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Reservoir Simulation - Lecture No02 – Introduction to Reservoir Simulation

Objectives:

- What is Reservoir Simulation and why is it needed?
- What are the application of Reservoir Simulation
- How Reservoir Simulation can help to Manage Oil and Gas Reservoirs?

Introduction

Reservoir simulation in the oil industry has become the standard for solving reservoir engineering problems. Simulators for various recovery processes have been developed and continue to be developed for new oil recovery processes. ***Reservoir simulation is the art of combining physics, mathematics, reservoir engineering, and computer programming to develop a tool for predicting hydrocarbon reservoir performance under various operating strategies***. There major steps involved in the development of a reservoir simulator: **formulation, discretization, well representation, linearization, solution, and validation**. *Formulation: outlines the basic assumptions inherent to the simulator, states these assumptions in precise mathematical terms, and applies them to a control volume in the reservoir*. The result of this step is a set of coupled, nonlinear partial differential equations (PDEs) that describes fluid flow through porous media. The PDEs derived during the formulation step, if solved analytically, would give reservoir pressure, fluid saturations, and well flow rates as continuous functions of space and time. Because of the highly nonlinear nature of the PDEs, however, analytical techniques cannot be used, and solutions must be obtained with numerical methods. In contrast to analytical solutions, numerical solutions give the values of pressure and fluid saturations only at discrete points in the reservoir and at discrete times. *Discretization: is the process of converting PDEs into algebraic equations*. Several numerical methods can be used to discretize the PDEs; however, the most common approach in the oil industry today is the finite difference method. The most commonly used finite-difference approach essentially builds on Taylor series expansion and neglects terms that are considered to be small when small difference in space parameters is considered. This expanded form is a set of algebraic equations. Finite element method, on the other hand, uses various functions to express variables in the governing equation. These functions lead to the development of an error function that is minimised in order to generate solutions to the governing equation. To carry out discretization, a PDEis written for a given point in space at a given time level. The choice of time level (oldtime level, current time level, intermediate timelevel) leads to the explicit, implicit, or Crank-Nicolson formulation method. The discretization process results in a system of nonlinear algebraic equations. These equations generally cannot be solved with linear equation solvers, and the *linearization of such equations becomes a necessary step before solutions can be obtained*. *Well representation:* is used to incorporate fluid production and injection into the nonlinear algebraic equations. *Linearization: involves approximating nonlinear terms (transmissibilities, production and injection, and coefficients of unknowns in the accumulation terms) in both space and time*. Linearization results in a set of linear algebraic equations. Any one of several linear *equation solvers can then be used to obtain the solution, which comprises pressure and fluid saturation distributions in the reservoir and well flow rates*. *Validation: of a reservoir simulator is the last step in developing a simulator, after which the simulator can be used for practical field*

applications. The validation step is necessary to make sure that no errors were introduced in the various steps of development or in computer programming. This validation is distinct from the concept of conducting experiments in support of a mathematical model.

Validation of a reservoir simulator merely involves testing the numerical code.

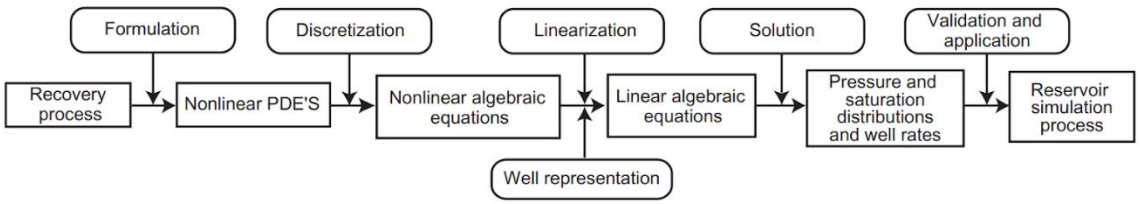


Figure 1: Major steps used to develop reservoir simulator *(modified from Odeh, A.S., 1982. An overview of mathematical modelling of the behaviour of hydrocarbon reservoirs. SIAM Rev. 24(3), 263.*).

