Mohr's Circle for Plane Stress

Dr. Azealdeen Salih Al-Jawadi

Department of Mining Engineering

College of Petroleum and Mining Engineering

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The German civil engineer Otto Christian Mohr (1835–1918) developed a useful graphical interpretation of the stress transformation equation. This method is known as Mohr’s circle.

Utility of Mohr’s Circle

Mohr’s circle can be used to determine stresses acting on any plane passing through a point. It is quite convenient for determining principal stresses and maximum shear stresses (both in-plane and absolute maximum shear stresses). If Mohr’s circle is plotted to scale, measurements taken directly from the plot can be used to obtain stress values. However, Mohr’s circle is probably most useful as a pictorial aid for the analyst who is performing analytical determinations of stresses and their directions at a point.

Sign Conventions Used in Plotting Mohr's Circle

normal stresses	horizontal coordinates	σ axis
shear stresses	vertical coordinates	τ axis

Normal Stresses. Tensile normal stresses are plotted on the right side of the τ axis, and compressive normal stresses are plotted on the left side of the τ axis. In other words, tensile normal stress is plotted as a positive value (algebraically) and compressive normal stress is plotted as a negative value.

Shear Stresses. A unique sign convention is required to determine whether a particular shear stress plots above or below the σ axis. The shear stress τ_{xy} acting on the x face must always equal the shear stress τ_{yx} acting on the y face. If a positive shear stress acts on the x face of the stress element, then a positive shear stress will also act on the y face, and vice versa. For shear stress, therefore, an ordinary sign convention (such as positive τ plots above the σ axis and negative τ plots below the σ axis) is not sufficient because

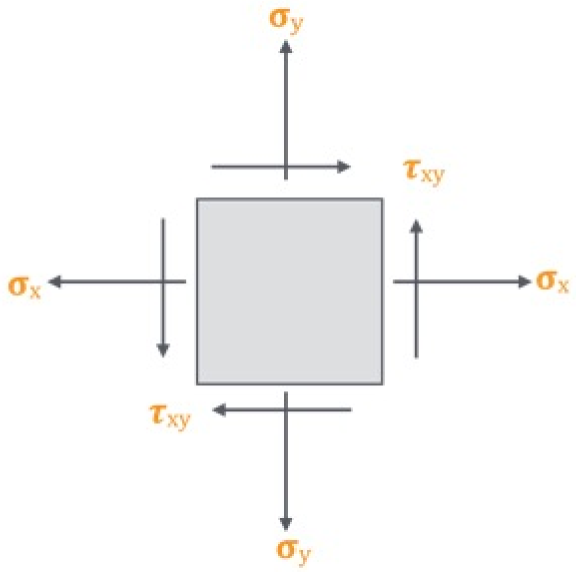
(a) the shear stresses on both the x and y faces will always have the same sign and

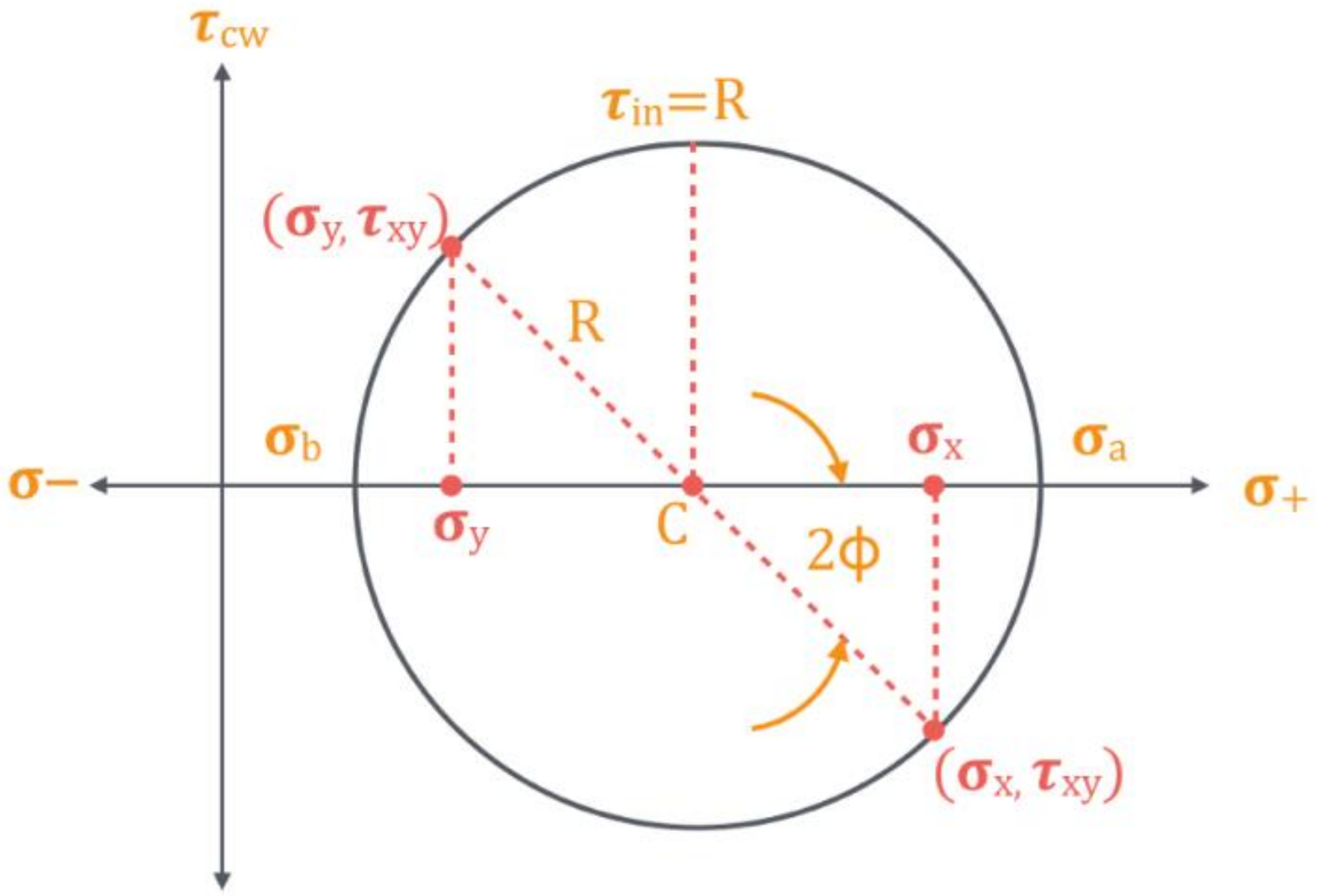
(b) the center of Mohr's circle must be located on the σ axis.

