# **Rock Slope Engineering 2**

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## 2. Sensitivity analysis

sensitivity analyses using upper and lower bound values



friction angle (range 15–25°) and the water pressure fully drained to fully saturated

This plot shows that water pressures have more influence on stability than the friction angle. That is, a fully drained, vertical slope is stable for a friction angle as low as  $15^{\circ}$ , while a fully saturated slope is unstable at an angle of  $60^{\circ}$ , even if the friction angle is  $25^{\circ}$ 

### 3. Probabilistic design methods

Probabilistic design is a systematic procedure for examining the effect of the variability of each parameter on slope stability

A probability distribution of the factor of safety is calculated, from which the probability of failure (PF) of the slope is determined

There is sometimes reluctance to use probabilistic design when there is a limited amount of design data that may not be representative of the population

The assessment and analysis of available data, by an expert or group of experts in the field

Relationship between levels of annual probability of failure for a variety of engineering projects, and the consequence of failure in terms of lives lost



#### (a) Distribution functions

The most common type of function is the normal distribution in which the mean value is the most frequently occurring value

$$f(x) = \frac{1}{\text{SD}\sqrt{2\pi}} e^{-\frac{1}{2}[(x-\bar{x})/\text{SD}]^2}$$
(1.15)

where  $\bar{x}$  is the mean value given by

$$\bar{x} = \frac{\sum_{x=1}^{n} x}{n} \tag{1.16}$$

and SD is the standard deviation given by

$$SD = \left[\frac{\sum_{x=1}^{n} (x - \bar{x})^2}{n}\right]^{1/2}$$
(1.17)



Properties of the normal distribution: (a) density of the normal distribution with mean  $\underline{x} = 0$  and standard deviations (SD) of 0.25, 0.5 and 1.0; (b) distribution function  $\phi(z)$  of the normal distribution with mean 0 and standard deviation 1.0

The scatter in the data, as represented by the width of the curve, is measured by the standard deviation

### The total area under the curve is equal to 1.0

68% of the values will lay within a range of one standard deviation either side of the mean, and 95% will lay within two standard deviations either side of the mean

 $\phi(z)$  is the distribution function (probability of occurrence) with mean 0 and standard deviation 1.0

For example, a value which has a probability of being greater than 50% of all values is equal to the mean, and a value which has a probability of being greater than 16% of all values is equal to the mean plus one standard deviation

## (b) Probability of failure

The probability of failure is calculated in a similar manner to that of the factor of safety in that the relative magnitude of the displacing and resisting forces in the slope is examined. Two common methods of calculating the coefficient of reliability are the margin of safety method, and the Monte Carlo method.

*margin of safety* is the difference between the resisting and displacing forces, with the slope being unstable if the margin of safety is negative



If the resisting and displacing forces are mathematically defined probability distributions f D(r) and f D(d) respectively, then it is possible to calculate a third probability distribution for the margin of safety. There is a probability of failure if the lower limit of the resisting force distribution f D(r) is less than the upper limit of the displacing force distribution f D(d)

If the resisting and displacing forces are defined by normal distributions, the margin of safety is also a normal distribution, the mean and standard deviation of which are calculated as follows:

Mean, margin of safety = fr - fd (1.18)

Standard deviation, margin of safety =  $(SD_{r}^{2} + SD_{d}^{2})^{1/2}$  (1.19)

where fr and fd are the mean values, and  $SD_r$  and  $SD_d$  are the standard deviations of the distributions of the resisting and displacing forces respectively

For example, if the mean margin of safety is 2000 MN and the standard deviation is 1200 MN, then the margin of safety is zero at (2000 - 0)/1200, or 1.67 standard deviations. From Figure 10(b), where the margin of safety distribution is represented by  $\phi(z)$ , the probability of failure is 5%



The margin of safety concept can only be used where the resisting and displacing forces are independent variables. This condition would apply where the displacing force is the weight of the sliding mass, and the resisting force is the installed reinforcement <u>Monte Carlo</u> analysis is an alternative method of calculating the probability of failure which is more versatile than the margin of safety method

It is work with any mixture of distribution types, and any number of variables, which may or may not be independent of each other

## Monte Carlo steps

1 Estimate probability distributions for each of the variable input parameters.

2 Generate random values for each parameter; Figure 10(b) illustrates the relationship for a normal distribution between a random number between 0 and 1 and the corresponding value of the parameter.

3 Calculate values for the displacing and resisting forces and determine if the resisting force is greater than the displacing force.

4 Repeat the process N times (N > 100) and then determine probability of failure Pf from the ratio:

$$P_f = \frac{N - M}{N}$$

Where M is the number of times the resisting force exceeded the displacing force (i.e. the factor of safety is greater than 1.0)







### 4. Load and Resistance Factor Design

is based on the use of probability theory to develop a rational design basis for structural design that accounts for variability in both loads and resistance

# Limit States Design

First, the structure and its components must **have**, during the intended service life, an **adequate margin of safety** against collapse under the maximum loads that might reasonably occur

Second, the structure and its components must **serve** the designed functions **without excessive deformations and deterioration** 

## Service Levels

*Ultimate limit state*—Collapse of the structure and slope failure including instability due to sliding, toppling and excessive weathering. *Serviceability limit state*—Onset of excessive deformation and unacceptable deterioration.

(resistance) 
$$\phi_k R_{nk} \ge \sum \eta_{ij} \gamma_{ij} Q_{ij}$$
 (load) (1.21)

where  $\varphi$ k is the resistance factor and  $R_{nk}$  is the nominal strength for the *k*th failure mode or serviceability limit state (resistance),  $\eta_{ij}$  is the factor to account for the ductility, redundancy and operational importance of the element or system,  $\gamma_{ij}$  is the load factor and  $Q_{ij}$  the member load effect for the *I* th load type in the *j* th load combination under consideration (load) The objective is to produce a uniform margin of safety for steel and concrete structures such as bridges, and geotechnical structures such as foundations under different loading conditions Mohr–Coulomb equation for the shear resistance of a sliding surface

$$\tau = f_c c + (\sigma - f_U U) f_\phi \tan \phi \qquad (1.22)$$

The cohesion c, friction coefficient tan  $\varphi$  and water pressure U are all multiplied by partial factors with values less than unity, while the normal stress  $\sigma$  on the sliding surface is calculated using a partial load factor greater than unity applied to the slope weight and any applied loads. Actual values for the resistance factors will vary depending on such factors as the extent of testing during construction and ratio of the live load to the dead load. LRFD would usually only be used for slope design where the slope was a component of a bridge foundation, for example