Milestones for the engineering approach

The foundations for the engineering approach have been overlooked all these years. Traditionally, reservoir simulators were developed by first using a control volume (or elementary volume), such as that shown in below figure 2 for 1-D flow or in figure 3 for 3-D flow that was visualized by mathematicians to develop fluid flow equations. Note that point x in 1-D and point (x, y, z) in 3-D fall on the edge of control volumes. The resulting flow equations are in the form of PDEs. Once the PDEs were derived, early pioneers of simulation looked to mathematicians to provide solution methods. These methods started with the description of the reservoir as a collection of gridblocks, represented by points that fall within them (or grid points representing blocks that surround them), followed by the replacement of the PDEs and boundary conditions by algebraic equations, and finally the solution of the resulting algebraic equations. Developers of simulators were all the time occupied by finding the solution and, perhaps, forgot that they were solving an engineering problem. The engineering approach can be realised should one try to relate the terms in the discretized flow equations for any block to the block itself and to all its neighboring blocks.

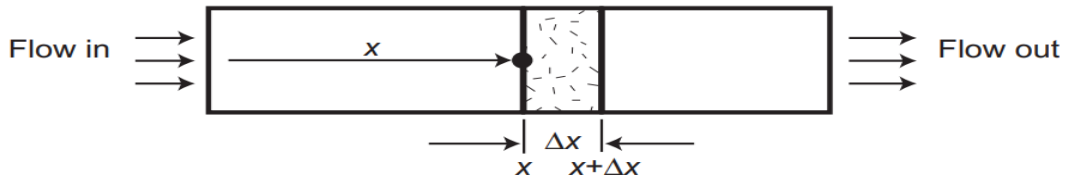


Figure 2: Control volume used by mathematicians for 1-D flow.

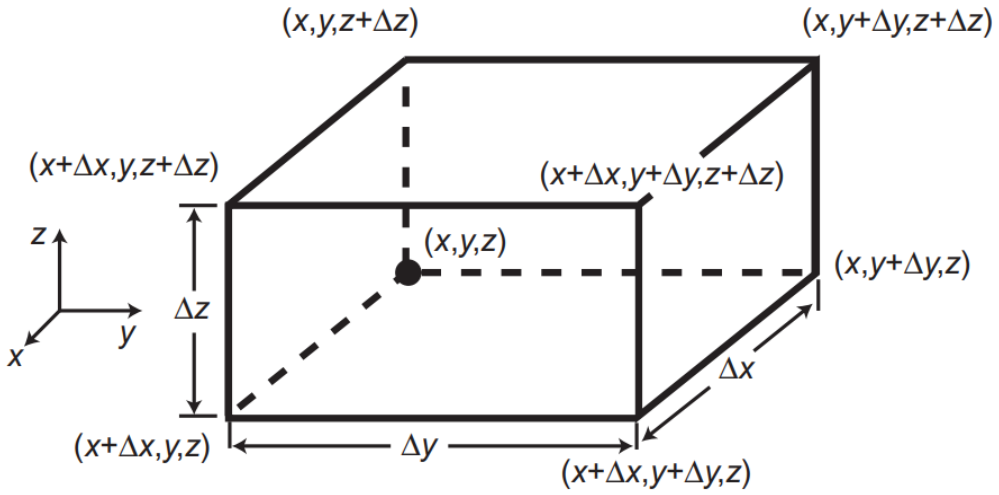


Figure 3: Control volume used by mathematicians for 3-D flow.

There were observations that the flow terms in the discretized form of governing equations were nothing but Darcy's law describing volumetric flow rate between any two neighbouring blocks. Making use of this observation coupled with an assumption related to the time level at which flow terms are evaluated, he developed the forward-central-difference equation and the backward-central-difference equation without going through the rigour of the mathematical approach in teaching reservoir simulation to undergraduate students. Control volume represented by a point at its centre in the mathematical approach as shown in Figure 4 for 1-D flow and Figure 5 for 3-D flow. This control volume is closer to engineer's thinking of representing blocks in reservoirs.

