FLUID MECHANICS

COLLEGE OF PETROLEUM AND MINING ENGINEERING

Dr. Ibrahim Adil Ibrahim Al-Hafidh
Mining Engineering Department
College of Petroleum and Mining Engineering
University of Mosul

Email: iibrahim@uomosul.edu.iq

LECTURE 5

Manometry

a) Piezometer Tube
b) U-Tube Manometer
c) Inclined-Tube Manometer

Dr.Ibrahim Al-Hafidh
Fluid Mechanics

Email: iibrahim@uomosul.edu.iq

Manometry

A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes.
Pressure-measuring device based on this technique are called *Manometer*.

The *mercury barometer* is an example of one type of manometer, but there are many other configurations possible depending on the particular application. Three common types of manometers include the piezometer tube, the *U-tube manometer*, and the *inclined-tube manometer*.

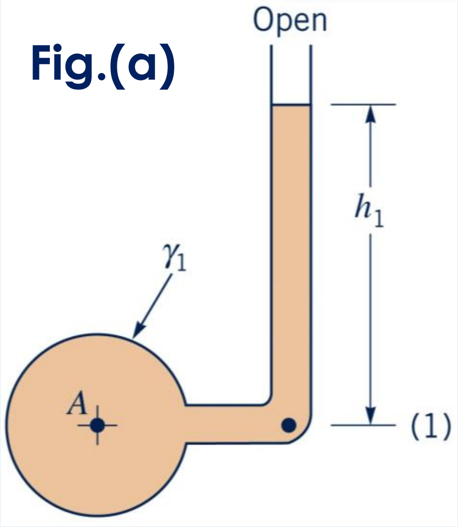
A- Piezometer Tube

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Figure (a).
The figure (b) in the margin shows an important device whose operation is based on this principle. It is a sphygmomanometer, the traditional instrument used to measure blood pressure.

Since manometers involve columns of fluids at rest, the fundamental equation describing their use is:

$$p = \gamma h + p_0$$

Fig.(a)



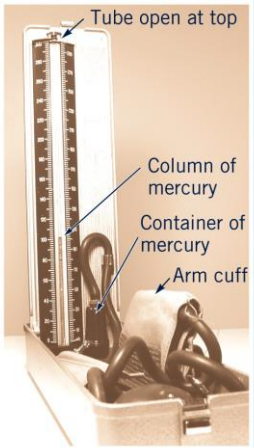


Fig.(b)

$$p = \gamma h + p_0$$

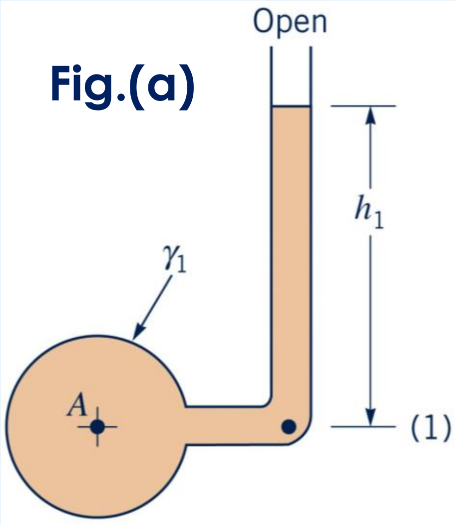
which gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure and the vertical distance *h* between *p* and p_o.

Note: The pressure in fluid at rest will increase as we move downward and will decrease as we move upward.

Application of this equation to the piezometer tube of Fig. (a) indicates that the pressure P_A can be determined by a measurement of through the relationship

$$p_A = \gamma_1 h_1$$

Fig.(a)



$$p_A = \gamma_1 h_1$$

where γ_1 is the specific weight of the liquid in the container.
Note: Since the tube is open at the top, the pressure *P_o* can be set equal to *zero*.
***h1*, is the height of fluid from upper surface to point (1).**

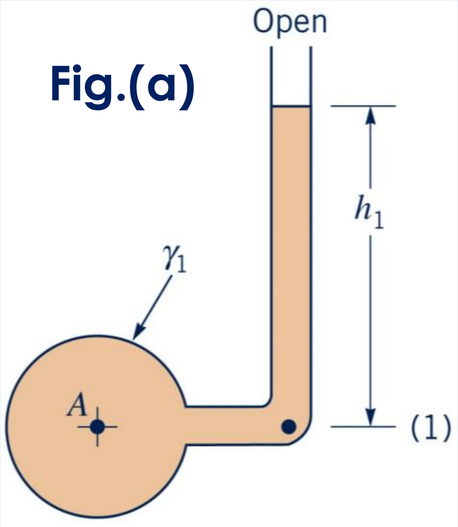


Fig.(a)

Although the piezometer tube is a very simple and accurate pressure-measuring device, it has ***several disadvantages***.

