INTRODUCTION TO STRUCTURE MOTION

In a broad sense, what one attempts to do in rock engineering is to anticipate the motion of a proposed structure under a set of given conditions. The main design objective is to calculate displacements, and as a practical matter, to see whether the displacements are acceptable. Very often restrictions on displacements are implied rather than stated outright. This situation is almost always the case in elastic design where the displacements of the structure of interest are restricted only to the extent that they remain within the range of elastic behavior. In rock mechanics, the "structure" of interest is simply the rock mass adjacent to a proposed excavation. The proposed excavation may be started at the surface, or it may be a deepening of an existing surface excavation, a start of a new underground excavation, or an enlargement of an existing underground excavation. In any case, the excavation plan, if actually carried out, would cause changes in the forces acting in the neighborhood of the excavation and would therefore cause deformation of the adjacent rock mass. Of course, the rock mass may also move in response to other forces such as those associated with earthquakes, equipment vibration, blasting, filling or draining of an adjacent water reservoir, temperature changes, and so on. Regardless of the specific identity of the forces acting, the associated motion must always be consistent with basic physical laws such as the conservation of mass and the balance of linear momentum. In this respect, rock is no different from other materials. Any motion must also be consistent with the purely geometrical aspects of translation, rotation, change in shape, and change in volume, that is, with kinematics. However, physical laws such as Newton's second law of motion (balance of linear momentum) and kinematics are generally not sufficient for the description of the motion of a deformable body. The number of unknowns generally exceeds the number of equations. This mathematical indeterminacy may be removed by adding to the system as many additional equations as needed without introducing additional unknowns. The general nature of such equations becomes evident following an examination of the internal mechanical reaction of a material body to the externally applied forces. The concepts of stress and strain arise in such inquiry and the additional equations needed to complete the system are equations that relate stress to strain. Stress-strain relationships represent a specific statement concerning the nature or "constitution" of material and are members of a general class of equations referred to as constitutive equations. Constitutive equations express material laws. Whereas physical laws and kinematics are common to all materials, constitutive equations serve to distinguish categories of material behavior. Hooke's law, for example, characterizes materials that respond elastically to load. A system of equations that describes the motion of a deformable body necessarily includes all three types of equations: physical laws, kinematics, and material laws. In reality, a system describing the motion of a material body is only an approximation. Mathematical complexities often dictate additional simplification and idealization. Questions naturally arise as to what simplifications should be made and, once made, how well the idealized representation corresponds to reality. Questions of this type relate more to the art than to the science of engineering design and have no final answers. Experience can, of course, be a great aid in this regard, when such experience is informed by a clear understanding of the fundamental concepts.

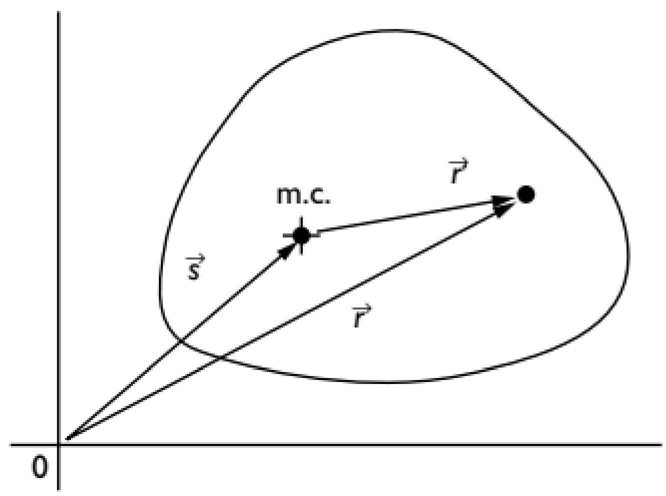


Figure 1 Position relative to the mass center of a body.

