## INTRODUCTION

1-1 The Art and Science<br>of Surveying<br>Basis of Surveying<br>Importance of Surveying<br>1-2 The Surveying Method<br>Defining Horizontal and Vertical Directions<br>Measuring Distances and Angles: An Overview<br>Plane and Geodetic Surveying

This textbook is intended to serve as an introduction to the fundamentals of surveying. The purposes of this chapter, and the following two chapters of Part 1, are to present a broad overview of the surveying method, to discuss the importance of surveying as a profession, and to cover some basic concepts regarding measurement, computation, and surveying mathematics. This will give the beginning student a foundation for effective study of the traditional and modern surveying instruments, field and office procedures, and surveying applications that are presented in the following parts of the book.

## 1-1 THE ART AND SCIENCE OF SURVEYING

Simply stated, surveying involves the measurement of distances and angles. The distances may be horizontal or vertical in direction. Similarly, the angles may be measured in a horizontal or vertical plane. Usually distances are measured on a slope, but they must eventually be converted to a corresponding horizontal distance. Vertical distances are also called elevations. Horizontal angles are used to express the directions of land boundaries and other lines.

There are two fundamental purposes for measuring distances and angles. The first is to determine the relative positions of existing points or objects on or near the surface of the earth. The second is to lay out or mark the desired positions of new points or objects that are to be placed or constructed on or near the earth's surface. There are many specific applications of surveying that expand upon these two basic purposes; these applications are outlined in Section 1-3.

Surveying measurements must be made with precision to achieve a maximum of accuracy with a minimum expenditure
of time and money. (We will discuss the terms precision and accuracy in more detail in Section 2-4.)

The practice of surveying is an art because it is dependent upon the skill, judgment, and experience of the surveyor. Surveying may also be considered an applied science because field and office procedures rely upon a systematic body of knowledge, related primarily to mathematics and physics. An understanding of the art and science of surveying is, of course, necessary for surveying practitioners, as well as for those who must use and interpret surveying data (architects, construction contractors, geologists, and urban planners, as well as civil engineers).

## Basis of Surveying

Surveying is based on the use of precise measuring instruments in the field and on systematic computational procedures in the office. The instruments may be traditional or electronic. The computations (primarily of position, direction, area, and volume) involve applications of geometry, trigonometry, and basic algebra.

Electronic handheld calculators and digital computers are used to perform office computations. In the past, surveyors had to perform calculations using trigonometric and logarithmic tables, mechanical calculators, and slide rules. Today, the availability of relatively low-cost electronic calculators, desktop computers, and surveying software (computer programs) relieves the modern-day surveyor from many hours of tedious computations. But it is still very important for the surveyor to understand the underlying mathematical procedures and to be able to perform the step-by-step computations by applying and solving the appropriate formulas.


FIGURE 1-1. Traditional surveying instruments: (a) Theodolite (Courtesy of CST/Berger, Illinois), (b) level, (c) a level rod, and (d) a steel tape.

The traditional measuring instruments used in the field are the transit or theodolite (to measure angles), the level and level rod (to measure vertical distances or elevations), and the steel tape (to measure horizontal distances). They are illustrated in Figure 1-1. The use of these types of instruments is described in detail in subsequent chapters.

Electronic measuring devices have largely replaced traditional instruments in surveying field work. One of the most advanced of these modern instruments is the electronic recording tacheometer, or total station, as it is also called (see Figure 1-2a). It comprises an electronic distance measuring (EDM) device, an electronic theodolite to measure angles, and an automatic data recorder. Many companies provide a "field-to-finish" system (Figure 1-2b), complete with the
computer hardware and software needed to analyze and plot the survey data.

The total station and other modern instruments will be discussed again later on in the text. But the fundamental principles of surveying remain the same, whether the electronic or the more traditional instruments are used. The beginning student must still learn these basic principles before using sophisticated modern instruments. In any event, the steel tape, the transit, and the level are still used for many construction and small-scale surveys. In fact, we shall see later on that the steel tape is more accurate than most electronic devices when it comes to measuring relatively small horizontal distances.