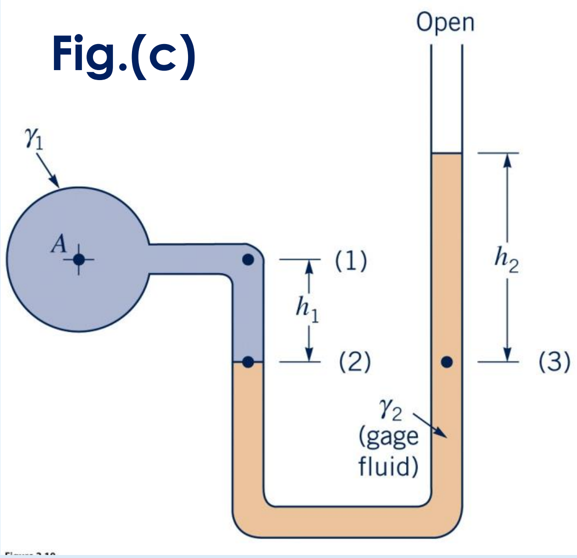
- It is suitable only if the pressure in the container is greater than atmospheric pressure (otherwise air would be sucked into the system),
- The pressure to be measured must be relatively small so the required height of the column is reasonable.
- The fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

B- U-Tube Manometer

another type of manometer which is widely used consists of a tube formed into the shape of a U, as is shown in Figure (c).

The fluid in the manometer is called the *gage fluid*.

Fig.(c)

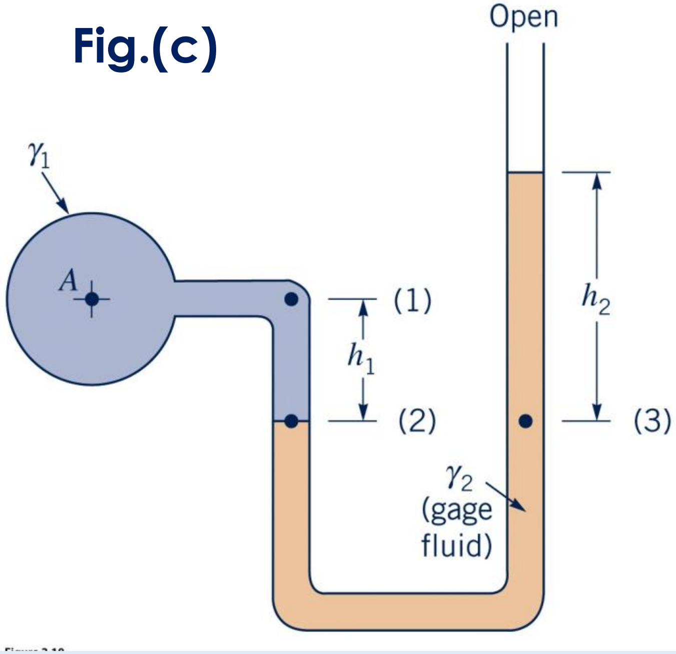


To find the pressure in P_A terms of the various column heights, we start at one end of the system and work our way around to the other end,

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

The pressure at points A and (1) are the same, move from point (1) to (2) the pressure will increase.
The pressure at point (2) is equal to the pressure at point (3), since the pressures at equal elevations in a continuous mass of fluid at rest must be the same.

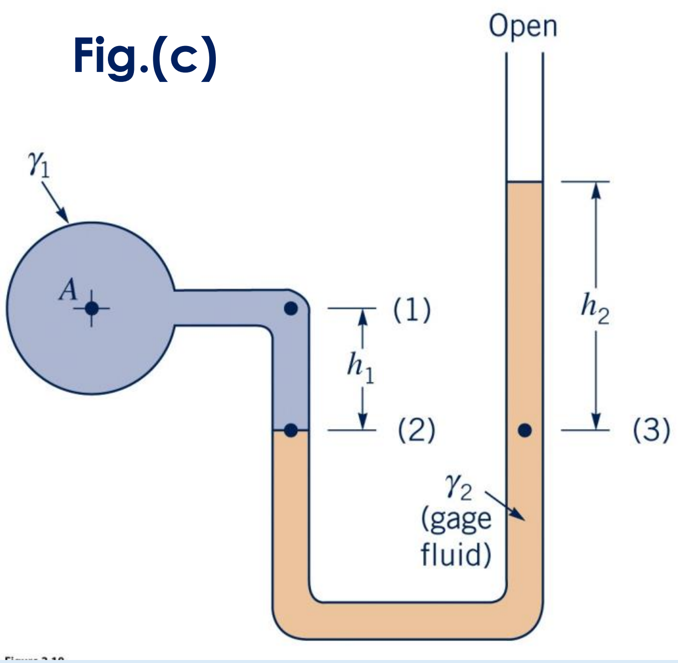
Fig.(c)



