FLUID MECHANICS

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LECTURE 4

Fluid Statics

1- Pressure at a Point *(Pascal's law)*

2- Pressure Variation in a Fluid at Rest

A- Incompressible Fluid

B- Compressible Fluid

3- Standard Atmosphere

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- **Density (الكثافة)**

$$\rho = \frac{Mass}{Volume}, \qquad Mass = \rho * Volume, \qquad Volume = \frac{Mass}{\rho}$$

- **Weight (الوزن)**

$$Weight = Mass * g \qquad 1\,N = (1Kg)\,(1\,m/s^2)$$

$$Weight = \rho * Volume * \frac{\gamma}{\rho}; \qquad Weight = \gamma * Volume$$

- **Specific weight (الوزن النوعي)**

$$\gamma = \rho\, g, \qquad \gamma = \frac{Mass}{Volume} * g, \qquad \gamma = \frac{Weight}{Volume}$$

- **Pressure**

$$P = \frac{Force}{Area} = \frac{F}{A}; \frac{N}{m^2}; \qquad Force = Pressure * area; \qquad F = PA$$

- Newton's second law, $Force = mass * acceleration$
- $PA = Mass * g;$ $\qquad P = \frac{Mass * g}{A}; \frac{\rho * Vol. * g}{A}; \frac{\gamma * Vol.}{A}$

$$\therefore P = \gamma * h$$

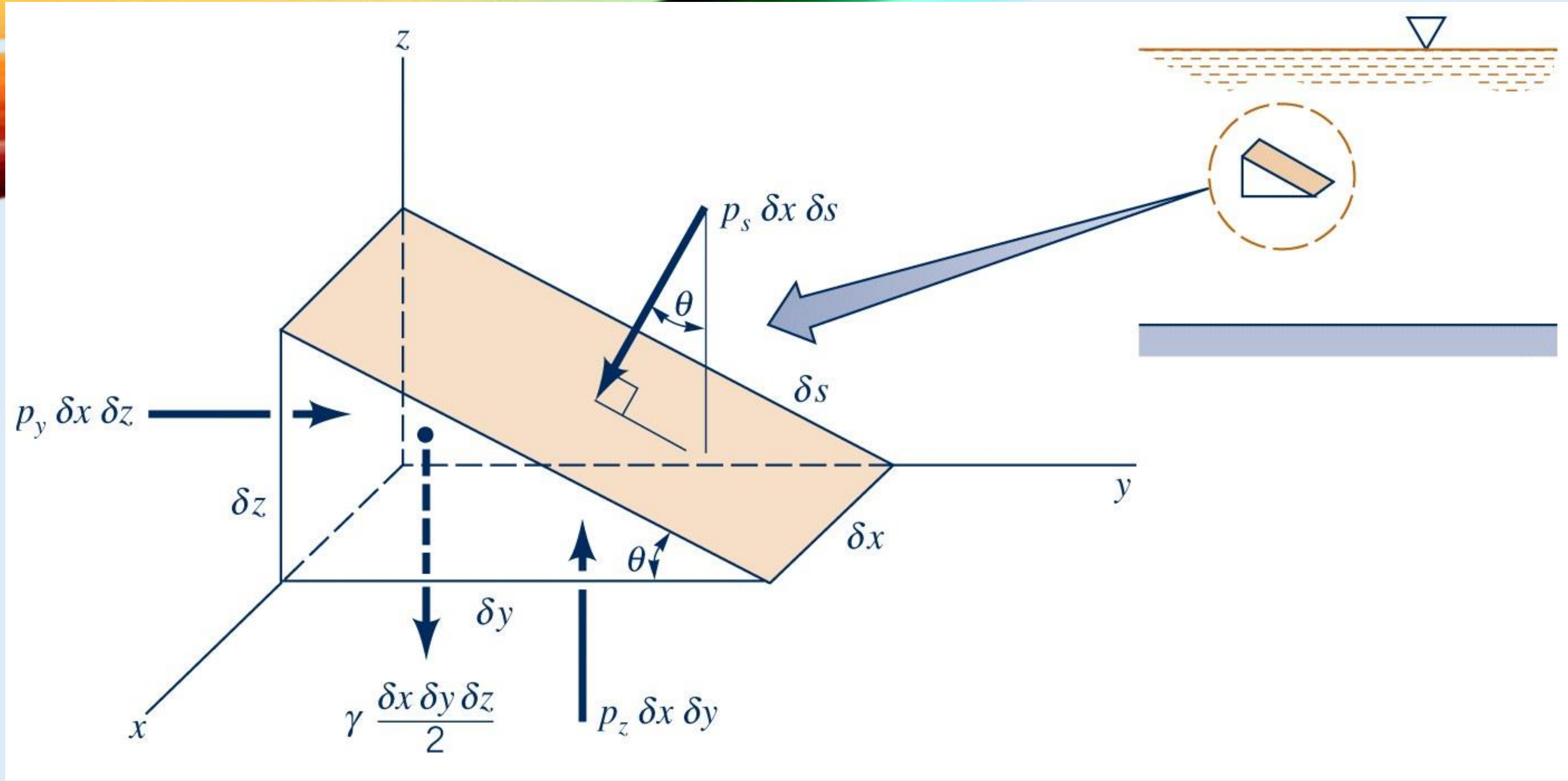
1- Pressure at a Point (الضغط في نقطة ضمن سائل في حالة السكون)