Example 1. The mass center of a body is defined by

$$M\vec{s} = \int_V \vec{r}\,\mathrm{d}m$$

where $\vec{s}$ and $\vec{r}$ are vectors shown in Figure 1. Differentiation twice with respect to time gives the interesting result

$$M\ddot{\vec{s}} = \int_B \vec{a}\,\mathrm{d}m = \int_V \rho\vec{a}\,\mathrm{d}V = \dot{\vec{P}} = F$$

that shows that the mass center moves as if it were a particle accelerating *a* according to Newton's second law (resultant of external forces = time rate of change of linear momentum). Thus, even though one cannot determine the acceleration everywhere in the body of interest at this juncture, there is a possibility of at least following the motion of the center of mass of the body. This fact remains true even if the body disintegrates. Consider a mass M of rock in a landslide or avalanche and suppose that the external forces are: (i) the weight *W* of the slide mass and (ii) the contact force *R* acting between the slide mass and the parent rock mass from which the slide mass has become detached. The contact force *R* may be frictional, viscous, and displacement-dependent, that is, $R = R_0 + R_1 ds/dt - R_2 s$. *R* increases with speed but decreases with a displacement of the mass center and resists the downhill component of weight $D = D0$. According to the previous result

$$F = D - R = M\ddot{s}$$

Hence

$$M\ddot{s} + R_1\dot{s} - R_2 s = D_0 - R_0$$

describes the motion of the slide mass center. The coefficients in this equation may depend on time and position (except *M*). Over a short period of time, however, a reasonable assumption is that they are constant in which case the form of the solution is known. Reasonable initial conditions are that the slide mass is at rest or moving at a constant speed (steady creep). The logic followed in Example 1 illustrates in a very compact form how one proceeds from physical laws (conservation of mass, Newton's second law) through problem simplification (look at mass center motion only, disregard deformation, disintegration, the motion of individual elements) and material idealization (assumptions concerning resistance) to a mathematically tractable representation of the original problem (landslide dynamics). Of course, simplification is a relative notion. Here simplification means one has progressed from an essentially hopeless situation to a situation where useful information may be extracted. In this example, useful information might refer to the estimation of slide mass travel in conjunction with zoning regulations for geologic hazards. The solution effort required may still be considerable. However, there may also be unexpected benefits. In this example, the "triggering" of catastrophic landslides under load level fluctuations that were formerly safe becomes understandable in relatively simple physical terms.

A PRACTICAL DESIGN OBJECTIVE

A tacit assumption in rock mechanics that is often made in the absence of inelastic behavior is that large displacements accompanying failure are precluded. Under these circumstances, design is essentially an analysis of safety and stability. A practical design objective is then to calculate a factor of safety appropriate for the problem at hand. An appropriate factor of safety depends on the problem and is an empirical index to "safety" or "stability." Safety and stability are often used interchangeably in rock mechanics, although strictly speaking, they are not synonymous. Stability often connotes a possibility of fast failure or the onset of large displacements below the elastic limit. An example is strata buckling where kinks in thin laminations may form suddenly below the yield point. Safety typically relates to strength and nearness to the elastic limit or yield point. If forces are of primary concern, then a ratio of forces resisting the motion to forces that tend to drive the motion is an appropriate safety factor. If rotation is of primary concern, then a ratio of resisting to driving moments would be an appropriate safety factor. When yielding at a point is of interest, then a ratio of "strength" to "stress" defines a useful safety factor when measures of "strength" and "stress" are well defined.

Example 2. Consider a rock mass high on a steep canyon wall that may pose a threat to the facilities below. A reasonable index to stability is a factor of safety *FS* defined as a ratio of forces tending to drive the slide mass downhill to forces resisting the motion. Show that a safety factor greater than one implies safety.

Solution: By definition, $FS = R/D$. The mass center then moves according to

$$F = D - R = D\,(1 - FS) = M\ddot{s}$$

Hence, a safety factor less than one implies downhill acceleration, while a safety factor greater than one, implies stability (uphill acceleration is physically meaningless in this situation).

Example 3. Stress concentration about vertical shaft results in compressive stress of 8,650 psi (59.66 MPa) at the shaft wall where the rock mass has an unconfined compressive strength of 12,975 psi (89.48 MPa). Determine the shaft wall safety factor at the considered point.

Solution: An appropriate safety factor at the shaft wall is the ratio of strength to stress.