With skillful use of surveying instruments and with proficient application of field and office procedures, almost


FIGURE 1-2. (a) An electronic total-station surveying instrument that can be used to measure and record distances and angles and compute coordinates. (Courtesy of Leica Geosystems, Inc.) (b) In a field-to-finish system, data may be stored electronically. The data can be "dumped" into the office desktop computer for computations and plotting or printing.


FIGURE 1-3. Practically every line recorded on this photograph was laid out with a transit, a steel tape, and a level-the primary equipment of the surveyor. (Courtesy of New Jersey Department of Environmental Protection)
any measurement problem can be solved. Conversely, it is difficult to solve any problem requiring relatively large and accurate measurements without resorting to proper surveying methods and instruments.

## Importance of Surveying

Surveying plays an essential role in the planning, design, layout, and construction of our physical environment and infrastructure. The term infrastructure is commonly used to represent all the constructed facilities and systems that allow human communities to function and thrive productively.

Surveying is the link between design and construction. Roads, bridges, buildings, water supply, sewerage, drainage systems, and many other essential public-works projects could never be built without surveying technology. Figure 1-3 shows a bird's-eye view of a typical urban environment that depends on accurate surveying for its existence. Nearly every detail seen on that photograph was positioned by surveying methods.

In addition to its customary applications in construction and land-use projects, surveying is playing an increasingly important role in modern industrial technology. Some activities that would be nearly impossible without accurate surveying methods include testing and installing accelerators
for nuclear research and development, industrial laser equipment, and other sensitive precision instruments for manufacturing or research. The precise construction of rocket-launching equipment and guiding devices is also dependent on modern surveying.

Without surveying procedures, no self-propelled missile could be built to the accuracy necessary for its operation. Its guiding devices could not be accurately installed; its launching equipment could not be constructed; it could not be placed in position or oriented on the pad; and its flight could not be measured for test or control. Moreover, its launch position and the position of its target would be a matter of conjecture. Surveying is an integral part of every project of importance that requires actual construction.

## 1-2 THE SURVEYING METHOD

The earth, of course, is spherical in shape. This fact, which we take for granted today, was an issue of great debate only a few hundred years ago. But despite the unquestionable roundness of the earth, most surveying activities are performed under the tacit assumption that measurements are being made with reference to a flat horizontal surface. This requires some further explanation.


FIGURE 1-4. The vertical direction is defined as the direction of the force of gravity.

## Defining Horizontal and Vertical Directions

The earth actually has the approximate shape of an oblate spheroid, that is, the solid generated by an ellipse rotated on its minor axis. Its polar axis of rotation is slightly shorter than an axis passing through the equator. But for our purposes, we can consider the earth to be a perfect sphere with a constant diameter. In fact, we can ignore, for the time being, surface irregularities like mountains and valleys. And we can consider that the surface of the sphere is represented by the average level of the ocean, or mean sea level.

By definition, the curved surface of the sphere is termed a level surface. The direction of gravity is perpendicular or normal to this level surface at all points, and gravity is used as a reference direction for all surveying measurements. The direction of gravity is easily established in the field by a freely suspended plumb line, which is simply a weight, or plumb
bob, attached to the end of a string. The direction of gravity is different at every position on the earth's surface. As shown in Figure 1-4, the direction of all plumb lines converge at the center of the earth; at no points are the plumb lines actually parallel.

The vertical direction is taken to be the direction of gravity. Therefore, it is incorrect to define vertical as simply "straight up and down," as many beginning students tend to do. The vertical direction varies from point to point on the earth's surface. The only common factor is the direction of gravity.

By definition, the horizontal direction is the direction perpendicular (at an angle of $90^{\circ}$ ) to the vertical direction of gravity. Because the vertical direction varies from point to point, the horizontal direction also does. A horizontal length or distance, then, is not really a perfectly straight line. It is curved like the surface of the earth. This is illustrated in Figure 1-5.


