Petroleum Reservoir Simulation

Introduction

Hydrocarbons are mixtures of organic compounds that can exist in different phases as fluids (liquid and gas), according to the pressure and temperature conditions at what they are found. When hydrocarbons are produced from a reservoir and transported to the surface, they experience changes in their pressure and temperature conditions affecting their flowing characteristics and their composition. Understanding of this is necessary for predicting how fluids would behave at any position and conditions of the production system facilities.

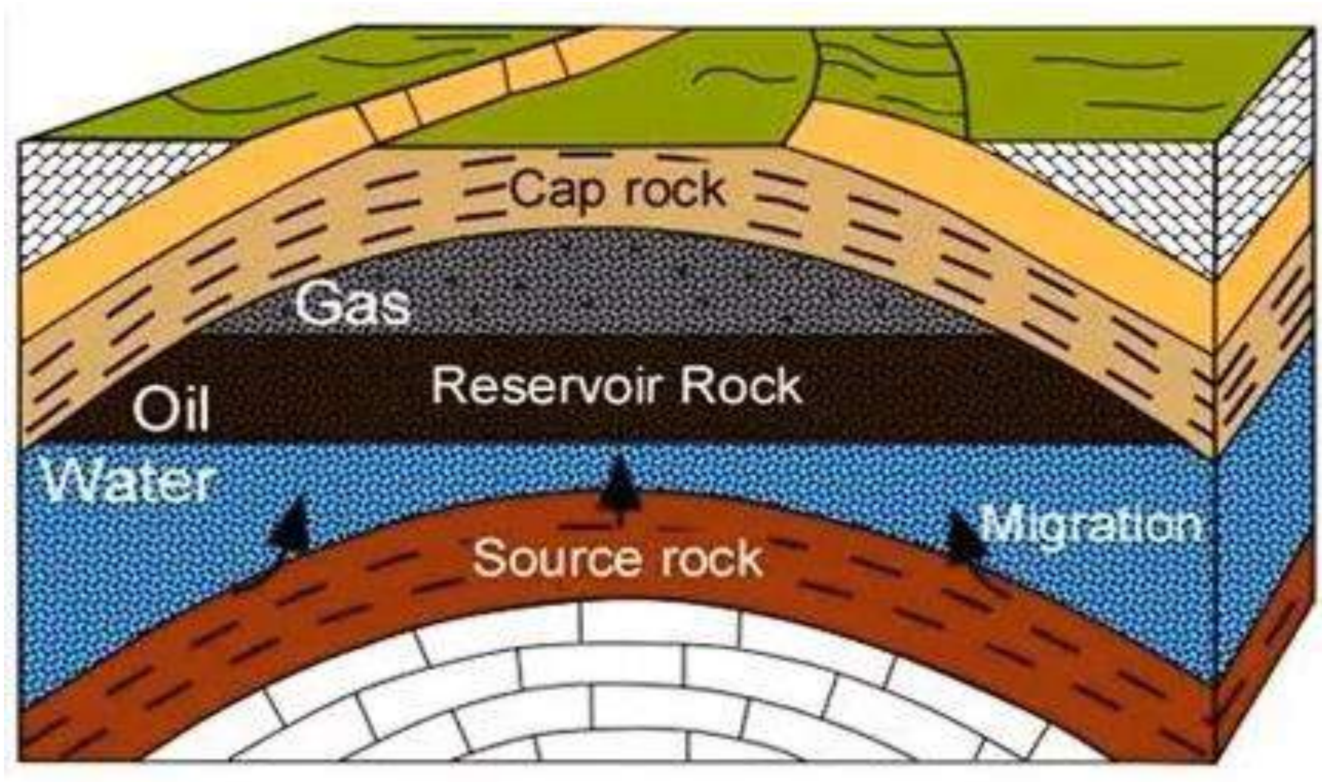
Due to their petro-physical characteristics (porosity, permeability, and water saturation), oil and gas accumulation occurs mainly in sedimentary rocks as limestone, dolomite, and sandstone, where fluids are stored and distributed according to their densities (Figure 1)

Definitions

A petroleum reservoir or oil and gas reservoir: is a subsurface accumulation of hydrocarbons contained in porous or fractured rock formations

Or Is a body of porous and permeable containing oil and gas through which fluids may move toward recovery openings under the pressure existing or that may be applied.