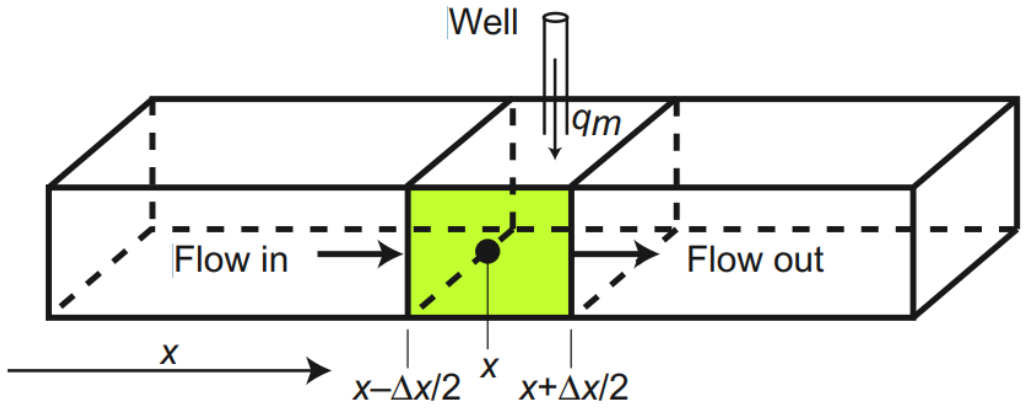


Figure 4: Control volume for 1-D flow.

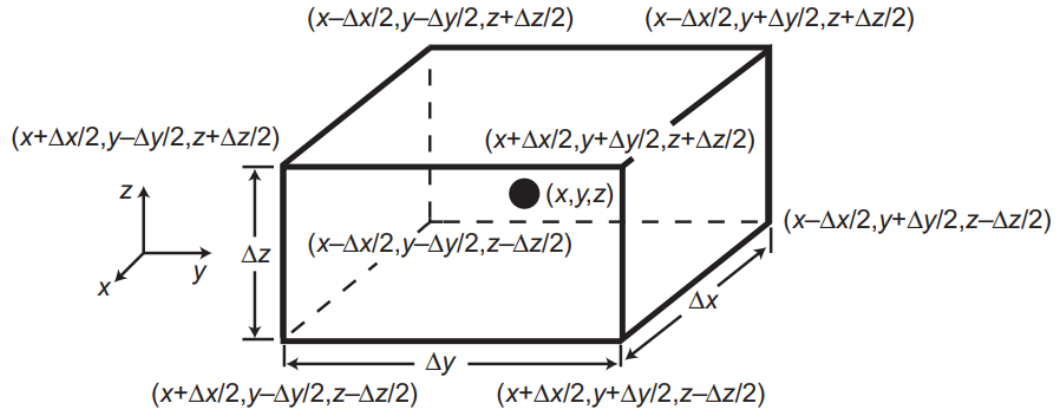


Figure 5: Control volume for 3-D flow.

Reservoir Simulation in simple words:

- As most of the reservoirs are not homogenous, reservoir simulation plays an important role to mimic the real situation. For example, if we want to calculate the flow rate using Darcy law, we will divide the reservoir into different blocks and the permeability value is different from one place to another within the blocks. So to calculate the flow as per Darcy we will take the length of the block and pressure difference; however if we calculate as one block, we will make some errors because we do not know which permeability value should be considered. That way we divided the reservoir into different blocks. Then we build the model based on each cell/block properties.

- Another example to consider in the reservoir simulation is the size of the reservoir. For example, if we deal with a 2 km square in size reservoir and we are calculating centminers of blocks we are distributing the results into the whole reservoir. For instance, plotting time vs. pressure over time as the calculated values using mathematical equations must be matched with the real values taken from the field (history matching).

- Any reservoir simulation starts with building a model using geological information using contouring maps as a top surface then we put the logs information to build the subsurface model. Then we put the flow status in the reservoir using the well information in 2D and 3D figures. So to understand the variations in the pressures, permeability, porosity, saturation we need to solve it numerically using differential equations.
- The reservoir simulation is always close to reality but not 100 real. For example, pressure vs time measured in reality is slightly different from the numerical model. So subtracting pressure taken from the field and pressure taken from the numerical model is the difference between them and it is close to reality.
- Reservoir simulation includes different components including:
    - Numerical Model
    - Mathematical Model
    - Computer Model
    - Geometric Model

- The Geometric Model defines the shape, dimensions and status and layer distribution.
  Mathematical and numerical models using differential equations to solve the problems
  Computer model is a programmed simulators using all above equations
- Main objectives of reservoir simulation
  1- To build a model of the reservoir and to examine its performance in terms of production and pressure
  2- To predict future performance.
  3- To find ways to increase ultimate recovery hydrocarbons more economically
- Methodology of Reservoir Simulation
  1- The reservoir is divided into a number of blocks (or grid blocks)
  2- Basic geological and reservoir data is provided for each block
  3- Wells are positioned within the arrangement of blocks
  4- The target rate and well pressure are specific as a function of time
  5- The appropriate equations derived from Darcy's Law are solved to give the pressure and saturation of each block as well as production and injection for each well.