$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

Equation 2.14

Note: We could not simply "jump across" from point (1) to a point at the same elevation in the right-hand tube since these would not be points within the same continuous mass of fluid.



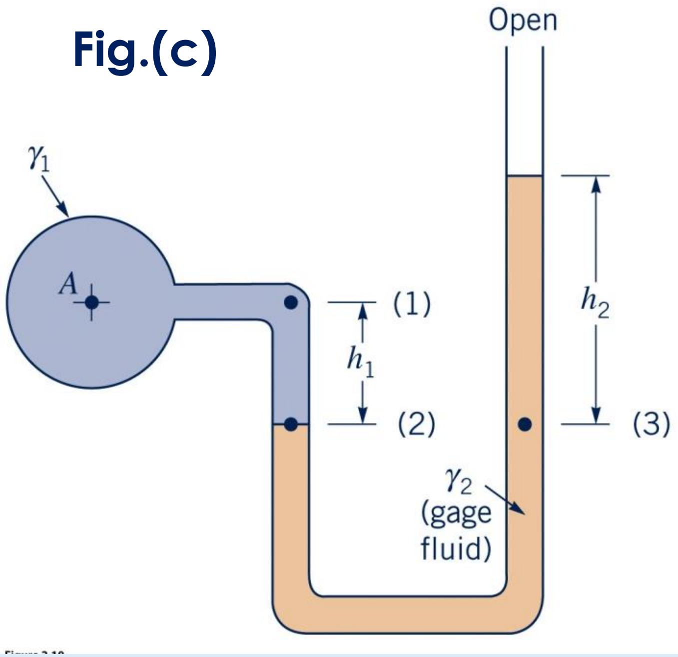
With the pressure at point (3) specified, we now move to the open end where the pressure is zero.
As we move vertically upward the pressure decreases by amount $\gamma_2 h_2$.

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

A major advantage of the U-tube manometer lies in the fact that the gage fluid can be different from the fluid in the container in which the pressure is to be determined. For example, the fluid in A in Fig. (c) can be either a liquid or a gas. If *A* does contain a gas, the contribution of the gas column, is almost always negligible so that , and the equation is become:

Fig.(c)



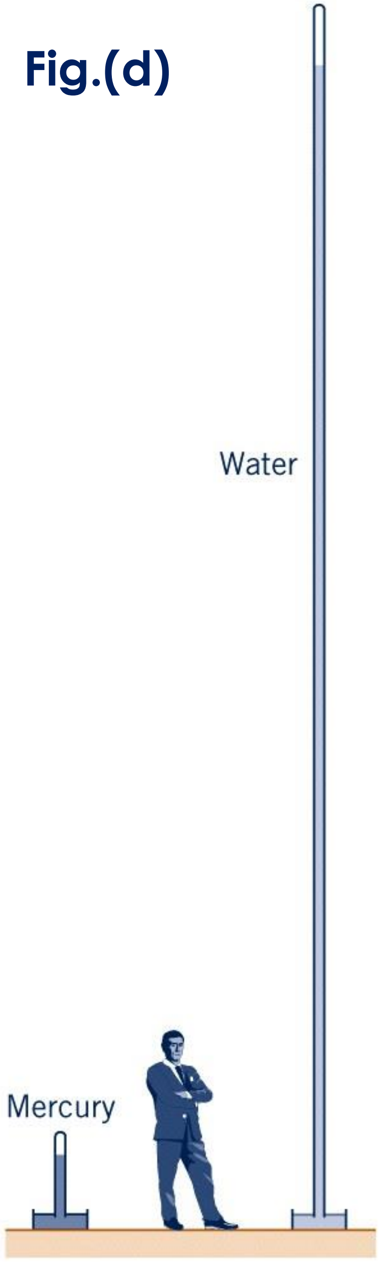
$$p_A = \gamma_2 h_2$$

For a given pressure the height, is governed by the *specific weight (γ)*, of the gage fluid used in the manometer. If the pressure is large, then a heavy gage fluid, such as mercury, can be used and a reasonable column height (not too long) can still be maintained.