Thus,

$$\begin{aligned} FS_c &= \frac{\text{Strength}}{\text{Stress}} \\ &= \frac{C_o}{\sigma_c} \\ &= \frac{12{,}975}{8{,}650} = \frac{89.48}{59.66} \\ FS_c &= 1.50 \end{aligned}$$

PROBLEM SOLVING

This text has been written from the point of view that whenever the main physical features of a problem are well known, as they are in the determination of tunnel support requirements, for example, then the strength of materials background should be sufficient for the development of quantitative analysis procedures. The emphasis throughout is upon the time-tested engineering approach to problem-solving requiring (i) a brief statement as to what is being required; (ii) a listing of related known data; (iii) a sketch of the "structure" for analysis shows, in particular, the applied loads and reactions; (iv) the equations and assumptions used; and (v) an outline of the major calculation steps taken in obtaining the desired results. Some of these steps may be combined as conditions allow.

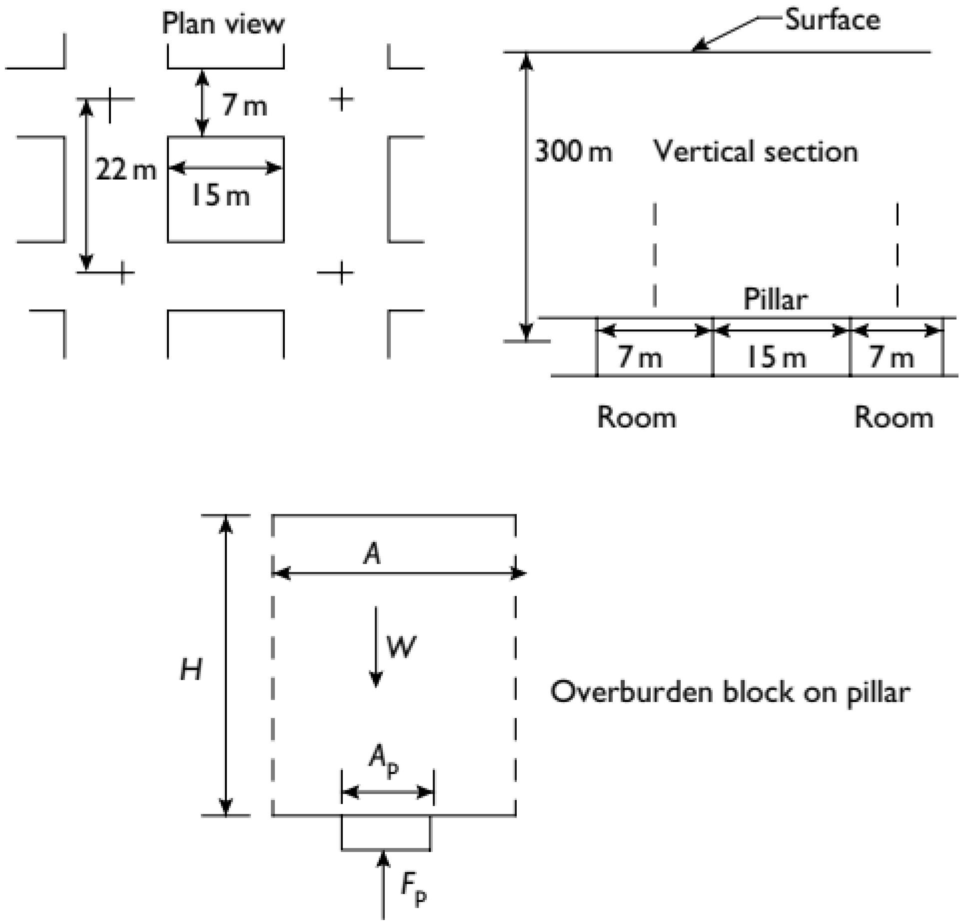
Example 4. A large array of square support pillars is formed by excavating rooms in a horizontal stratum 5 m (16.4 ft) thick at a depth of 300 m (984 ft). The pillars are 15 m (49.2 ft) on edge and are spaced on 22 m (72.2 ft) centers. Determine the average vertical stress in the pillars.

Solution: A large array implies that the pillars in the array are similar, so consideration of the equilibrium of one pillar should reveal the relationship of forces acting at equilibrium. The pillars are the materials that remain after the rooms have been excavated, and must carry the weight of the overburden that, prior to excavation, was supported by all materials in the seam.