FIGURE 1-5. A true horizontal distance is actually curved, like the surface of the earth.

## Measuring Distances and Angles: An Overview

As shown in Figure 1-5, a horizontal distance or length is measured along a level surface. At every point along that length, the line tangent to the level surface is horizontal. Horizontal distances may be measured by stretching a steel tape between a series of points along a horizontal line. Electronic distance meters, which use infrared light waves and measure very long distances almost instantaneously, are also used. For most surveys, the curvature of the earth can be neglected, as will be discussed in more detail in the next section. Taping and the use of EDM instruments are discussed in Chapter 4.

A vertical distance is measured along the direction of gravity and is equivalent to a difference in height between two points. When the height is measured with reference to a given level surface such as mean sea level, it is called an elevation.

Vertical distances are usually measured with wooden or fiberglass rods held vertically and graduated in centimeters or hundredths of a foot. An instrument called a level is used to observe the rod at different points. A level consists of a telescopic line of sight, which can be made horizontal by adjusting an attached sensitive spirit bubble. The instrument can be turned in various directions around a stationary vertical axis. As shown in Figure 1-6, the difference in the readings on the rod at two points is equivalent to the difference in height or elevation between the points.

The relative vertical positions of several points separated by long distances can be determined by a continuous series of level rod observations, as illustrated in Figure 1-7. This procedure is called leveling. The line of sight of the level is horizontal at each observation. Because most level rod observations are made with relatively short line-of-sight distances (less than about 300 ft or 90 m ), the effect of the earth's curvature is not at all noticeable. This is explained more thoroughly in the following discussion of plane surveying. In any case, proper leveling methods will compensate for the effects of curvature, as well as for possible instrumental errors. Leveling theory and field procedures are discussed in detail in Chapter 5.


FIGURE 1-6. Measuring a difference in height between a rail and a platform. The difference here is $5.82-1.71=4.11 \mathrm{ft}$.

A horizontal angle is measured in a plane that is horizontal at the point of measurement, as illustrated in Figure 1-8. When a horizontal angle is measured between points that do not lie directly in the plane, like points $A$ and $B$ in Figure 1-8, it is measured between the perpendiculars extended to the plane from those points. (Actually, angles are measured between lines, not points. We will discuss this more thoroughly in the part of the book on angular measurement.)

A vertical angle is measured in a plane that is vertical at the point of observation or measurement. Either the horizontal direction (horizon) or vertical direction (zenith) may be used as a reference line for measuring a vertical angle. In Figure 1-8, $V_{1}$ is the vertical angle between the horizon and the instrument line of sight to point $A$, and $V_{2}$ is the vertical angle between the horizon and the line of sight to point $B$. Both vertical and horizontal angles are discussed in more detail in Chapter 6.

Horizontal and vertical angles are measured with an instrument called a transit or theodolite. This type of instrument consists essentially of an optical line of sight, which is perpendicular to and is supported on a horizontal axis.


FIGURE 1-7. The relative vertical positions of two or more points are determined by leveling.


FIGURE 1-8. Measurement of horizontal and vertical angles.

Theodolites are generally finer in quality and performance (and are more expensive) than transits.

As shown in Figure 1-9, the horizontal axis of the instrument is perpendicular to a vertical axis, about which it can rotate. Spirit levels are used to make the vertical axis coincide with the direction of gravity. Modern instruments use an electronic leveling system. In older instruments, graduated metal circles with verniers or glass circles with micrometers are used to read the angles. In modern theodolites, the circles


FIGURE 1-9. Transit essentials. Schematic diagram of an alidade, which is the upper part of a transit.
are scanned electronically, and the value of the angle is displayed digitally.

## Plane and Geodetic Surveying

We mentioned in the preceding section that most surveying measurements are carried out as if the surface of the earth were perfectly flat. In effect, this means that we make our measurements as if the lines of force due to gravity were everywhere parallel to each other, and as if underneath the irregular ground surface there existed a flat, horizontal reference plane. This is illustrated in Figure 1-10.