To determine how a shear stress value should be plotted, one must consider both the face that the shear stress acts on and the direction in which the shear stress acts:

• If the shear stress acting on the face of the stress element tends to rotate the stress element in a clockwise direction, then the shear stress is plotted above the σ axis.

• If the shear stress tends to rotate the stress element in a counterclockwise direction, then the shear stress is plotted below the σ axis.



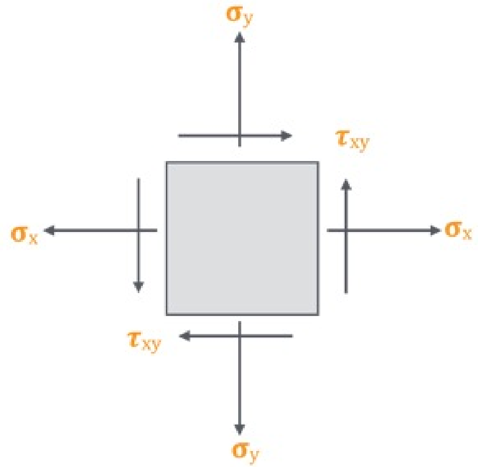


τ_{cw}
τ_{in}=R
(σ_y, τ_{xy})
R
σ_b
σ_x
σ_a
$\sigma-$
$\sigma+$
σ_y
C
2ϕ
(σ_x, τ_{xy})

