathcal{W} = mg$ , Eq. 1 can be written as

$$F_f = m\left(g + a\right)$$

Before substituting any number into Eq. 2, we must decide on a system of units, and then be sure all of the data are expressed in these units. Since we want  $F_{\ell}$  in newtons, we will use SI units so that

 $F_f = 36 \text{ kg} [9.81 \text{ m/s}^2 + (15 \text{ ft/s}^2)(0.3048 \text{ m/ft})]$ = 518 kg · m/s<sup>2</sup>

Since  $1 \text{ N} = 1 \text{ kg} \cdot \text{m} / \text{s}^2$ , it follows that





Figure E1.2a (Photograph courtesy of NASA.)

(Ans)

(1)

(2)

The direction is downward since the force shown on the free-body diagram is the force of the support *on the tank* so that the force the tank exerts *on the support* is equal in magnitude but opposite in direction.

**COMMENT** As you work through a large variety of problems in this text, you will find that units play an essential role in arriving at a numerical answer. Be careful! It is easy to mix units and cause large errors. If in the above example the acceleration had been left as  $15 \text{ ft/s}^2$  with *m* and *g* expressed in SI units, we would have calculated the force as 893 N and the answer would have been 72% too large!

31