Alternatively, if the pressure is small, a lighter gage fluid, such as water, can be used so that a relatively large column height can be achieved.

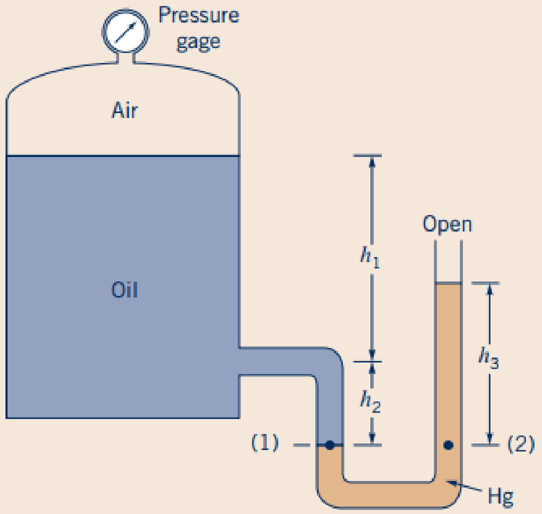
Specific weight for Mercury = 133 KN/m^3
Specific weight for Water = 9.8 KN/m^3

Fig.(d)



GIVEN A closed tank contains compressed air and oil ($SG_{oil} = 0.90$) as is shown in Fig. E2.4. A U-tube manometer using mercury ($SG_{Hg} = 13.6$) is connected to the tank as shown. The column heights are $h_1 = 36$ in., $h_2 = 6$ in., and $h_3 = 9$ in.

FIND Determine the pressure reading (in psi) of the gage.



■ **Figure E2.4**

SOLUTION

Following the general procedure of starting at one end of the manometer system and working around to the other, we will start at the air–oil interface in the tank and proceed to the open end where the pressure is zero. The pressure at level (1) is

$$p_1 = p_{air} + \gamma_{oil}(h_1 + h_2)$$

This pressure is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. As we move from level (2) to the open end, the pressure must decrease by $\gamma_{Hg}h_3$, and at the open end the pressure is zero. Thus, the manometer equation can be expressed as

$$p_{air} + \gamma_{oil}(h_1 + h_2) - \gamma_{Hg}h_3 = 0$$

or

$$p_{air} + (SG_{oil})(\gamma_{H_2O})(h_1 + h_2) - (SG_{Hg})(\gamma_{H_2O})\,h_3 = 0$$

For the values given

$$p_{air} = -(0.9)(62.4 \text{ lb/ft}^3)\left(\frac{36 + 6}{12} \text{ ft}\right) + (13.6)(62.4 \text{ lb/ft}^3)\left(\frac{9}{12} \text{ ft}\right)$$

so that

$$p_{air} = 440 \text{ lb/ft}^2$$

Since the specific weight of the air above the oil is much smaller than the specific weight of the oil, the gage should read the pressure we have calculated; that is,

$$p_{gage} = \frac{440 \text{ lb/ft}^2}{144 \text{ in.}^2/\text{ft}^2} = 3.06 \text{ psi} \qquad \textbf{(Ans)}$$

COMMENTS Note that the air pressure is a function of the height of the mercury in the manometer and the depth of the oil (both in the tank and in the tube). It is not just the mercury in the manometer that is important.