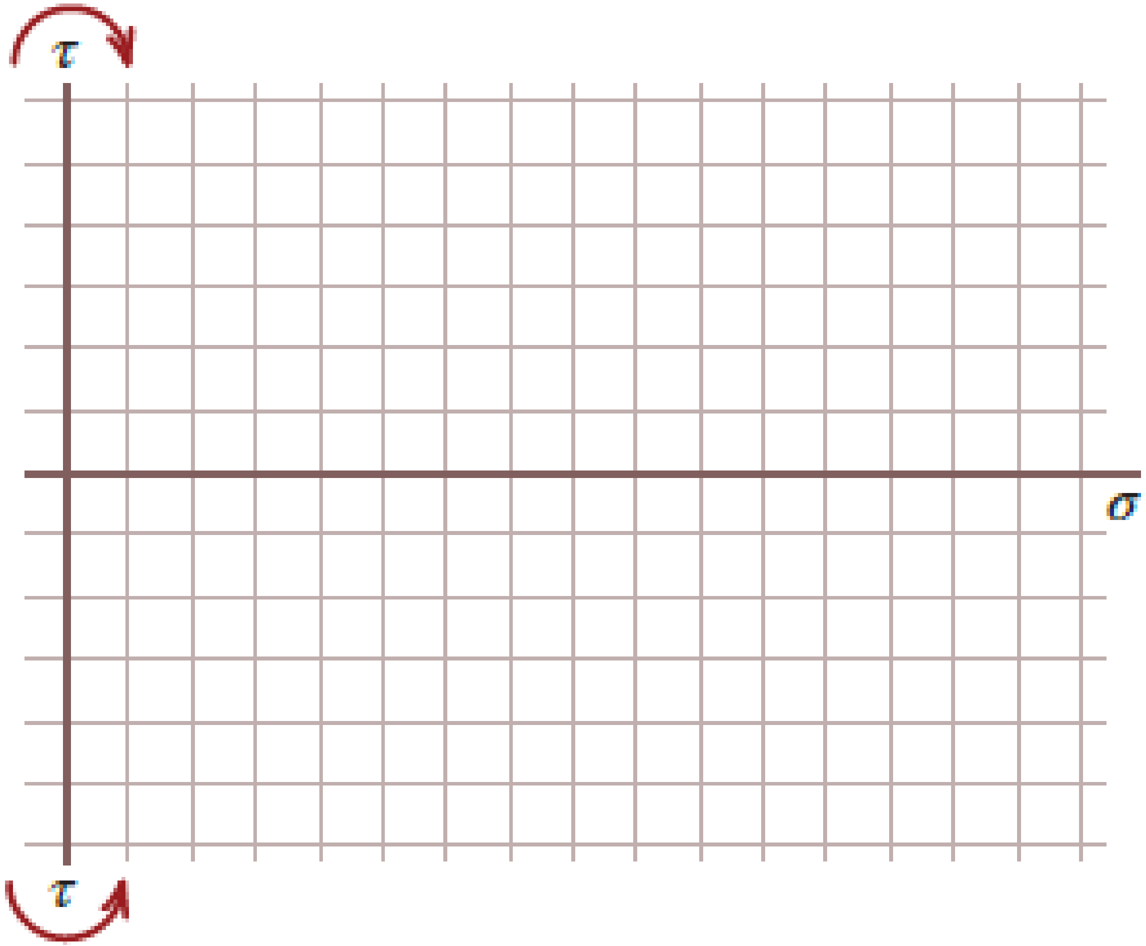
Basic Construction of Mohr's Circle

Mohr's circle can be constructed in several ways, depending on which stresses are known and which stresses are to be found. To illustrate the basic construction of Mohr's circle for plane stress, assume that stresses σx, σy, and τxy are known. Then, the following procedure can be used to construct the circle:

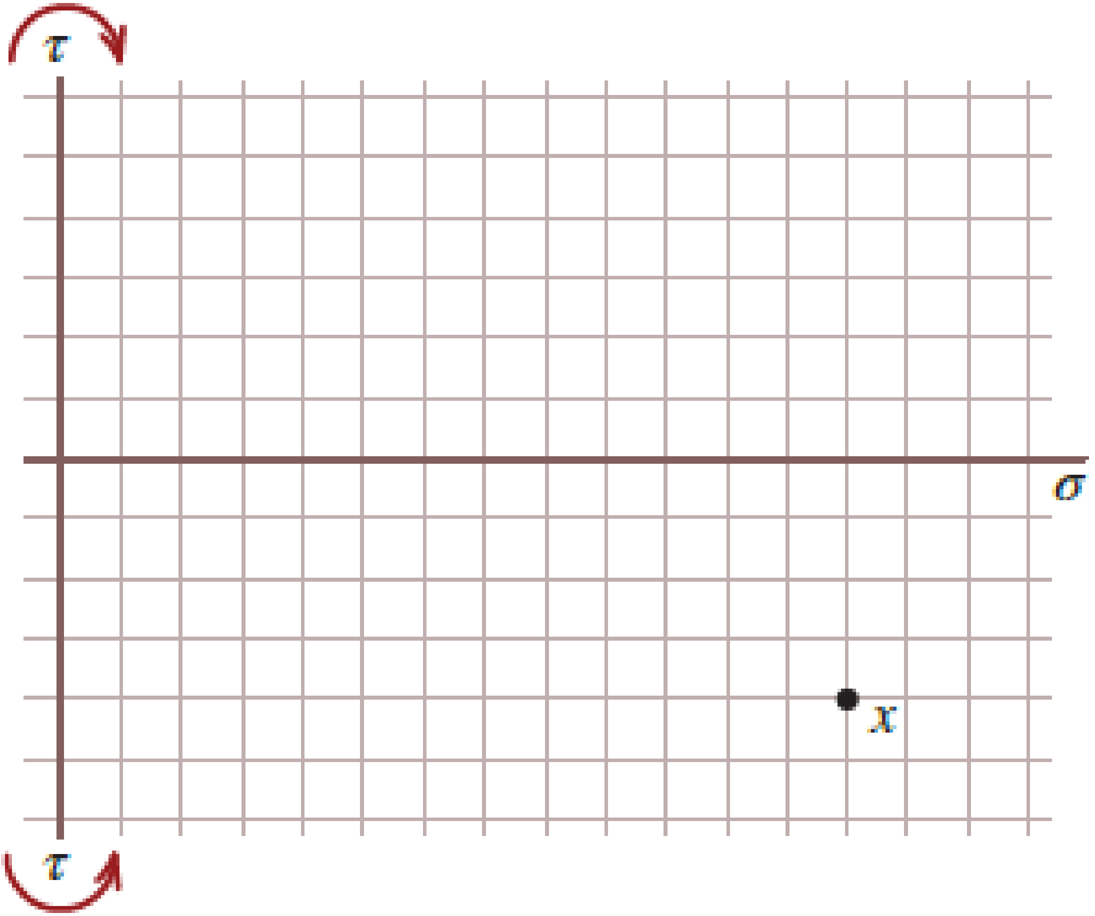
1. Identify the stresses acting on orthogonal planes at a point. These are usually the stresses σx, σy, and τxy acting on the x and y faces of the stress element. It is helpful to draw a stress element before beginning the construction of Mohr's circle.