1 Sketch the geometry of the problem in plan view and vertical section.

2 Apply force equilibrium in the vertical direction. Thus, $W = Fp$ as shown in the sketch where W = (specific weight) (volume) and Fp = (average vertical pillar stress) (pillar area). One may estimate the overburden-specific weight as, say, 24.8 kN/m^3(158 pcf).

3 Do calculations. (24.8) (300) (22) (22) = *Sp* (15) (15), so *Sp* = 16 kN/m^2 (2,320 psi).



Sketches not to scale

UNITS

No one system of units is more "scientific" than another, many texts now use metric (SI) units and English engineering units. Both are also used in this text.

Both view Newton's second law of motion as $F = ma$ where F is the result of external forces (pound-force, Newton), m is a mass (slug, kilogram) and a is acceleration (feet/second^2, meter/second^2). A 1-pound force results when 1 slug is accelerated 1 ft/s^2. Thus, 1 lbf = slug.ft/s^2. (A slug is a 32.174 pound mass and 1 lbf = 1 lbm.ft/s^2.) A 1 N force results when 1 kg is accelerated 1 m/s^2, so 1 N = kg.m/s^2. Sometimes both units are given with one in parentheses following the other (Example 4). Tables of data may be given in either system. Sometimes conversion factors are given with a table of data when both units are not presented, but not always. Some useful conversion factors are given in Table 1.

Example 5. An estimate of pre-excavation vertical stress is 1 psi per foot of depth. This estimate is based on an assumed overburden-specific weight of 144 pcf. An improved estimate would use 158 pcf or 1.1 psi/ft, that is, $Sv = 1.1h$ where Sv is the vertical stress in psi and h is depth in feet. Modify this last equation to give Sv in kN/m^2 with h in meters.

Solution: The estimate in detail is:

Sv (kN/m^2) = 1.1(psi/ft) 6894.9 (N/m^2/psi) 3.281(ft/m) 10^{-3} (kN/N) h(m), that is,

Sv (kN/m^2) = 24.9h(m).

Table 1 Conversion factors[a]

Feet (ft)[b]	0.3048	Meters (m)
Inches (in.)	2.54	Centimeters (cm)
Meters (m)	3.2808	Feet (ft)
Centimeters (cm)	0.3937	Inches (in.)
Pound force (lbf)	4.4482	Newton (N)[c]
Newton (N)	0.2248	Pound force (lbf)
lbf/square inch (psi)	6894.8	Newton/square meter
Newton/square meter	$1.4504\ (10^{-4})$	lbf/square inch (psi)
lbf/ft^3 (pcf)	157.09	N/m^3
N/m^3	$6.366\ (10^{-3})$	lbf/ft^3 (pcf)

Notes

a Multiply units on the left by the number in the middle column to obtain units on the right.

b Abbreviations for units are not terminated with a period with the exception of inches.

c $N/m^2 = 1$ Pascal, $1\ kN/m^2 = 1$ kPa, $1\ MN/m^2 = 1$ MPa, k = kilo (10^3), M = mega (10^6). Also: 1 bar = 14.504 psi = 100 kPa, 1 atm = 14.7 psi.

Alternatively, from Table 1 and the given data (158 pcf) (157.09) = 24.8 (103) N/m^3 = 24.8 kPa/m within truncation and round off error.

Example 6. Given the specific gravity (SG) of a rock sample as 2.67, determine the specific weight γ in lbf/ft^3 and kN/m^3.

Solution: By definition, *SG* is the ratio of a given mass to the mass of water occupying the same volume at the same temperature. This definition is thus the ratio of the mass density of the given material ρ to the mass density of water ρw at the given temperature. Thus, $SG = \rho/\rho w$. Also by definition specific weight γ is the weight of a unit volume of material, and weight is the force of gravity acting on the given mass, so that $\gamma = \rho g$. Hence, $\gamma = \rho g = (SG)\ (\rho w)\ g = (SG)\ (\gamma w)$ where the last term is the specific weight of water which is 62.43 lbf/ft^3. Thus, $\gamma = 2.67(62.43) =$ 167 lbf/ft^3 and from the conversion factor in Table 1 $\gamma = (167\ lbf/ft^3)\ (157.09)\ (10^{-3})$ $= 26.2\ kN/m^3$.