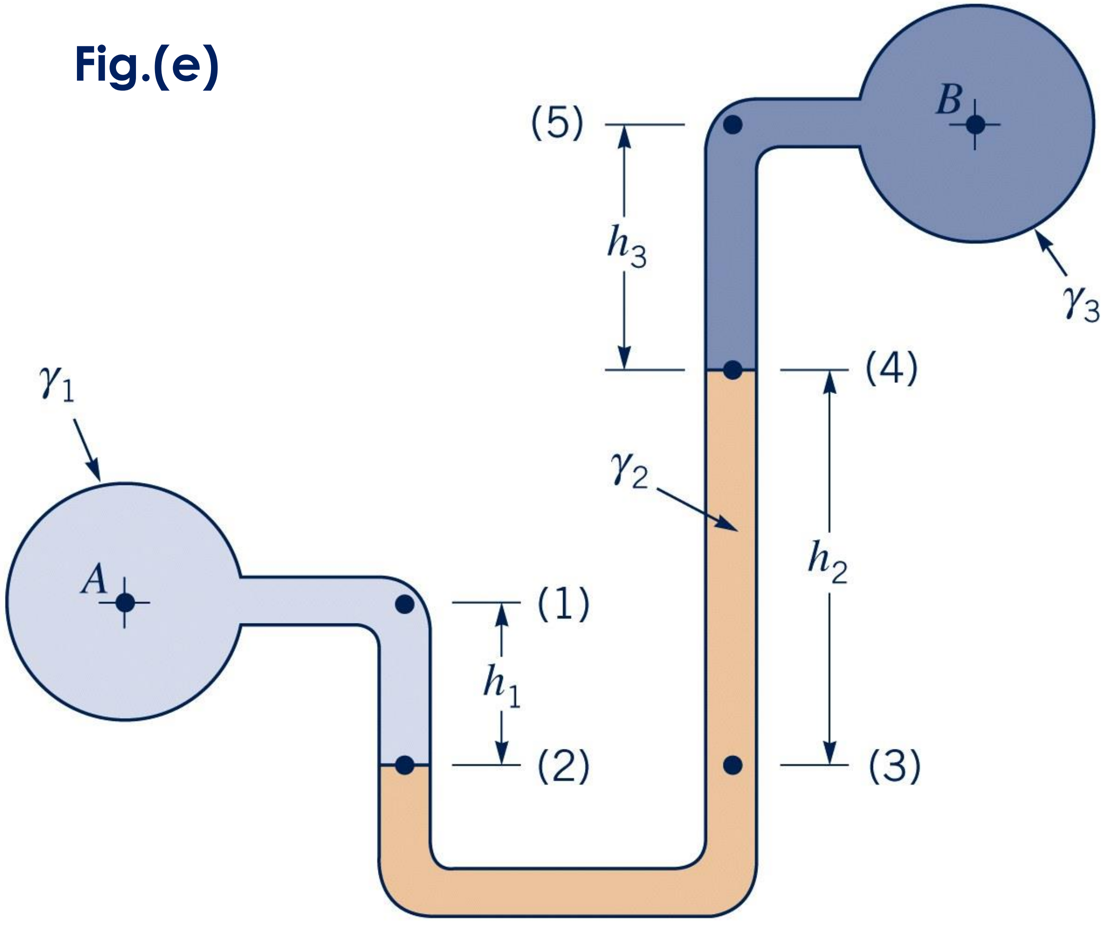
Assume that the gage pressure remains at 3.06 psi, but the manometer is altered so that it contains only oil. That is, the mercury is replaced by oil. A simple calculation shows that in this case the vertical oil-filled tube would need to be $h_3 = 11.3$ ft tall, rather than the original $h_3 = 9$ in. There is an obvious advantage of using a heavy fluid such as mercury in manometers.

The U-tube manometer is also widely used to measure the difference in pressure between two containers or two points in a given system.

Consider a manometer connected between containers A and B as is shown in Figure (e).