The term pressure is used to indicate the normal force per unit area at a given point acting on a given plane within the fluid mass of interest.

How the pressure at a point varies with the orientation of the plane passing through the point. ?????

كيف يختلف الضغط عند نقطة ما, مع اتجاه مستوي يمر عبر النقطة ؟؟؟
هل الضغط في نقطة لمائع ساكن متساوي في جميع الاتجاهات ؟؟؟

To answer this question, consider the free-body diagram as shown in the next slide……..

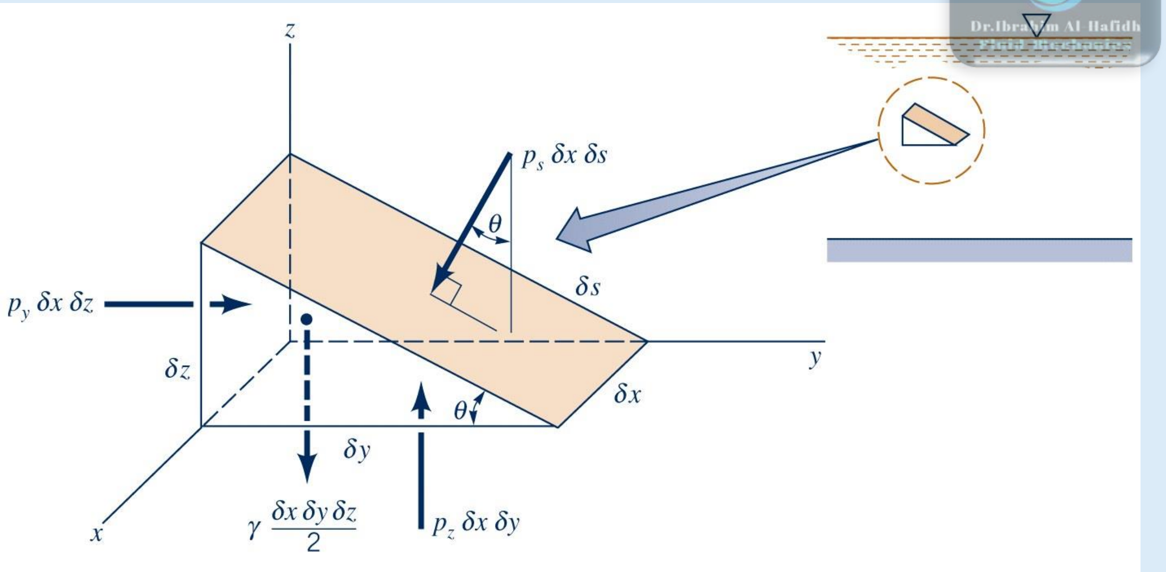


Since we are considering the situation in which there are no shearing stresses, the only external forces acting on the wedge are due to the pressure and the weight.

For simplicity the forces in the x direction are not shown, and the z axis is taken as the vertical axis.

The weight acts in the negative z direction.