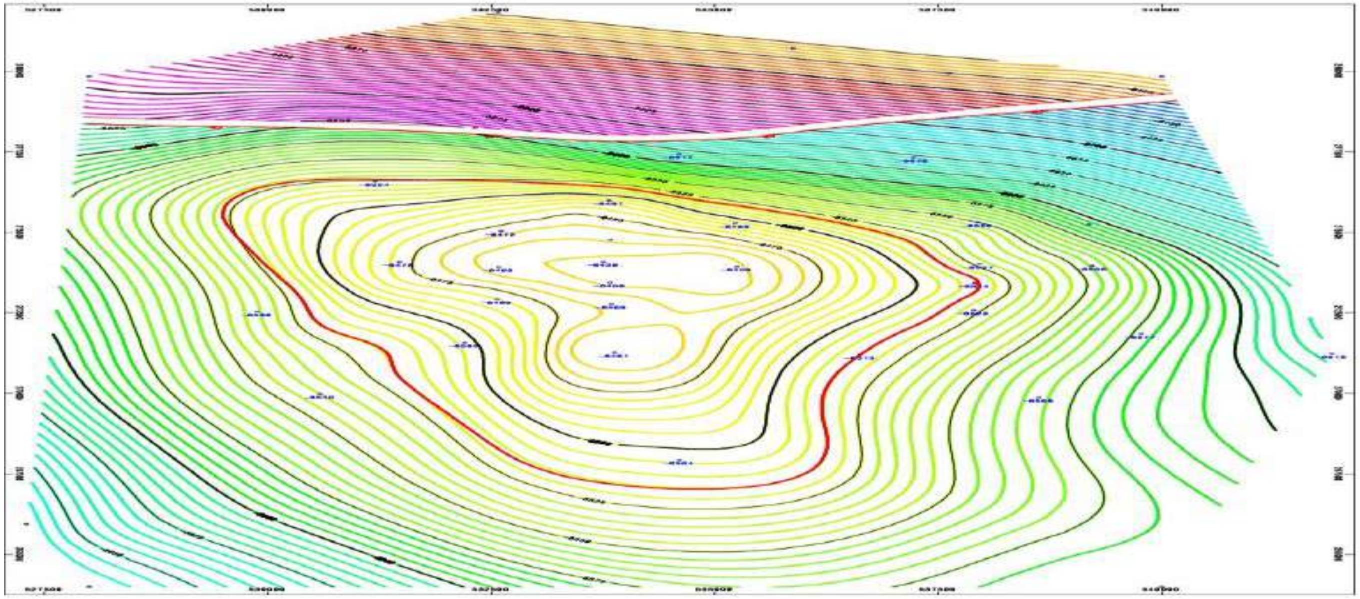
A petroleum reservoir is a term that is used to describe the accumulation of crude oil in a defined location. Usually, the location where the crude oil may have formed is often underground or beneath the sea or ocean floor. These formations are the result of the decomposition of organic matter over the course of centuries.