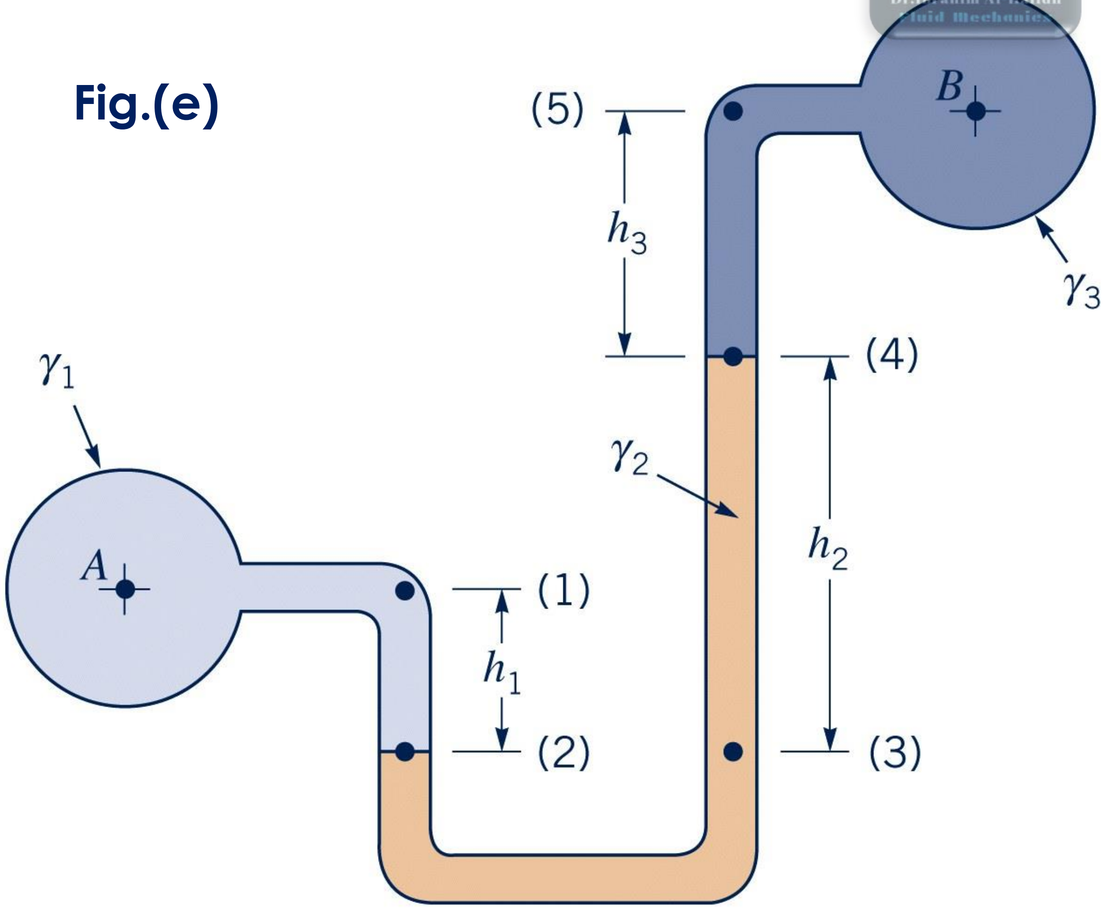
The difference in pressure between A and B can be found by again starting at one end of the system and working around to the other end.

Fig.(e)



At (*A*) the pressure is (P_A), which is equal to (P_1) and as we move to point (2) the pressure *increases* by $\gamma_1 h_1$. The pressure at (P_2) is equal to (P_3) and as we move upward to point (4) the pressure *decreases* by $\gamma_2 h_2$. Similarly, as we continue to move upward from point (4) to (5) the pressure *decreases* by $\gamma_3 h_3$. Finally, $P_5 = P_B$ since they are at equal elevations. Thus

Fig.(e)

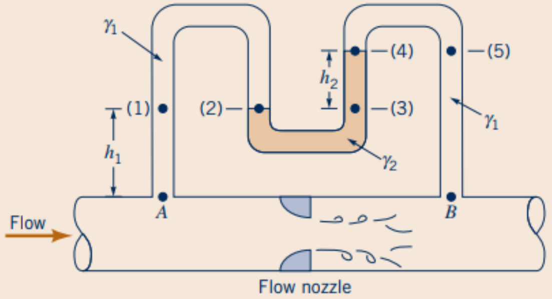


$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

GIVEN As will be discussed in Chapter 3, the volume rate of flow, Q, through a pipe can be determined by means of a flow nozzle located in the pipe as illustrated in Fig. E2.5*a*. The nozzle creates a pressure drop, $p_A - p_B$, along the pipe that is related to the flow through the equation $Q = K\sqrt{p_A - p_B}$, where K is a constant depending on the pipe and nozzle size. The pressure drop is frequently measured with a differential U-tube manometer of the type illustrated.

FIND **(a)** Determine an equation for $p_A - p_B$ in terms of the specific weight of the flowing fluid, γ_1, the specific weight of the gage fluid, γ_2, and the various heights indicated. **(b)** For $\gamma_1 = 9.80 \text{ kN/m}^3$, $\gamma_2 = 15.6 \text{ kN/m}^3$, $h_1 = 1.0$ m, and $h_2 = 0.5$ m, what is the value of the pressure drop, $p_A - p_B$?



■ **Figure E2.5*a***

Solution

(a) Although the fluid in the pipe is moving, the fluids in the columns of the manometer are at rest so that the pressure variation in the manometer tubes is hydrostatic. If we start at point A and move vertically upward to level (1), the pressure will decrease by $\gamma_1 h_1$ and will be equal to the pressure at (2) and at (3). We can now move from (3) to (4) where the pressure has been further reduced by $\gamma_2 h_2$. The pressures at levels (4) and (5) are equal, and as we move from (5) to B the pressure will increase by $\gamma_1(h_1 + h_2)$. Thus, in equation form