We are primarily interested in fluids at rest. However, to make the analysis as general as possible, we will allow the fluid element to have **accelerated motion**.
But still The assumption of zero shearing stresses will be valid.

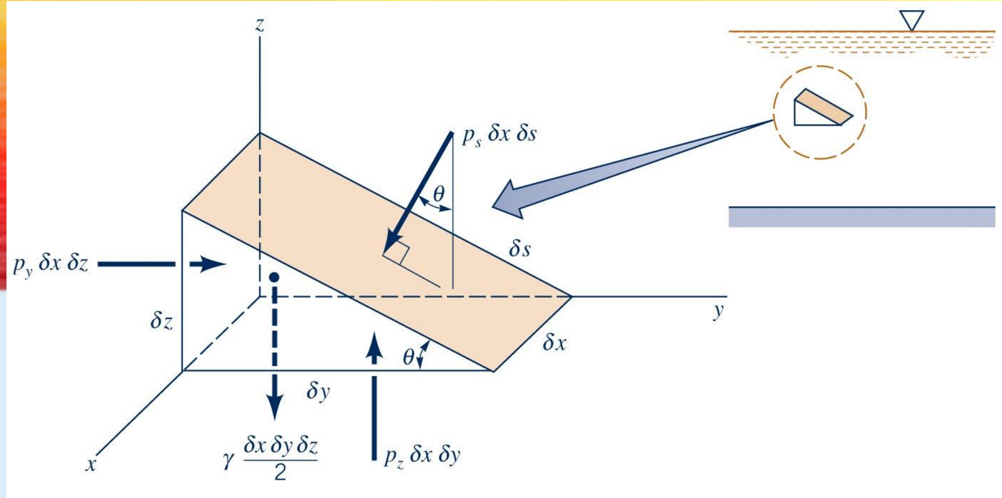


Newton's second law, Force = mass * acceleration, *Mass = ρ * Volume*, *Weight = γ * Volume*

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s \sin\theta = \rho \frac{\delta x\,\delta y\,\delta z}{2} a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s \cos\theta - \gamma \frac{\delta x\,\delta y\,\delta z}{2} = \rho \frac{\delta x\,\delta y\,\delta z}{2} a_z$$

Force = ρ * *Volume** acceleration



Where:

Ps, P_y, **and** **P_z** **are the average pressures on the faces.**

γ is a specific weight of the fluid

ρ is a specific density of the fluid.

a_y & a_z the acceleration.

Note that a pressure must be multiplied by an appropriate area to obtain the force generated by the pressure. It follows from the geometry that

$$\delta y = \delta s \cos\theta \qquad \delta z = \delta s \sin\theta$$

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s\sin\theta = \rho\,\frac{\delta x\,\delta y\,\delta z}{2}\,a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s\cos\theta - \gamma\,\frac{\delta x\,\delta y\,\delta z}{2} = \rho\,\frac{\delta x\,\delta y\,\delta z}{2}\,a_z$$

$$p_y - p_s = \rho a_y\,\frac{\delta y}{2}$$

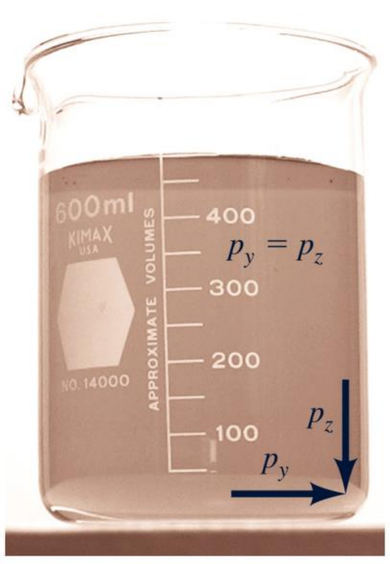
$$p_z - p_s = (\rho a_z + \gamma)\,\frac{\delta z}{2}$$

Since we are really interested in what is happening at a point, we take the limit as $\delta x, \delta y, and\ \delta z$ approach *zero* (while maintaining the angle θ), and it follows that

$$P_y = P_s \quad , \quad P_z = P_s$$

Or

$$P_s = P_y = P_z$$



The angle θ was arbitrarily chosen. We can conclude that;

The pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present.