The method of surveying based upon this assumption is called plane surveying. In plane surveying, we neglect the curvature of the earth, and we use the principles of plane geometry and plane trigonometry to compute the results of our surveys.

The use of plane surveying methods simplifies the work of the surveyor. And for surveys of limited extent, very little accuracy is lost. Within a distance of about 12 mi , or 20 km , the effect of the earth's curvature on our measurements is so small that we can hardly measure it. In other words, a horizontal distance measured between two points along a truly level (or curved) line is, for practical purposes, the same distance measured along the straight chord connecting the two points. In fact, over a distance of about 12 mi , the difference between the length of arc and the chord length is only about 0.25 in .

This textbook is designed primarily as an introduction to plane surveying, which, for the reason described previously, is suitable for surveys extending over distances less than about 12 mi . But as it turns out, the vast majority of ordinary private surveys are performed well within these limits. Certain public surveys, however, are conducted by federal or state agencies and cover large areas or distances.


FIGURE 1-10. In plane surveying, the curvature of the earth is neglected and vertical distances are measured with reference to a flat plane.

Such large-scale surveys must account for the true shape of the earth so that the required degree of accuracy is not lost in the results.

A survey that takes the earth's curvature into account is called a geodetic survey. These types of surveys are usually conducted by federal agencies such as the U.S. Geological Survey and the U.S. National Geodetic Survey. Various river basin commissions and large cities also perform geodetic surveys. Such surveys generally use very precise instruments and field methods and make use of advanced mathematics and spherical trigonometric formulas to adjust for curvature. In some cases, the instruments and field methods used in a geodetic survey do not differ from those used in a plane survey, but spherical trigonometry must always be used to reduce the geodetic survey data.

The geometry and trigonometry of figures on a curved surface differ considerably from the geometry and trigonometry of plane or flat figures. For example, in a plane triangle, the interior angles always add up to $180^{\circ}$. But this is not the case with a triangle on a curved surface. The triangle shown on the sphere in Figure 1-11, for instance, must contain more than $180^{\circ}$. The sides of that triangle change direction by $90^{\circ}$ at each corner, $A$ and $B$, on the equator. With angle $C$ added to $A$ and $B$, the sum is clearly more than $180^{\circ}$. Spherical trigonometry, then, takes into account the properties of geometric shapes on curved surfaces.

Geodetic surveying methods are generally used to map large areas and to establish large-scale networks of points on the earth for horizontal and vertical control. The relative positions of these points are measured with a high degree of precision and accuracy, both in longitude and in latitude, ${ }^{*}$ as well as in elevation. They are used as points of reference for many other local surveys that require a lower degree of accuracy.

[^0]

FIGURE 1-11. On a curved surface, the sum of the angles in a triangle is more than $180^{\circ}$.

## 1-3 SURVEYING APPLICATIONS

As we mentioned at the beginning of this chapter, the two fundamental purposes for surveying are to determine the relative positions of existing points and to mark the positions of new points on or near the surface of the earth.

Within this framework, many different kinds of surveys are performed. Some specific applications or types of surveys are outlined briefly in this section and are discussed in more detail in Part 3 of the text. Generally, these different types of surveys require different field procedures and varying degrees of precision for carrying out the work.

## Property Survey

A property survey is performed to establish the positions of boundary lines and property corners. It is also referred to as a land survey, title survey, or a boundary survey. Property surveys

# MEASUREMENTS AND COMPUTATIONS 

## CHAPTER OUTLINE

2-1 Units of Measurement
Angles
Distance
Area
Volume
Conversion to SI Metric

2-2 Computations<br>Tools for Computation<br>Significant Figures<br>2-3 Mistakes and Errors<br>Blunders<br>Systematic and Accidental Errors

2-4 Accuracy and Precision
torror of Closure and Relative Accuracy
Questions for Review
Practice Problems

Measurement of distances and angles is the essence of surveying. One of the purposes of this chapter is to discuss the appropriate units of measure for those, and for other related quantities (such as area and volume). Surveyors in the United States must now be able to work with both U.S. Customary units and metric units.