$$p_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1(h_1 + h_2) = p_B$$

or

$$p_A - p_B = h_2(\gamma_2 - \gamma_1) \qquad \textbf{(Ans)}$$

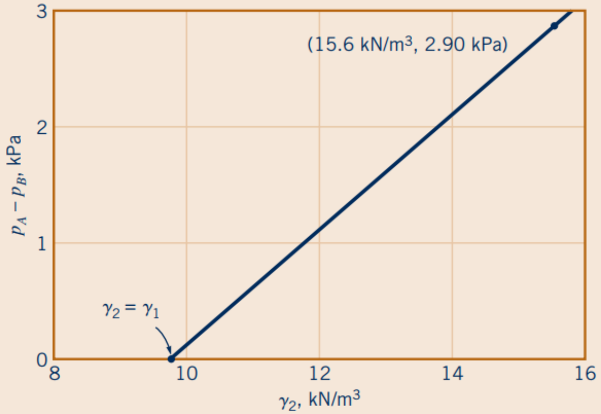
COMMENT It is to be noted that the only column height of importance is the differential reading, h_2. The differential manometer could be placed 0.5 or 5.0 m above the pipe ($h_1 = 0.5$ m or $h_1 = 5.0$ m), and the value of h_2 would remain the same.

(b) The specific value of the pressure drop for the data given is

$$p_A - p_B = (0.5 \text{ m})(15.6 \text{ kN/m}^3 - 9.80 \text{ kN/m}^3)$$
$$= 2.90 \text{ kPa} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for manometer fluids with different specific weights, γ_2, the results shown in Fig. E2.5*b* are obtained. Note that relatively small pressure differences can be measured if the manometer fluid has nearly the same specific weight as the flowing fluid. It is the difference in the specific weights, $\gamma_2 - \gamma_1$, that is important.

Hence, by rewriting the answer as $h_2 = (p_A - p_B)/(\gamma_2 - \gamma_1)$ it is seen that even if the value of $p_A - p_B$ is small, the value of h_2 can be large enough to provide an accurate reading provided the value of $\gamma_2 - \gamma_1$ is also small.

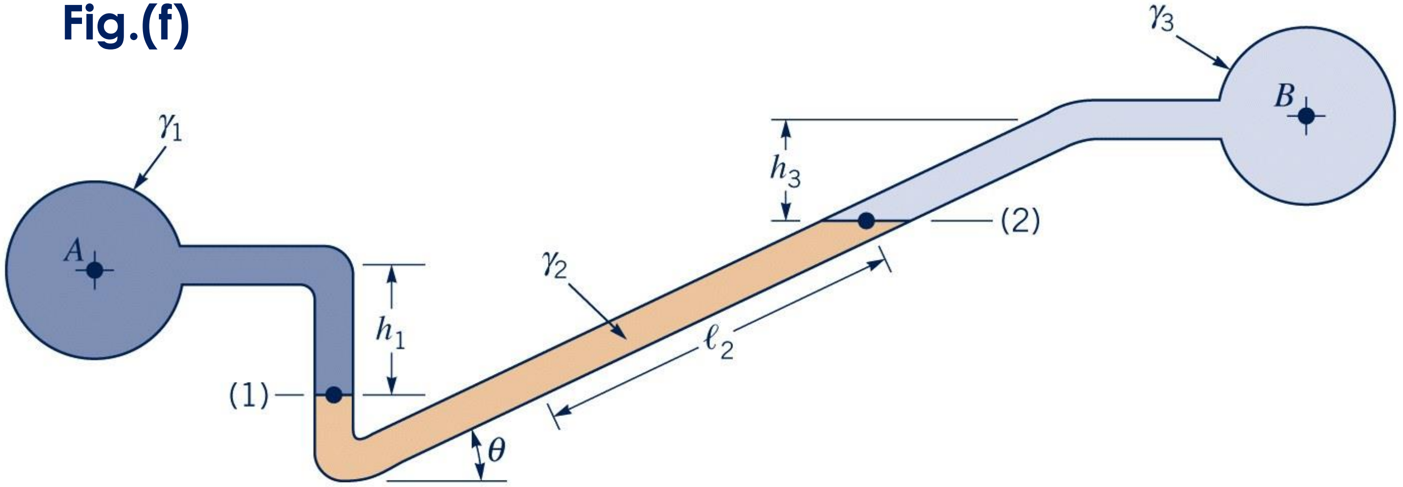


■ **Figure E2.5*b***

C- Inclined-Tube Manometer

To measure small pressure changes, a manometer of the type shown in Fig. (f) is frequently used. One leg of the manometer is inclined at an angle *ϑ* and the differential reading *l_2* is measured along the inclined tube. The difference in pressure *P_A-P_B* can be expressed as