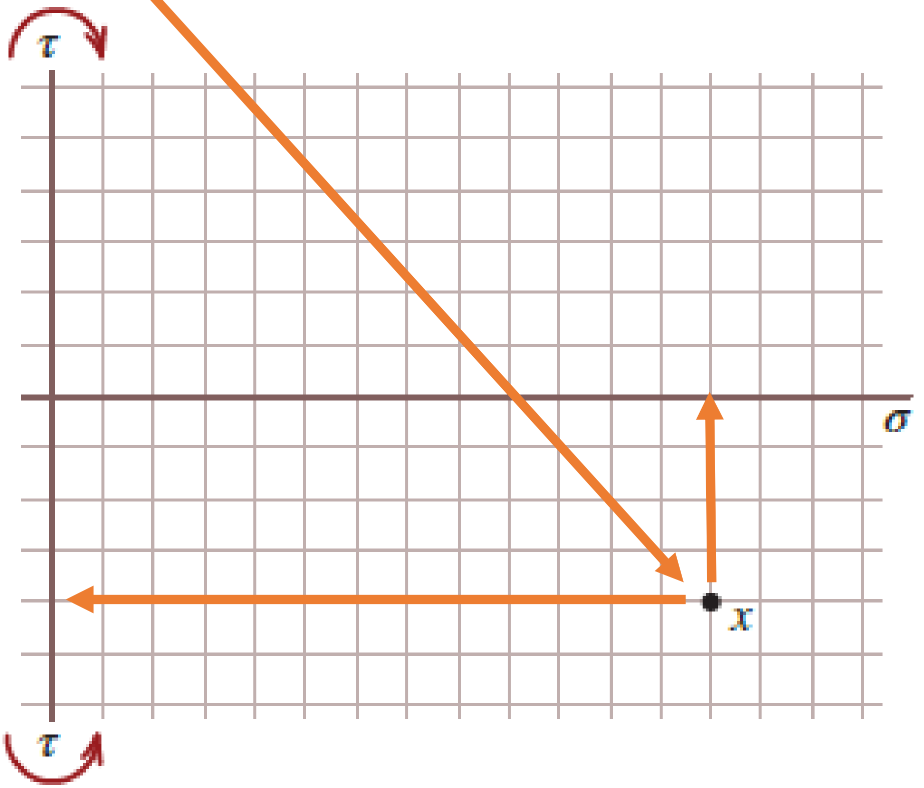
2. Draw a pair of coordinate axes. The σ axis is horizontal. The τ axis is vertical. It is not mandatory, but it is helpful, to construct Mohr's circle at least approximately to scale. Pick an appropriate stress interval for the data, and use the *same interval* for both σ and τ. Label the upper half of the τ axis with a clockwise arrow. Label the lower half with a counterclockwise arrow. These symbols will help you remember the sign convention used in plotting shear stresses.



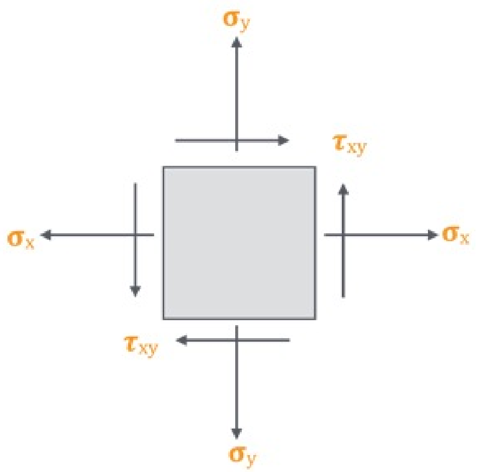
3. Plot the state of stress acting on the x face. If σx is positive (i.e., tension), then the point is plotted to the right of the τ axis. Conversely, a negative σx plots to the left of the τ axis. Correctly plotting the value of τxy is easier if you use the clockwise–counterclockwise sign convention. Look at the shear stress arrow on the x face. If this arrow tends to rotate the stress element clockwise, then plot the point above the σ axis. For the stress element shown here, the shear stress acting on the x face tends to rotate the element counterclockwise; therefore, the point should be plotted below the σ axis.