Figure(1): Initial fluids distribution in an oil reservoir

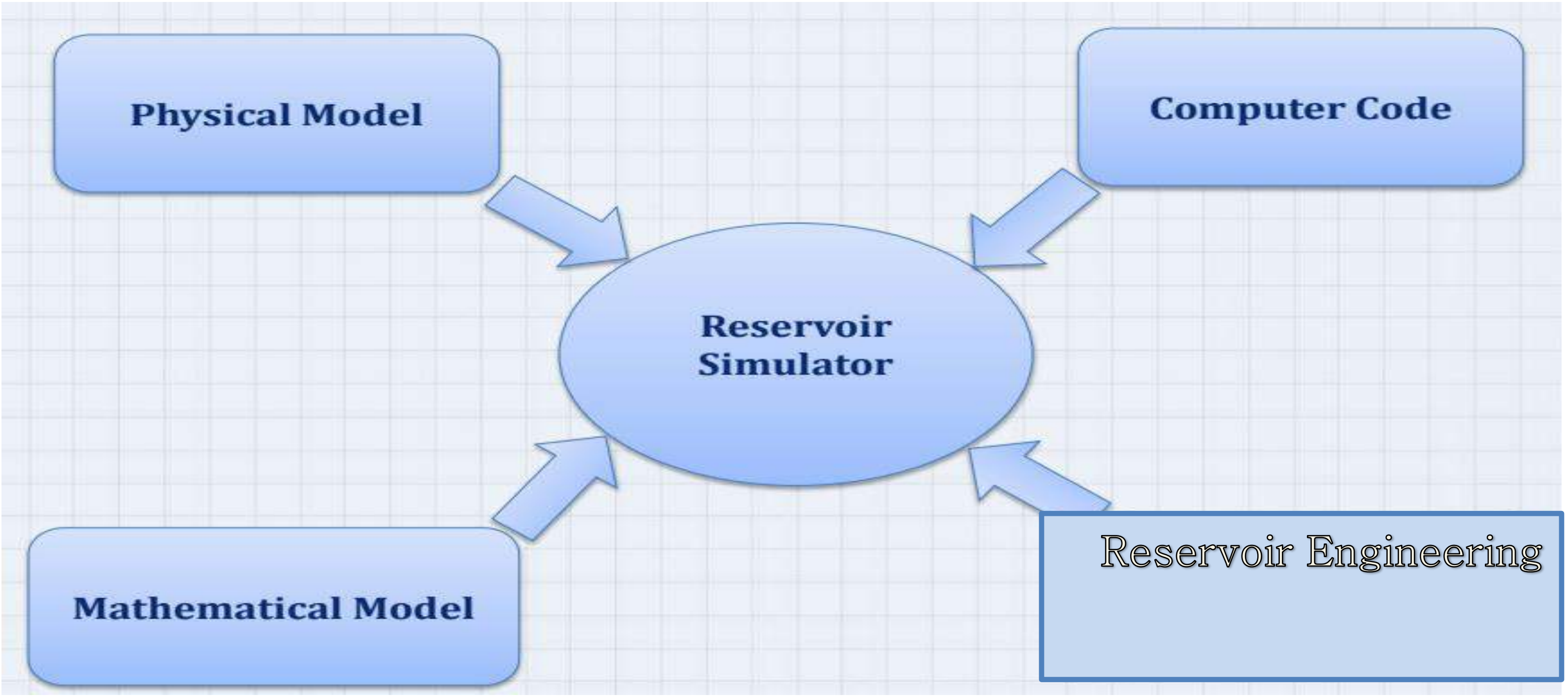
Reservoir simulation

A method of modeling the flow of fluids (typically, oil, water, and gas) through porous materials, and is widely used in the petroleum industry for predicting the behavior of large oil and gas fields as they are exploited. Reservoir simulation is the art of combining: Physics mathematics reservoir engineering, computer programming to develop a tool for predicting



Figure(2): A structure map, generated by contour map software for an 8,500-ft-deep gas and oil reservoir in the Erath field, Erath, Louisiana.

Components of a Reservoir Simulator



Figure(3): Components of reservoir simulator

Reservoir simulation is a widely used tool for making decisions on the development of new fields, the location of infill wells, and the implementation of enhanced recovery projects. It is the focal point of an integrated effort of geosciences, petrophysics, reservoir, production and facilities engineering, computer science, and economics.

Also, this can be used to determine the number and location of wells used to extract material from a reservoir, and to predict which methods of advanced extraction will be needed to fully exploit the reservoir in the most economical fashion.

Reservoir simulation in the oil industry has become the standard for solving reservoir engineering problems and is used for two main purposes:

1- To optimize development plans for new fields
2- To assist with operational and investment decisions.