Computation (or data reduction) is also an essential part of surveying; the surveyor must understand the concept of significant figures in the computed, as well as in the measured, quantities. These subjects, as well as the use of modern tools for computation, are discussed in this chapter. We will also discuss the basic types of mistakes and errors that a surveyor must eliminate or minimize in field work. And because no measurement is perfect, we must clarify the meaning and use of the terms accuracy and precision.

## 2-1 UNITS OF MEASUREMENT

Most countries of the world use SI metric units of measurement; SI stands for "Système International." In the United States, a gradual transition from the English or U.S. Customary units to SI units is still in progress. This transition will have a continuing impact on surveying practice. Surveyors in the United States must be able to work in both systems and readily convert from one to the other.

Most measurements and computations in surveying are related to the determination of angles (or directions), distance, area, and volume. The appropriate units of measure for these quantities are discussed here briefly.

## Angles

An angle is simply a figure formed by the intersection of two lines. It may also be viewed as being generated by the rotation of a line about a point, from an initial position to a
terminal position. The point of rotation is called the vertex of the angle. Angular measurement is concerned with the amount of rotation or the space between the initial and terminal positions of the line.

In surveying, of course, the lines do not actually rotate-they are defined by fixed points on or near the ground. It is the line of sight of a transit or theodolite that is rotated about a vertical (or horizontal) axis, located at the vertex of the angle being measured. Angles must be identified properly and labeled clearly, as illustrated in Figure 2-1, to avoid confusion.

Degrees, Minutes, and Seconds There are several systems of angular measurement. The most common is the sexagesimal system, in which a complete rotation of a line (or a circle) is divided into 360 degrees of arc. In this system, 1 degree is divided into 60 minutes, and 1 minute is further divided into 60 seconds of arc.

The symbols for degrees, minutes, and seconds are ${ }^{\circ}$, ', and ", respectively. Some theodolites can measure an angle as small as 1 second of arc. An angle measured and expressed to the nearest second would, for example, be written as $35^{\circ}$ $17^{\prime} 46^{\prime \prime}$ ( 35 degrees, 17 minutes, 46 seconds). A right angle,


Angle A-BC


FIGURE 2-1. The designation $A-B C$ or $A-C B$ shows which of the two angles at point $A$ is being measured or referred to. Clockwise rotation is generally assumed. To simply write "angle $A$ " is usually not sufficient.
the space between two perpendicular lines, is equal to exactly $90^{\circ} 00^{\prime} 00^{\prime \prime}$.

If two angles such as $35^{\circ} 17^{\prime} 46^{\prime \prime}$ and $25^{\circ} 47^{\prime} 36^{\prime \prime}$ are to be added together, the degrees, minutes, and seconds are first combined separately, resulting in $60^{\circ} 64^{\prime} 82^{\prime \prime}$. But this must be converted to $61^{\circ} 05^{\prime} 22^{\prime \prime}$ because $82^{\prime \prime}=01^{\prime} 22^{\prime \prime}$ and $65^{\prime}=$ $1^{\circ} 05^{\prime}$. When subtracting angles, it may be necessary to first "borrow" 60 minutes from a degree and 60 seconds from a minute. For example, to subtract $35^{\circ} 17^{\prime} 46^{\prime \prime}$ from $90^{\circ} 00^{\prime}$ $00^{\prime \prime}$, we must write

$$
\begin{array}{r}
89^{\circ} 59^{\prime} 60^{\prime \prime} \\
-35^{\circ} 17^{\prime} 46^{\prime \prime} \\
\hline 54^{\circ} 42^{\prime} 14^{\prime \prime}
\end{array}
$$