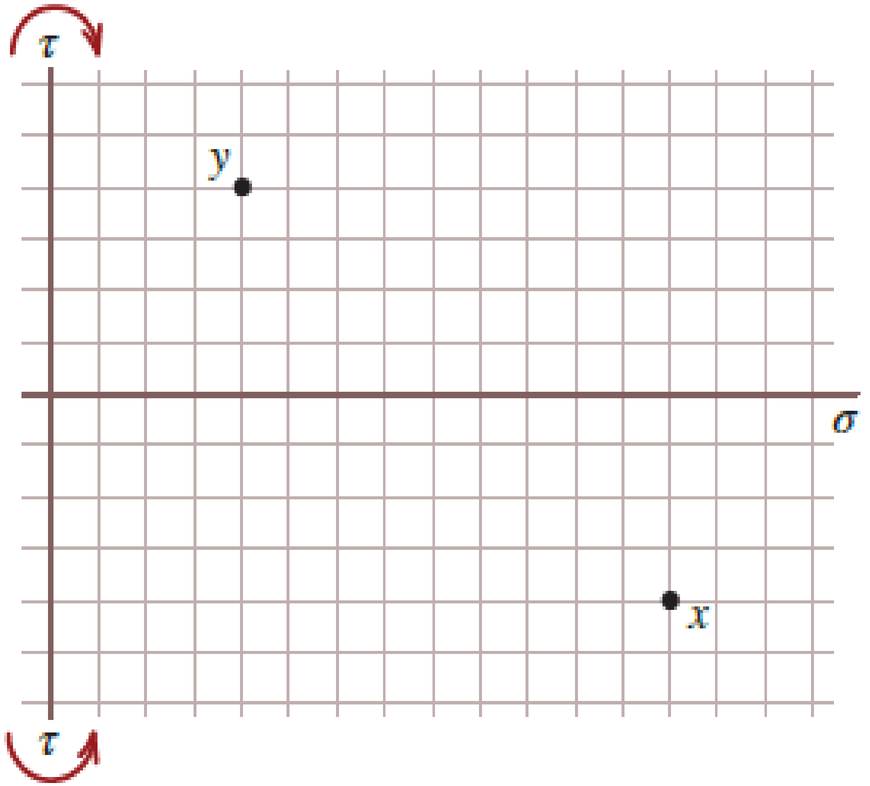


4. Label the point plotted in step 3 point x. This point represents the combination of normal and shear stress on a specific plane surface, specifically the x face of the stress element. Keep in mind that the coordinates used in plotting Mohr's circle are not spatial coordinates like x and y distances, which are more commonly used in other settings. Rather, the coordinates of Mohr's circle are σ and τ. To establish orientations of specific planes by using Mohr's circle, we must determine angles relative to some reference point, such as point x, which represents the state of stress on the x face of the stress element. Consequently, it is very important to label the points as they are plotted.

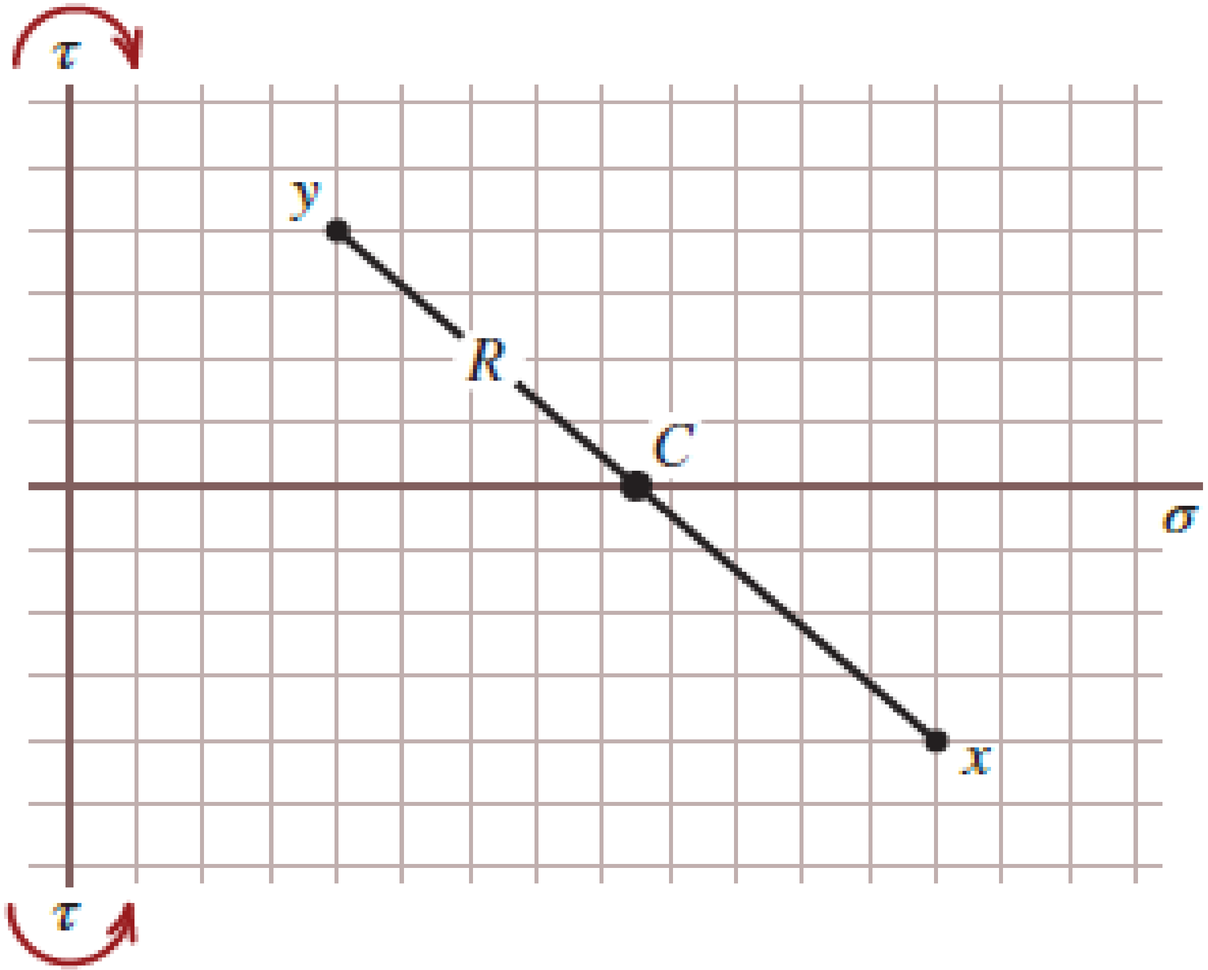


5. Plot the state of stress acting on the y face. Look at the shear stress arrow on the y face of the stress element shown in the previous figure. This arrow tends to rotate the element clockwise; therefore, the point is plotted above the σ axis. Label this point y, since it represents the combination of normal and shear stress acting on the y face of the stress element. Notice that points x and y are both the same distance away from the σ axis-one point is above the σ axis, and the other point is below. This configuration will always be true because the shear stress acting on the x and y faces must always have the same magnitude.