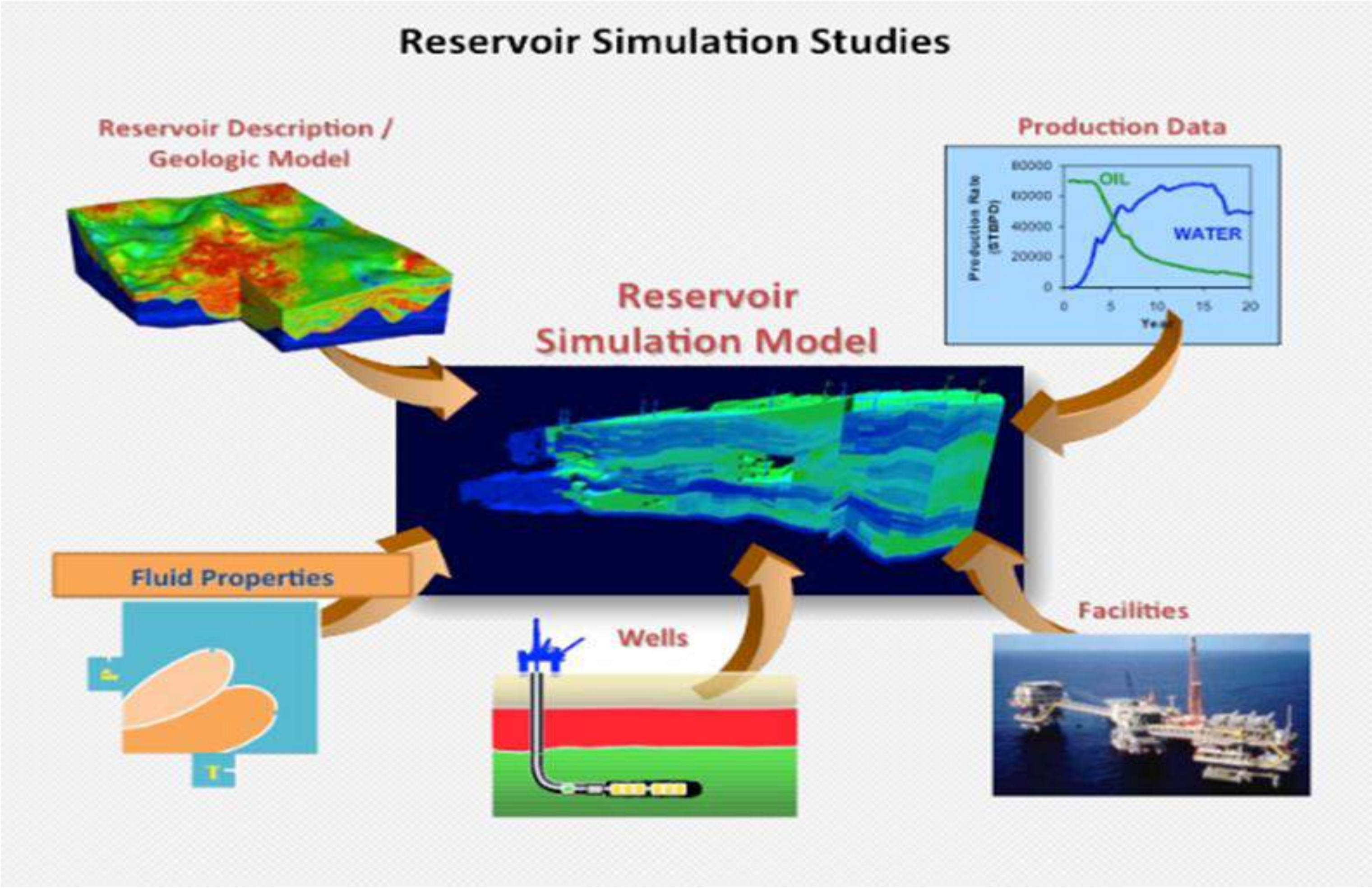
Reservoir simulation components

Basically, reservoir simulation consists of:

1- A geological model in the form of a volumetric grid with cell/face properties that describes the given porous rock formation

2- A flow model that describes how fluids flow in a porous medium, typically given as a set of partial differential equations expressing conservation of mass or volumes together with appropriate closure relations

3- A well model that describes the flow in and out of the reservoir, including a model for flow within the well bore and any coupling to flow control devices or surface facilities



Reservoir Simulation Studies
Reservoir Description / Geologic Model
Production Data
OIL
WATER
Production Rate (STBPD)
80000
60000
40000
20000
0
0
5
10
15
20
Year
Reservoir Simulation Model
Fluid Properties
P
T
Wells
Facilities

a- The complexity of the reservoir because of heterogeneous and anisotropic rock properties .

b- Regional variations of fluid properties and relative permeability characteristics.

c- The complexity of the hydrocarbon-recovery mechanism.

The major steps involved in the development of a reservoir simulator:

* Formulation outlines the basic assumptions inherent to the simulator, states these assumptions in precise mathematical terms, and applies them to a control volume in the reservoir.
* Discretization is the process of converting PDEs into algebraic equations. Several numerical methods can be used to discretize the PDEs; however, the most common approach in the oil industry today is the finite difference method.
* Well representation is used to incorporate fluid production and injection into the nonlinear algebraic equations.