This important result is known as *Pascal's law*.

Pascal's law: it states that the pressure inside the fluid is same through out in all direction when the fluid is in the rest.

2- Pressure Variation in a Fluid at Rest

For liquids or gases at rest, the pressure gradient in the vertical direction at any point in a fluid depends only on the specific weight (γ) of the fluid at that point.

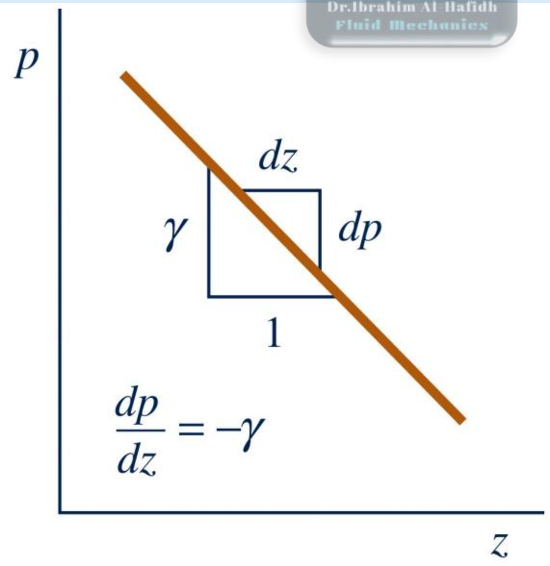
$$P = \gamma * h$$

$$\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = 0 \qquad \frac{\partial P}{\partial z} = -\gamma$$

These equations show that the pressure does not depend on x or y. Thus, as we move from point to point in a horizontal plane (any plane parallel to the x–y plane), the pressure does not change. Since p depends only on z, the last of Equation can be written as the ordinary differential equation

$$\frac{d P}{d z} = -\gamma$$

$$\frac{d\,P}{d\,z} = -\gamma$$



This is the fundamental equation for fluids at rest and can be used to determine how pressure changes with elevation.

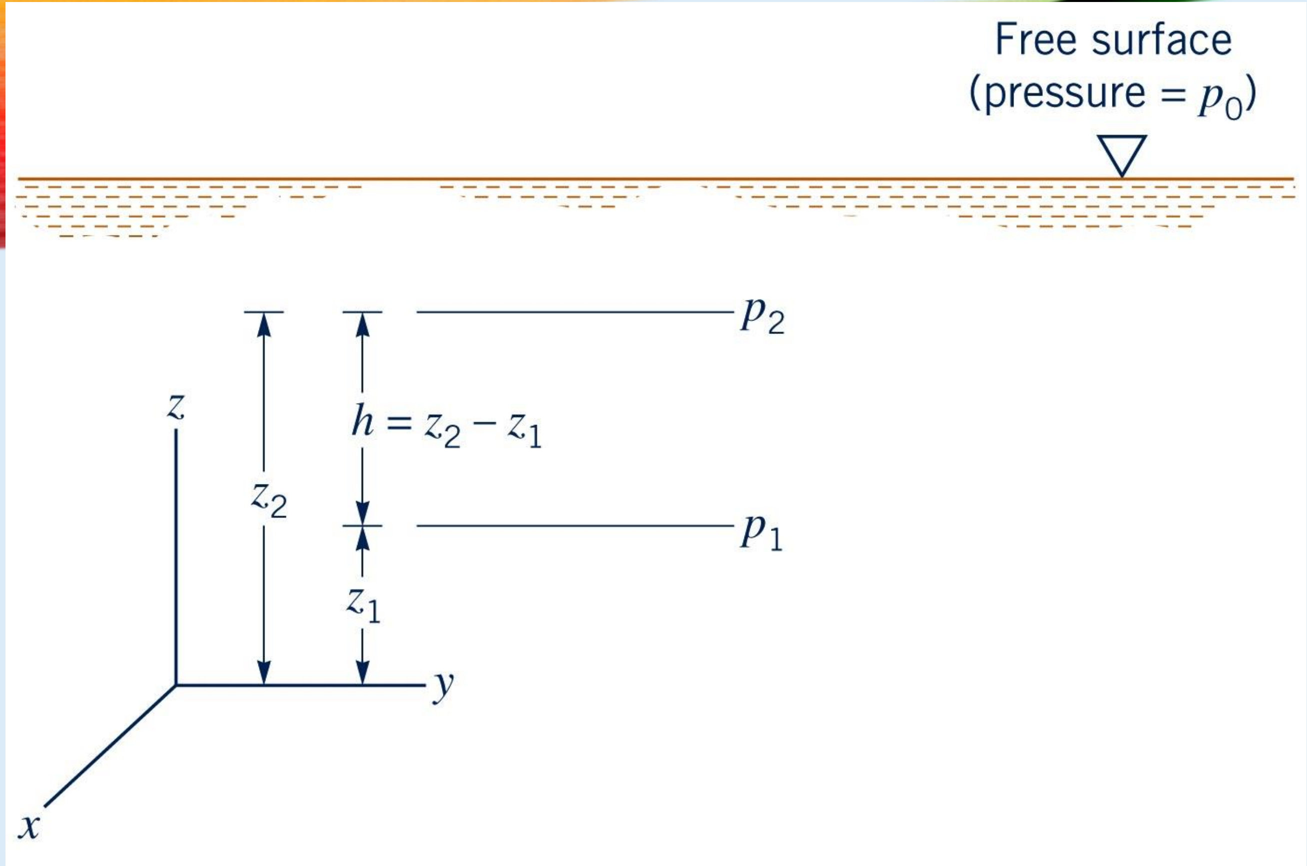
The pressure gradient in the vertical direction is negative; that is, the pressure decreases as we move upward in a fluid at rest.
There is no requirement that γ be a constant. Thus, it is valid for fluids with constant specific weight, such as liquids, as well as fluids whose specific weight may vary with elevation, such as air or other gases.