Some handheld calculators accept angular values expressed directly in degrees, minutes, and seconds. With many calculators, however, it is necessary to convert degrees, minutes, and seconds to degrees and decimal parts of a degree, or vice versa. For example, an angle of $35^{\circ} 30^{\prime}$ is equivalent to $35.5^{\circ}$, since $30^{\prime} / 60^{\prime}=0.5^{\circ}$. Likewise, an angle of $142.125^{\circ}$ is equivalent to $142^{\circ} 07^{\prime} 30^{\prime \prime}$, since $0.125^{\circ}=0.125 \times 60^{\prime}=7.5^{\prime}$ and $0.5=$ $0.5^{\prime} \times 60^{\prime \prime}=30^{\prime \prime}$.

Grads The centesimal system of angular measurement is used in some countries. Here, a complete rotation is divided into 400 grades, or grads, written as $400^{\circ}$. The grad is subdivided into 100 parts called centigrads $\left(1^{g}=100^{c}\right)$, and the centigrad is further subdivided into centi-centigrads ( $1^{\mathrm{c}}=$ $100^{\text {cc }}$ ). A right angle $\left(90^{\circ}\right)$ is equivalent to $100^{\circ}$. For an angle expressed as $139.4325^{9}$, the first two digits after the decimal point are centigrads $\left(0.43^{\mathrm{g}}=43^{\mathrm{c}}\right)$, and the second pair of digits represents centi-centigrads $\left(0.0025^{\mathrm{g}}=25^{\mathrm{cc}}\right)$.

Modern scientific handheld calculators can work with angles expressed in degrees or grads; the mode of angle measurement is usually displayed by the calculator as DEG or GRAD. It is most important, of course, to preset the appropriate mode of angle when using the calculator for computations. For conversions, $1^{\mathrm{g}}=0.9^{\circ}$.

Another mode of angular measurement programmed into most calculators is the radian, or rad. By definition, one radian is equivalent to the angle formed between two radii in a circle, when the arc length between the radii is the same as the radius. Since the circumference of a circle is equal to $2 \pi R$ (see Section 3-2), there must be $2 \pi$ (about $2 \times 3.14=6.28$ ) rad in a circle. Therefore, $6.28 \mathrm{rad}=360^{\circ}$ and $1 \mathrm{rad}=57.3^{\circ}$. Radians are used primarily in mathematical formulas and certain surveying computations, but not in field work.

There are other systems for angular measurement that find use in astronomy, navigation, and military applications. In astronomical observations, for example, angles may be measured in terms of hours, minutes, and seconds of time (as a function of the rotation of the earth). This is of significance to the surveyor when "shooting" the sun, or the North Star, and making measurements to determine astronomical north. For military use, the mil is used, where one full circumference is equal to 6400 mils.

## Distance

In the U.S. Customary system, the basic unit for distance or length is the foot, abbreviated as $f t$. A foot is divided into inches and fractions of an inch ( $1 \mathrm{ft}=12 \mathrm{in}$ ), but when the U.S. Customary system is applied in surveying practice, decimal fractions of a foot are typically used instead of inches and fractions of an inch. A distance of 75 ft 3 in , for example, would be expressed by a surveyor as 75.25 ft , since $3 \mathrm{in}=0.25 \mathrm{ft}$. (There is a distinction between the U.S. Survey foot and the international foot, discussed below.)

In the international or SI system of units, the basic unit of length or distance is the meter, abbreviated as $m$. Divisions of the meter include the decimeter $(\mathrm{dm})$, which equals 0.1 m , the centimeter $(\mathrm{cm})$, which equals 0.01 m , and the millimeter $(\mathrm{mm})$, which equals 0.001 m . Decimal fractions of a meter are typically applied in surveying practice, rather than the units of decimeters and centimeters. For example, a distance would be expressed as 26.75 m rather than 27 m 75 cm . Relatively large distances are typically expressed in units of kilometers, abbreviated as $\mathrm{km}(1 \mathrm{~km}=1000 \mathrm{~m})$. A distance of $123,400 \mathrm{~m}$, for example, may be expressed as 123.4 km (see Appendix B on units and conversions).