*** Linearization involves approximating nonlinear terms (transmissibility's, production and injection, and coefficients of unknowns in the accumulation terms) in both space and time. Linearization results in a set of linear algebraic equations.**
*** Solution: Any one of several linear equation solvers can then be used to obtain the solution, which comprises pressure and fluid saturation distributions in the reservoir and well flow rates.**
*** Validation of a reservoir simulator is the last step in developing a simulator, after which the simulator can be used for practical field applications. The validation step is necessary to make sure that no errors were introduced in the various steps of development or in computer programming**

To run a reservoir simulation model

you must:

a- Gather and input the fluid and rock (reservoir description) data

b- Choose certain numerical features of the grid (number of grid, blocks, time, etc).

c- Set up the correct field well controls (injection rates, bottom hole pressure, etc).

d- Choose which output you would like to have printed to file.

The simulation model computes the saturation change of three phases (oil, water and gas) and pressure of each phase in each cell at each time step. As a result of declining pressure as in a reservoir depletion study, gas will be liberated from the oil. If pressures increase as a result of water or gas injection, the gas is re-dissolved into the oil phase.

A simulation project of a developed field, usually requires “history matching” where historical field production and pressures are compared to calculated values. The model’s parameters are adjusted until a reasonable match is achieved on a field basis and usually for all wells. Commonly, producing water cuts or water-oil ratios and gas-oil ratios are matched

DERIVATION OF FLUID FLOW EQUATION IN POROUS MEDIA

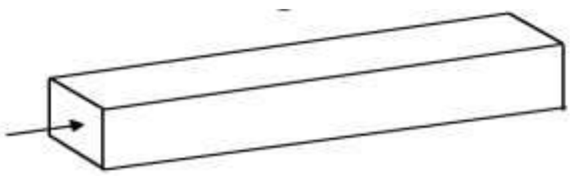
Review of basic steps

Generally speaking, flow equations for flow in porous materials are based on a set of mass, momentum and energy conservation equations, and constitutive equations forthe fluids and the porous material involved. For simplicity, we will in the following assume isothermal conditions, so that we not have to involve an energy conservation equation. However, in cases of changing reservoir temperature, such as in the case of cold water injection into a warmer reservoir, this may be of importance.

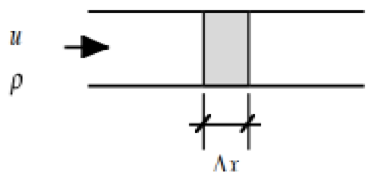
Below, equations are initially described for single phase flow in linear, one dimensional, horizontal systems, but are later on extended to multi-phase flow in two and three dimensions, and to other coordinate systems

Conservation of mass

Consider the following one dimensional rod of porous material



Mass conservation may be formulated across a control element of the slab, with one fluid of density ρ is flowing through it at a velocity u



The mass balance for the control element is then written as

$$\begin{Bmatrix}\text{Mass into the}\\ \text{element at } x\end{Bmatrix} - \begin{Bmatrix}\text{Mass out of the}\\ \text{element at } x+\Delta x\end{Bmatrix} = \begin{Bmatrix}\text{Rate of change of mass}\\ \text{inside the element}\end{Bmatrix},$$

or

$$\{u\rho A\}_x - \{u\rho A\}_{x+\Delta x} = \frac{\partial}{\partial t}\{\phi A \Delta x \rho\}.$$

Dividing by Δx, and taking the limit as Δx approaches zero, we get the conservation of mass, or continuity equation

$$-\frac{\partial}{\partial x}(A\rho u) = \frac{\partial}{\partial t}(A\phi\rho).$$

:

For constant cross sectional area, the continuity equation simplifies to

$$-\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial t}(\phi \rho).$$

Next, we need to replace the velocity term by an equation relating it to pressure gradient and fluid and rock properties, and the density and porosity terms by appropriate pressure dependent functions.