a- Incompressible Fluid (السوائل الغير مضغوطة)

Since the **specific weight** is equal to the product of **fluid density** and **acceleration of gravity** ($\gamma = \rho g$), changes in are caused by a change in either ρ or g.

For most engineering applications the variation in g is negligible, so our main concern is with the possible variation in the fluid density ρ.

In general, a fluid with constant density (ρ) is called an *incompressible fluid*.



$$p_1 - p_2 = \gamma(z_2 - z_1)$$

$$p_1 - p_2 = \gamma h$$

$$p_1 = \gamma h + p_2$$

Where P_1 and P_2 are pressures at the vertical elevations as is illustrated in figure above.
And, *h* is the distance, z_2-z_1, which is the depth of fluid measured downward from the location of P_2.
This type of pressure distribution is commonly called a *hydrostatic distribution*.

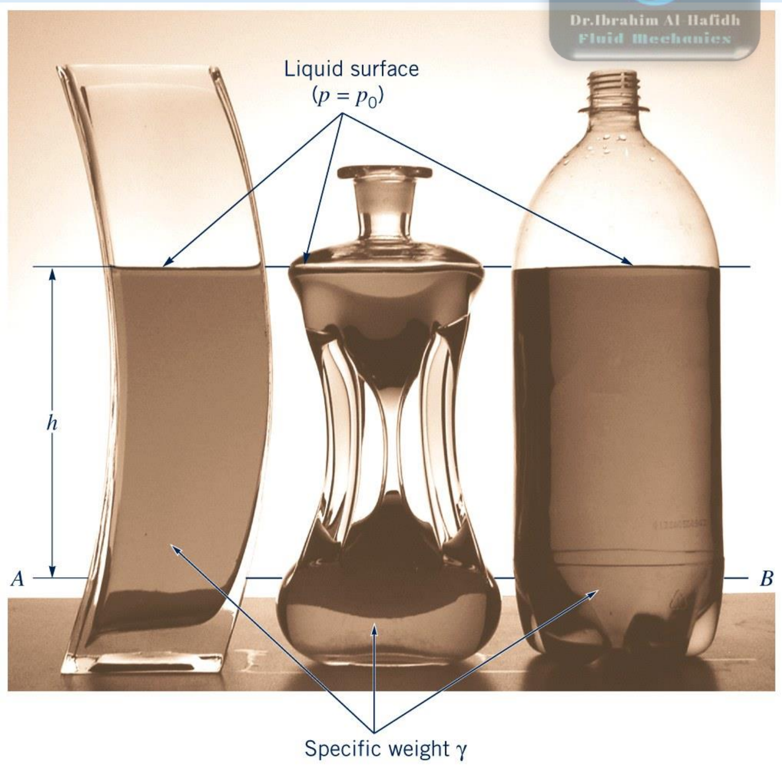
$$h = \frac{p_1 - p_2}{\gamma}$$

In this case *h* is called the *pressure head* and is interpreted as the height of a column of fluid of specific weight γ required to give a pressure difference $P_1 - P_2$.