Fig.(f)



$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

Or

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

$$\sin\theta = \frac{Opposite}{Hypotenuse} = \frac{\text{المقابل}}{\text{الوتر}} = \frac{\mathbf{h}}{l_2}$$

$$h = l_2 * \sin\theta$$

$$l_2 = \frac{\mathbf{h}}{\sin\vartheta}$$

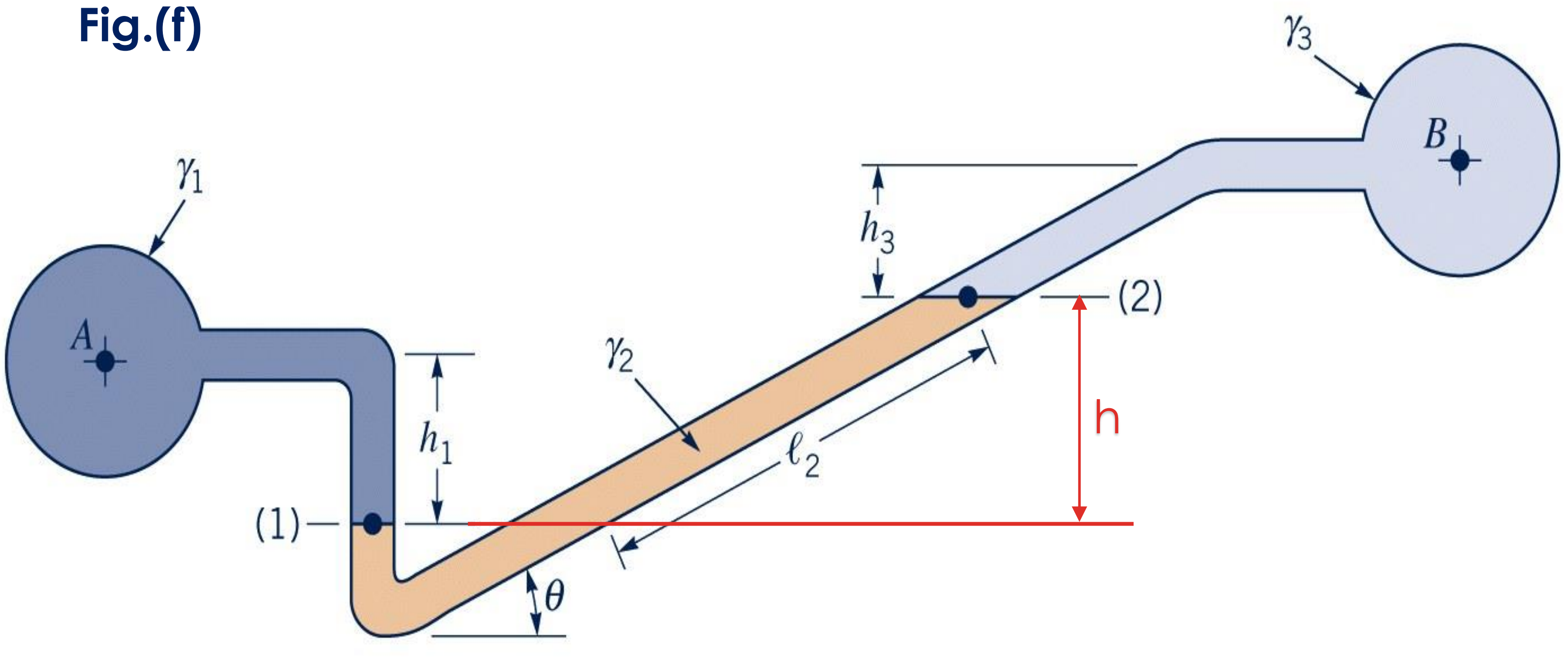
If θ = 90º , sin90=1

$$l_2 = h$$

If θ = 30º , sin30=0.5

$$l_2 = 2h$$

Fig.(f)



where it is to be noted the pressure difference between points (1) and (2) is due to the *vertical* distance between the points, which can be expressed as (***l_2* sinϑ**).

Thus, for relatively small angles the differential reading along the inclined tube can be made large even for small pressure differences.

The inclined-tube manometer is often used to measure small differences in gas pressures so that if pipes A and B contain a gas, then

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta$$

Or

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$

where the contributions of the gas columns h_1 and h_3 have been neglected.