6. Draw a line connecting points x and y. The location where this line crosses the σ axis marks the center C of Mohr's circle. The radius R of Mohr's circle is the distance from center C to point x or to point y. Moreover, as shown by Equation (6), the center C of Mohr's circle will always lie on the σ axis.



$$\boxed{(\sigma_n - C)^2 + \tau_{nt}^2 = R^2} \quad \text{Equation (6)}$$

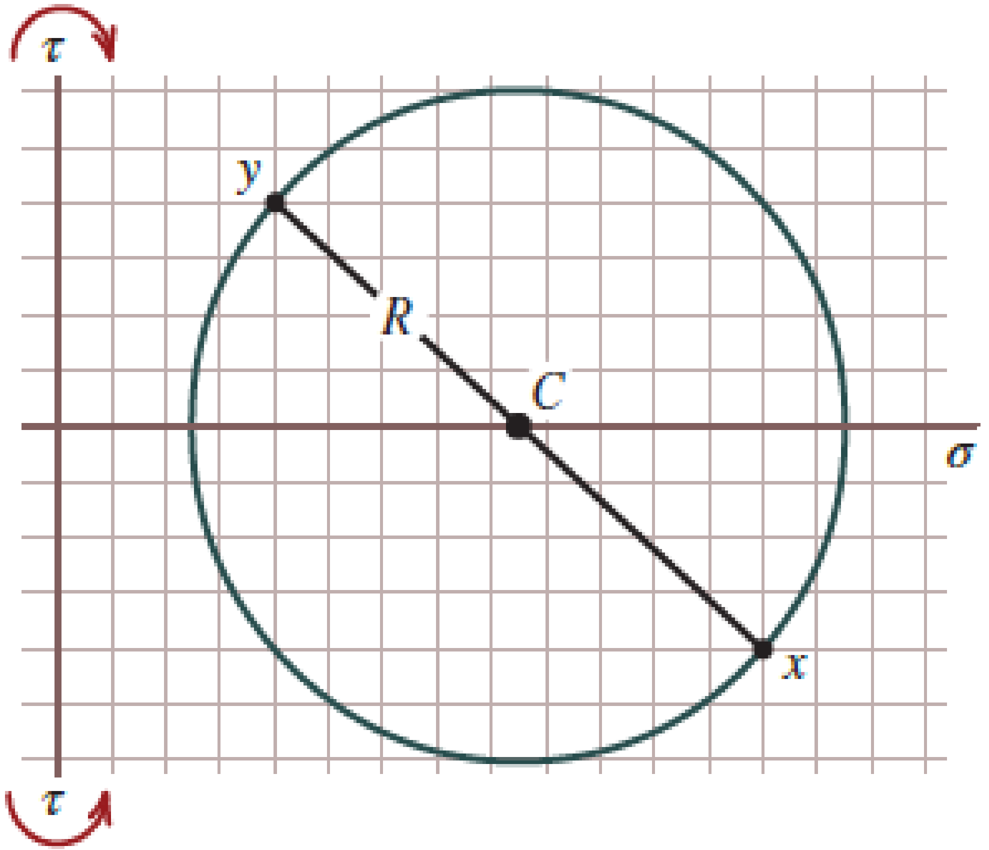
7. Draw a circle with center C and radius R. Every point on the circle represents a combination of σ and τ that exists at some orientation. The equations used to derive Mohr's circle [Equations (1) and (2)] were expressed in terms of double-angle trigonometric functions. Consequently, all angular measures in Mohr's circle are double angles 2θ. Points x and y, which represent stresses on planes 90° apart in the x–y coordinate system, are 180° apart in the σ-τ coordinate system of Mohr's circle. Points at the ends of any diameter represent stresses on orthogonal planes in the x–y coordinate system.

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

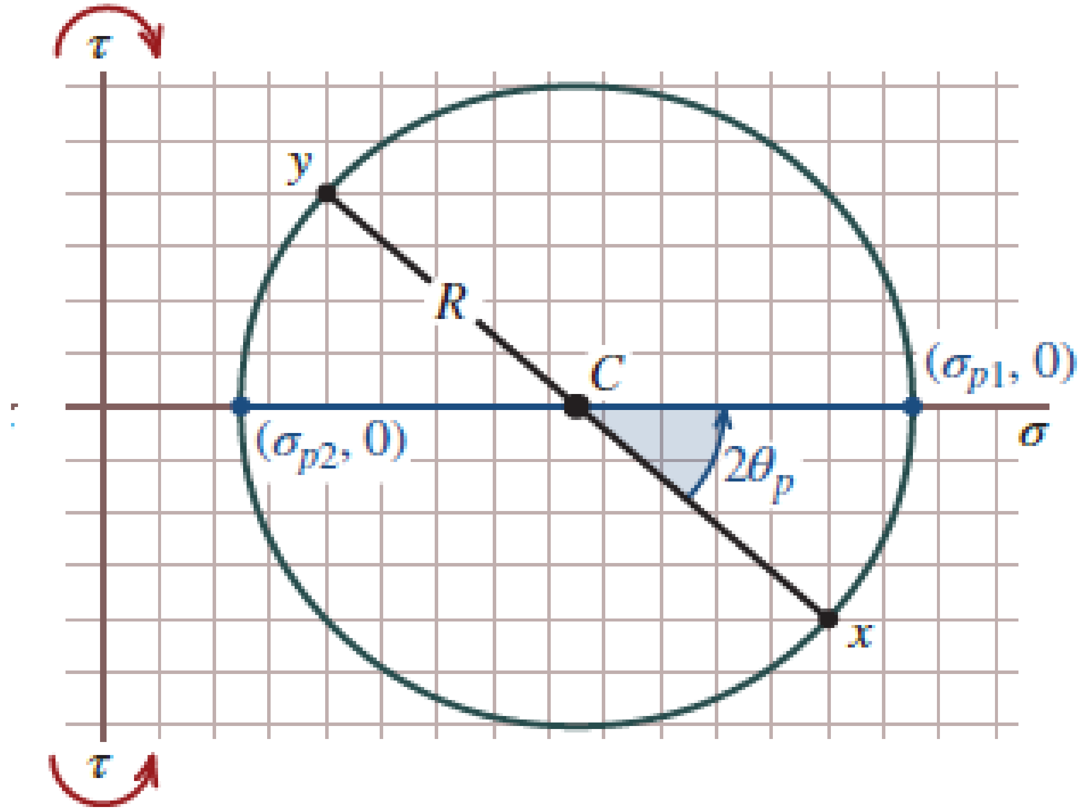
Equation (1)