When one works with liquids there is often a free surface, and it is convenient to use this surface as a reference plane.
The reference pressure P_0 would correspond to the pressure acting on the free surface (which would frequently be atmospheric pressure), and thus if we let $P_2 = P_0$. it follows that the pressure p at any depth h below the free surface is given by the equation:

$$p = \gamma h + p_0$$

The pressure in a homogeneous, incompressible fluid at rest **depends** on the **depth of the fluid** relative to some reference plane, and **it is not influenced by the size or shape** of the tank or container in which the fluid is held.

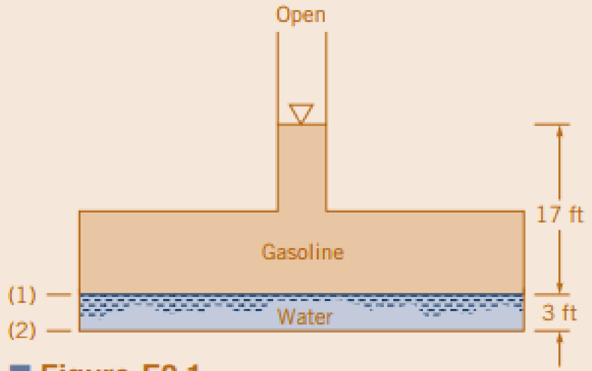


The pressure is the same at all points along the line **AB**, even though the containers have very irregular shapes. The actual value of the pressure along **AB** depends only on the depth, *h*, the surface pressure P_0, and the specific weight, **γ** of the liquid in the container.

EXAMPLE 2.1 Pressure–Depth Relationship

GIVEN Because of a leak in a buried gasoline storage tank, water has seeped in to the depth shown in Fig. E2.1. The specific gravity of the gasoline is $SG = 0.68$.

FIND Determine the pressure at the gasoline–water interface and at the bottom of the tank. Express the pressure in units of lb/ft^2, lb/in.^2, and as a pressure head in feet of water.



■ **Figure E2.1**

SOLUTION

Since we are dealing with liquids at rest, the pressure distribution will be hydrostatic, and therefore the pressure variation can be found from the equation:

$$p = \gamma h + p_0$$

With p_0 corresponding to the pressure at the free surface of the gasoline, then the pressure at the interface is

$$\begin{aligned} p_1 &= SG\gamma_{H_2O} h + p_0 \\ &= (0.68)(62.4\ \text{lb/ft}^3)(17\ \text{ft}) + p_0 \\ &= 721 + p_0\ (\text{lb/ft}^2) \end{aligned}$$

If we measure the pressure relative to atmospheric pressure (gage pressure), it follows that $p_0 = 0$, and therefore

$$p_1 = 721\ \text{lb/ft}^2 \qquad \textbf{(Ans)}$$

$$p_1 = \frac{721\ \text{lb/ft}^2}{144\ \text{in.}^2/\text{ft}^2} = 5.01\ \text{lb/in.}^2 \qquad \textbf{(Ans)}$$

$$\frac{p_1}{\gamma_{H_2O}} = \frac{721\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} = 11.6\ \text{ft} \qquad \textbf{(Ans)}$$

It is noted that a rectangular column of water 11.6 ft tall and 1 ft^2 in cross section weighs 721 lb. A similar column with a 1-in.^2 cross section weighs 5.01 lb.