The international meter was originally defined in 1791 by the French Academy of Sciences as being equivalent to one ten-millionth $(1 / 10,000,000)$ of the distance from the Earth's equator to the North Pole. With improvements in technology, the definition of the meter has evolved over time. In 1983, the meter was officially defined by scientists to be the distance traveled by light in a vacuum in $1 / 299,792,458$ second; this provides a very precise, constant, and universal standard of length for scientists as well as surveyors.

In the United States, the SI system of units is mandatory only for the design of federal government facilities and for geodetic surveys conducted by federal agencies such as the National Geodetic Survey. Many states also use the SI system for highway design and construction layout, but by and large, switching from Customary units to SI units is a voluntary process by surveying and mapping practitioners in the U.S. Because of this, it is often necessary for surveyors to convert distances and coordinates from one system of units to another.

Originally, the conversion relationship between the foot and the international meter was $1 \mathrm{ft}=1200 / 3937 \mathrm{~m}=$ 0.03048006096 m . That is called the U.S. Survey foot. In 1959, the relationship was redefined to be $1 \mathrm{ft}=0.3048 \mathrm{~m}$ (exactly). That is called the international foot. The difference between the two standards is very small (about 2 parts per million or 8 inches in 60 miles), and is of little or no consequence for ordinary plane surveys. Although this textbook is primarily concerned with plane surveying methods, it is important for students to be aware of these refinements in standards of linear measure.

They must be taken into account when making unit conversions related to the coordinates and elevations of points in horizontal and vertical control surveys.

One of the disadvantages of the U.S. Customary system of units is the wide variety of terms used for linear measure. The Gunter's chain, for example, has long been used as a unit of linear measure for land surveys in the United States. One chain is equivalent to 66 ft . One quarter of a chain is called a rod, perch, or pole; each is equivalent to 16.5 ft . The chain contains exactly 100 links.

In the past, the standard width of public roads was set at 2 rd , or 33 ft . Many old deeds state the distances of land boundaries in terms of chains and its fractions, and the entire U.S. Public Land Survey is based on Gunter's chain (see Section 8-1). And in the southwest part of the United States, another unit, called the vara (equivalent to about 33 in), was used in many past surveys.

Following are the relationships among several units of distance in the U.S. Customary system. (These, along with other metric relationships and conversions, are also tabulated in Appendix B for easy reference.)

$$
\begin{aligned}
& 1 \text { foot }(\mathrm{ft})=12 \text { inches }(\mathrm{in} .) \\
& 1 \text { yard }(\mathrm{yd})=3 \text { feet } \\
& 1 \text { mile }(\mathrm{mi})=5280 \text { feet }=80 \text { chains }(\mathrm{ch}) \\
& 1 \text { chain }=66 \text { feet } \\
& 1 \text { rod }(\mathrm{rd})=0.25 \text { chain }=16.5 \text { feet } \\
& 1 \text { link }(\mathrm{lk})=0.01 \text { chain }=7.92 \text { inches }=0.66 \text { feet }
\end{aligned}
$$

Metric Prefixes In the SI metric system, certain prefixes are used along with the meter to define different lengths. For example, the prefix kilo stands for 1000 and the prefix milli stands for $1 / 1000$, or 0.001 . The following SI relationships are useful in surveying practice:

$$
\begin{aligned}
& 1 \text { kilometer }(\mathrm{km})=1000 \text { meters }(\mathrm{m}) \\
& 1 \text { millimeter }(\mathrm{mm})=0.001 \text { meter } \\
& 1 \text { centimeter }(\mathrm{cm})=0.01 \text { meter } \\
& 1 \text { decimeter }(\mathrm{dm})=0.1 \text { meter } \\
& 1 \mathrm{~m}=10 \mathrm{dm}=100 \mathrm{~cm}=1000 \mathrm{~mm}
\end{aligned}
$$

## Area

The unit for measuring area, which expresses the amount of two-dimensional space encompassed within the boundary of a closed figure or shape, is derived from the basic unit of length. In the U.S. Customary system, this is the square foot (sq ft or $\mathrm{ft}^{2}$ ). For land areas, the more common U.S. term for area is the acre (ac), where $1 \mathrm{ac}=43,560 \mathrm{ft}^{2}$.