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

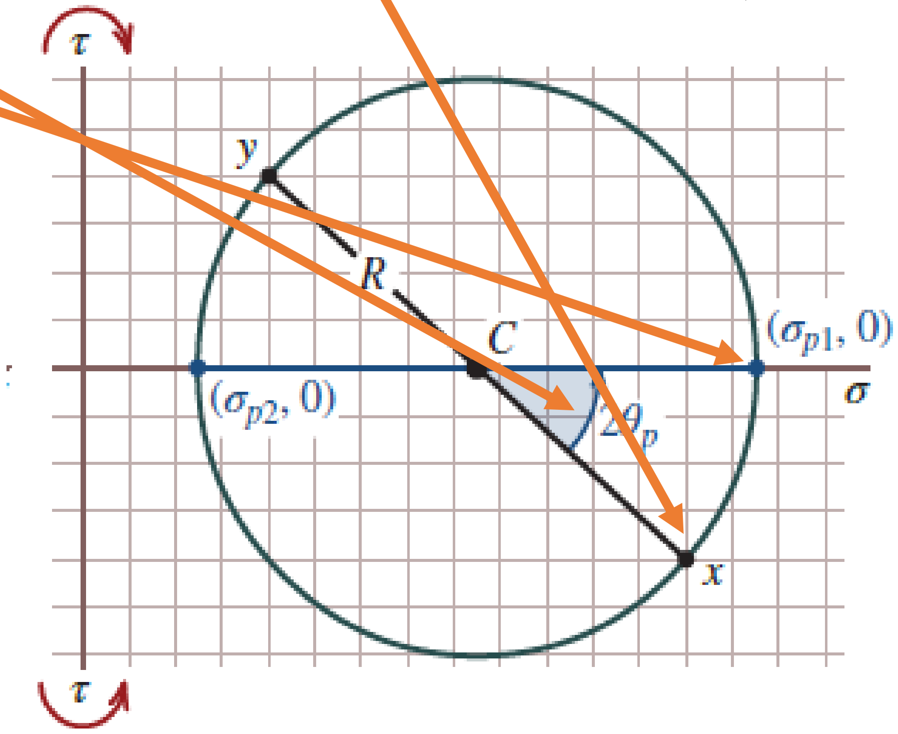
Equation (2)



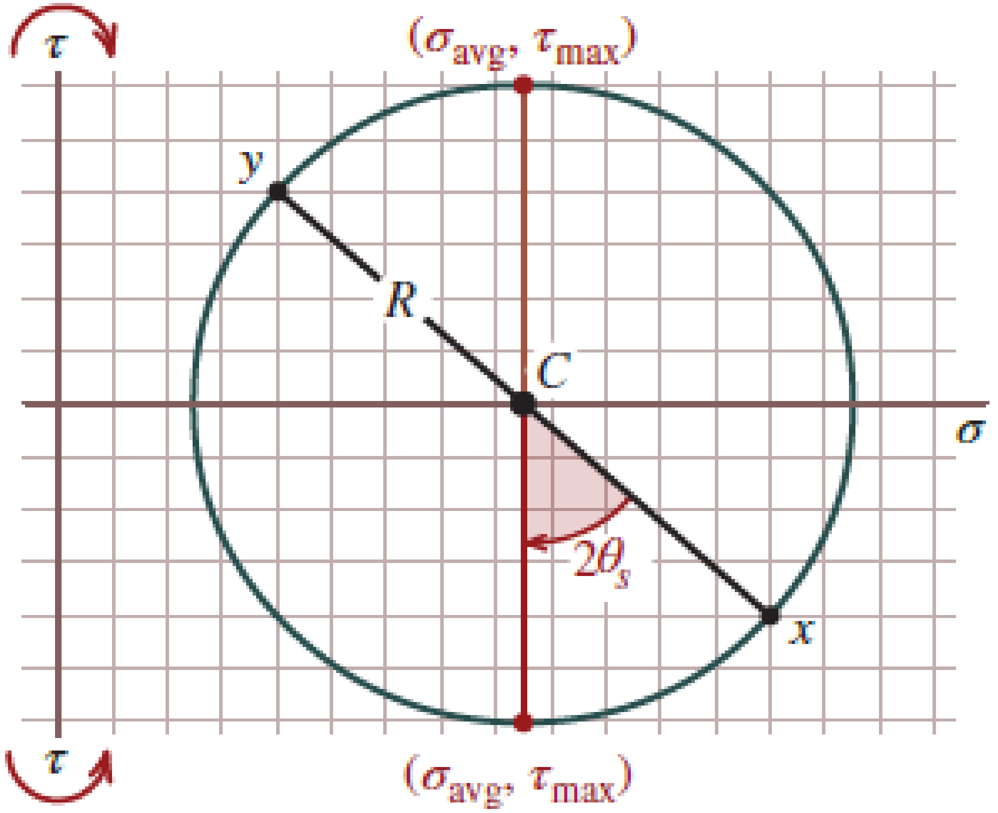
8. Several points on Mohr's circle are of particular interest. The principal stresses are the extreme values of the normal stress that exist in the stressed body, given the specific set of stresses σx, σy, and τxy that act in the x and y directions. From Mohr's circle, the extreme values of σ are observed to occur at the two points where the circle crosses the σ axis. The more positive point (in an algebraic sense) is σ p1, and the more negative point is σ p2. Notice that the shear stress τ at both points is zero. As discussed previously, the shear stress τ is always zero on planes where the normal stress σ has a maximum or a minimum value.