We can now apply the same relationship to determine the pressure at the tank bottom; that is,

$$\begin{aligned} p_2 &= \gamma_{H_2O} h_{H_2O} + p_1 \\ &= (62.4\ \text{lb/ft}^3)(3\ \text{ft}) + 721\ \text{lb/ft}^2 \qquad \textbf{(Ans)} \\ &= 908\ \text{lb/ft}^2 \end{aligned}$$

$$p_2 = \frac{908\ \text{lb/ft}^2}{144\ \text{in.}^2/\text{ft}^2} = 6.31\ \text{lb/in.}^2 \qquad \textbf{(Ans)}$$

$$\frac{p_2}{\gamma_{H_2O}} = \frac{908\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} = 14.6\ \text{ft} \qquad \textbf{(Ans)}$$

COMMENT Observe that if we wish to express these pressures in terms of *absolute* pressure, we would have to add the local atmospheric pressure (in appropriate units) to the previous results. A further discussion of gage and absolute pressure is given in Section 2.5.

b- compressible Fluid (السوائل المضغوطة)

We normally think of gases such as air, oxygen, and nitrogen as being compressible fluids because the density of the gas can change significantly with changes in pressure and temperature.

Or

Compressible flow is the branch of fluid mechanics that deals with flows having significant changes in fluid density.

The equation of state for an ideal (or perfect) gas

$$\rho = \frac{p}{RT}$$

Where p is the absolute pressure, R is the gas constant, and T is the absolute temperature.

This relationships can be combined

$$\frac{dp}{dz} = -\gamma$$

$$\rho = \frac{p}{RT}$$

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

Where: g and R are assumed to be constant over the elevation change from z_1 to z_2.

Although the acceleration of gravity, g, does vary with elevation, the variation is very small, and g is usually assumed constant at some average value for the range of elevation involved.

It must be specifying the nature of the variation of temperature with elevation. And if we assume that the temperature has a constant value T_o over the range (*isothermal conditions*)
(which the temperature of a system remains constant)

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

This equation provides the desired pressure–elevation relationship for an isothermal layer.

EXAMPLE 2.2 Incompressible and Isothermal Pressure–Depth Variations

GIVEN In 2010, the world's tallest building, the Burj Khalifa skyscraper, was completed and opened in the United Arab Emirates. The final height of the building, which had remained a secret until completion, is 2717 ft (828 m).

FIND **(a)** Estimate the ratio of the pressure at the 2717-ft top of the building to the pressure at its base, assuming the air to be at a common temperature of 59 °F. **(b)** Compare the pressure calculated in part (a) with that obtained by assuming the air to be incompressible with $\gamma = 0.0765\ \text{lb/ft}^3$ at 14.7 psi (abs) (values for air at standard sea level conditions).

Figure E2.2 (Figure courtesy of Emaar Properties, Dubai, UAE.)

SOLUTION

(a) For the assumed isothermal conditions, and treating air as a compressible fluid, Eq. 2.10 can be applied to yield

$$\frac{p_2}{p_1} = \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

$$= \exp\left\{-\frac{(32.2\ \text{ft/s}^2)(2717\ \text{ft})}{(1716\ \text{ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})[(59 + 460)^\circ\text{R}]}\right\}$$

$$= 0.906 \qquad \textbf{(Ans)}$$

(b) If the air is treated as an incompressible fluid we can apply Eq. 2.5. In this case

$$p_2 = p_1 - \gamma(z_2 - z_1)$$

or

$$\frac{p_2}{p_1} = 1 - \frac{\gamma(z_2 - z_1)}{p_1}$$

$$= 1 - \frac{(0.0765\ \text{lb/ft}^3)(2717\ \text{ft})}{(14.7\ \text{lb/in.}^2)(144\ \text{in.}^2/\text{ft}^2)} = 0.902 \qquad \textbf{(Ans)}$$

COMMENTS Note that there is little difference between the two results. Since the pressure difference between the bottom and top of the building is small, it follows that the variation in fluid density is small and, therefore, the compressible fluid and incompressible fluid analyses yield essentially the same result.

We see that for both calculations the pressure decreases by approximately 10% as we go from ground level to the top of this tallest building. It does not require a very large pressure difference to support a 2717-ft-tall column of fluid as light as air. This result supports the earlier statement that the changes in pressures in air and other gases due to elevation changes are very small, even for distances of hundreds of feet. Thus, the pressure differences between the top and bottom of a horizontal pipe carrying a gas, or in a gas storage tank, are negligible since the distances involved are very small.