An acre is also equivalent to 10 sq ch , that is, the area encompassed in a rectangle that is 1 ch wide and 10 ch long $\left(66 \mathrm{ft} \times 660 \mathrm{ft}=43,560 \mathrm{ft}^{2}\right)$. Very large areas are generally expressed in terms of square miles (sq mi or $\mathrm{mi}^{2}$ ). The square yard (sq yd or $\mathrm{yd}^{2}$ ) may be used to express areas for earthwork computations.

In SI metric units, the basic unit for area is the square meter $\left(\mathrm{m}^{2}\right)$. Large land areas may be expressed in terms of either square kilometers ( sq km or $\mathrm{km}^{2}$ ) or by hectares (ha), where 1 ha is equivalent to $10,000 \mathrm{~m}^{2}$. Another metric unit
for area is the are, where 1 are $=100 \mathrm{~m}^{2}$. The following is a summary of the relationships pertaining to area:

$$
\begin{aligned}
& 1 \text { square mile }\left(\mathrm{mi}^{2}\right)=640 \text { acres }(\mathrm{ac}) \\
& 1 \text { acre }=10 \text { square chains }(\mathrm{sq} \mathrm{ch})=43,560 \text { square feet }\left(\mathrm{ft}^{2}\right) \\
& 1 \text { square yard }\left(\mathrm{yd}^{2}\right)=3 \mathrm{ft} \times 3 \mathrm{ft}=9 \text { square feet }\left(\mathrm{ft}^{2}\right) \\
& 1 \text { hectare }(\text { ha })=100 \text { ares }=10,000 \text { square meters }\left(\mathrm{m}^{2}\right) \\
& 1 \text { square kilometer }\left(\mathrm{km}^{2}\right)=100 \text { hectares }=1,000,000 \\
& \text { square meters }\left(\mathrm{m}^{2}\right)
\end{aligned}
$$

The following approximate conversions are useful in surveying applications:

$$
\begin{aligned}
& 1 \mathrm{~km}^{2}=0.386 \mathrm{mi}^{2} \\
& 1 \mathrm{ha}=2.47 \mathrm{ac} \\
& 1 \mathrm{~m}^{2}=1.2 \mathrm{yd}^{2}=10.76 \mathrm{ft}^{2}
\end{aligned}
$$

In Figure 2-2, the relationship between the acre and the hectare is shown to scale. For surveyors in the United States, it is important to "think metric," and to develop an ability to quickly visualize such relationships between the two systems of units. It is better to remember approximate relationships between U.S. and SI units; the exact conversions can always be looked up in a table.

## Volume

The U.S. Customary unit of measure for the volume of a solid is cubic feet $\left(\mathrm{ft}^{3}\right)$, or, more often in surveying, cubic yards $\left(\mathrm{yd}^{3}\right)$. Volume is also expressed in terms of cubic meters $\left(\mathrm{m}^{3}\right)$ in the SI system. Measurement and computation of earthwork volumes to determine the amount of excavation (cut) and embankment (fill) needed for a roadway or site development project constitute a common surveying task. (When the expression "yards" of excavation or fill is used, it really means cubic yards.) It is important to note


FIGURE 2-2. Think metric! There are roughly 2.5 ac in 1 ha of area.


[^0]:    ${ }^{\text {* }}$ Longitude is the angular distance of a point on the earth's surface, measured east or west of the prime meridian at Greenwich, England. Latitude is the angular distance of a point on the earth's surface, measured north or south of the equator.