Conservation of momentum

Conservation of momentum is goverened by the Navier-Stokes equations, but is normally simplified for low velocity flow in porous materials to be described by the semi-empirical Darcy's equation, which for single phase, one dimensional, horizontal flow is:

$$u = -\frac{k}{\mu}\frac{\partial P}{\partial x}.$$

Alternative equations are the Forchheimer equation, for high velocity flow

$$-\frac{\partial P}{\partial x} = u\frac{\mu}{k} + \beta u^{n},$$

where n was proposed by Muscat to be 2, and the Brinkman equation, which applies to both porous and non-porous flow

$$-\frac{\partial P}{\partial x} = u\frac{\mu}{k} - \mu\frac{\partial^2 u}{\partial x^2}.$$

Brinkman's equation reverts to Darcy's equation for flow in porous media, since the last term then normally is negligible, and to Stoke's equation for channel flow because the Darcy part of the equation then may be neglected.

In the following, we assume that Darcy's equation is valid for flow in porous media.

Constitutive equation for porous materials

To include pressure dependency in the porosity, we use the following definition of rock compressibility, which for constant temperature is written:

$$c_r = (\frac{1}{\phi})(\frac{\partial \phi}{\partial P})_T.$$

Normally, we may assume that the bulk volume of the porous material is constant,
.i.e

the bulk compressibility is zero. This is not always true, as witnessed by the subsidence in the Ekofisk area.

Constitutive equation for fluids

Recall the familiar fluid compressibility definition, which applies to any fluid at constant temperature:

$$c_f = -\left(\frac{1}{V}\right)\left(\frac{\partial V}{\partial P}\right)_T .$$

Equally familiar is the gas equation, which for an ideal gas is:

$$pV = nRT ,$$

and for a real gas includes the deviation factor, Z:

$$pV = nZRT .$$

These descriptive equations for the fluids are frequently used in reservoir engineering applications. However, for more general purposes, such as in reservoir simulation models

Simple form of the flow equation and analytical solutions

In the following, we will briefly review the derivation of single phase, one dimensional, horizontal flow equation, based on continuity equation, Darcy's equation, and compressibility definitions for rock and fluid, assuming constant permeability and viscosity.

Let us substitute Darcy´s equation into the continuity equation derived above:

$$\frac{\partial}{\partial x}\left(\rho \frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}(\rho\phi)$$

The right hand side (RHS) of the equation may be expanded as:

$$\frac{\partial}{\partial t}(\rho\phi) = \rho\frac{\partial}{\partial t}(\phi) + \phi\frac{\partial}{\partial t}(\rho)$$

Since porosity and density both are functions of pressure only (assuming temperature to be constant), we may write:

and

$$\frac{\partial}{\partial t}(\phi) = \frac{d\phi}{dP}\frac{\partial P}{\partial t}$$

$$\frac{\partial}{\partial t}(\rho) = \frac{d\rho}{dP}\frac{\partial P}{\partial t}.$$

From the compressibility expressions we may obtain the following relationships:

$$\frac{d\rho}{dP} = \rho c_f \text{ and } \frac{d\phi}{dP} = \phi c_r.$$

By substituting these expressions into the equation, we obtain the following form of the right hand side of the flow equation:

$$\frac{\partial}{\partial t}(\rho\phi) = \phi\rho(c_f + c_r)\frac{\partial P}{\partial t}.$$

The left hand side of the flow equation may be expanded as follows:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right) + \frac{k}{\mu}\frac{\partial P}{\partial x}\frac{\partial}{\partial x}(\rho) = \rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right) + \frac{k}{\mu}\frac{\partial P}{\partial x}\frac{d\rho}{dP}\frac{\partial P}{\partial x}$$

For now, let us assume that k=constant and μ=constant. Let us also substitute for

$$\frac{d\rho}{dP} = \rho c_f$$

The LHS may now be written as:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\rho k}{\mu}\left[\frac{\partial^2 P}{\partial x^2} + c_f\left(\frac{\partial P}{\partial x}\right)^2\right].$$

Since c f is small, at least for liquids, and the pressure gradient is small for the low velocity flow we normally have in reservoirs, we make the following assumption:

$$c_f\left(\frac{\partial P}{\partial x}\right)^2 << \frac{\partial^2 P}{\partial x^2}.$$

Then, our LHS simplifies to:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\rho k}{\mu}\frac{\partial^2 P}{\partial x^2}.$$

The complete partial differential flow equation (PDE) for this simple rock-fluid system then becomes:

$$\frac{\partial^2 P}{\partial x^2} = (\frac{\phi \mu c}{k}) \frac{\partial P}{\partial t},$$