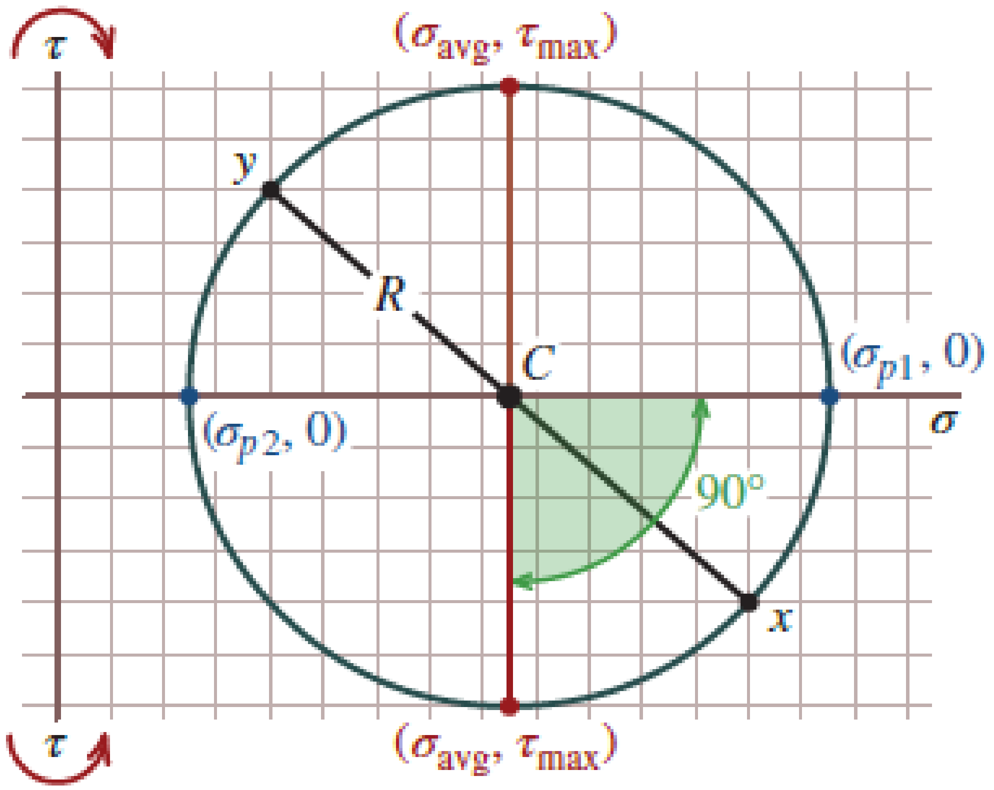
9. The geometry of Mohr's circle can be used to determine the orientation of the principal planes. From the geometry of the circle, the angle between point x and one of the principal stress points can be determined. The angle between point x and one of the principal stress points on the circle is 2θp. In addition to the magnitude of 2θp, the sense of the angle (either clockwise or counterclockwise) can be determined from the circle by inspection. The rotation of 2θp from point x to the principal stress point should be determined. In the x–y coordinate system of the stress element, the angle between the x face of the stress element and a principal plane is θp, where θp rotates in the same sense (either clockwise or counterclockwise) in the x–y coordinates the system as 2θp does in Mohr's circle.



10. Two additional points of interest in Mohr's circle are the extreme shear stress values. The largest shear stress magnitudes will occur at points located at the top and the bottom of the circle. Since the center C of the circle is always located on the σ axis, the largest possible value of τ is simply the radius R. Note that these two points occur directly above and directly below the center C. In contrast to the principal planes, which always have zero shear stress, the planes of maximum shear stress generally do have normal stress. The magnitude of this normal stress is identical to the σ coordinate of the center C of the circle.



11. Notice that the angle between the principal stress points and the maximum shear stress points on Mohr's circle is 90°. Since angles in Mohr's circle are doubled, the actual angle between the principal planes and the maximum shear stress planes will always be 45°.



The advantage offered by Mohr's circle is that it provides a concise visual summary of all stress combinations possible at any point in a stressed body. Since all stress calculations can be performed with the geometry of the circle and basic trigonometry, Mohr's circle provides an easy-to-remember tool for stress analysis.

Thank You!