Using the seismic and well logs information we can identify the horizons which have the same characterisitcs then divide them into blocks for all wells. Each block has its own properties so the simulation will distribute the properties into the whole reservoir.

- Elements of a Reservoir Simulation Study
  1- Setting up study
  2- Defining the study’s objectives
  3- Formulating a model
  4- Data preparation
  5- Collecting rock and fluid data
  6- Reservoir description
  7- Collect well performance and completion data
  8- Defining producing conditions
  9- History matching
  10- Adjusting reservoir parameters to match past performance
  11- Predicting performance
  12- Running the simulator and analysing the outcome
  12- Sensitivity analysis
  13- Identify critical parameters
  14- Evaluation alternative strategies for development

DERIVATION OF FLUID FLOW EQUATIONS

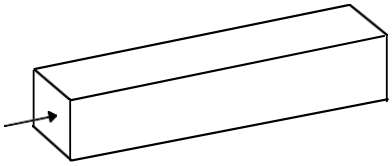
Review of basic steps

Generally speaking, flow equations for flow in porous materials are based on a set of mass, momentum and energy conservation equations, and constitutive equations for the fluids and the porous material involved. For simplicity, we will in the following assume isothermal conditions, so that we not have to involve an energy conservation equation. However, in cases of changing reservoir temperature, such as in the case of cold water injection into a warmer reservoir, this may be of importance.

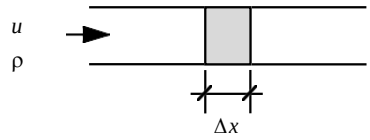
Below, equations are initially described for single phase flow in linear, one-dimensional, horizontal systems, but are later on extended to multi-phase flow in two and three dimensions, and to other coordinate systems.

Conservation of mass

Consider the following one dimensional rod of porous material:



Mass conservation may be formulated across a control element of the slab, with one fluid of density ρ is flowing through it at a velocity *u*:



The mass balance for the control element is then written as:

$$\left\{\begin{matrix}\text{Mass into the}\\ \text{element at } x\end{matrix}\right\} - \left\{\begin{matrix}\text{Mass out of the}\\ \text{element at } x+\Delta x\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of change of mass}\\ \text{inside the element}\end{matrix}\right\},$$

or

$$\left\{ u\rho A \right\}_{x} - \left\{ u\rho A \right\}_{x+\Delta x} = \frac{\partial}{\partial t}\left\{ \phi A \Delta x \rho \right\}.$$

Dividing by Δx, and taking the limit as Δx approaches zero, we get the conservation of mass, or continuity equation:

$$-\frac{\partial}{\partial x}(A\rho u) = \frac{\partial}{\partial t}(A\phi\rho).$$

For constant cross sectional area, the continuity equation simplifies to:

$$-\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial t}(\phi\rho).$$

Next, we need to replace the velocity term by an equation relating it to pressure gradient and fluid and rock properties, and the density and porosity terms by appropriate pressure dependent functions.

Conservation of momentum

Conservation of momentum is goverened by the Navier-Stokes equations, but is normally simplified for low velocity flow in porous materials to be described by the semi-empirical Darcy's equation, which for single phase, one dimensional, horizontal flow is:

$$u = -\frac{k}{\mu}\frac{\partial P}{\partial x}.$$

Alternative equations are the Forchheimer equation, for high velocity flow:

$$-\frac{\partial P}{\partial x} = u\frac{\mu}{k} + \beta u^n,$$

where *n* was proposed by Muscat to be 2, and the Brinkman equation, which applies to both porous and non-porous flow:

$$-\frac{\partial P}{\partial x} = u\frac{\mu}{k} - \mu\frac{\partial^2 u}{\partial x^2}.$$

Brinkman's equation reverts to Darcy's equation for flow in porous media, since the last term then normally is negligible, and to Stoke's equation for channel flow because the Darcy part of the equation then may be neglected.

In the following, we assume that Darcy's equation is valid for flow in porous media.

Constitutive equation for porous materials

To include pressure dependency in the porosity, we use the following definition of rock compressibility, which for constant temperature is written:

$$c_r = \left(\frac{1}{\phi}\right)\left(\frac{\partial \phi}{\partial P}\right)_T.$$

Normally, we may assume that the bulk volume of the porous material is constant, i.e. the bulk compressibility is zero. This is not always true, as witnessed by the subsidence in the Ekofisk area.

Constitutive equation for fluids

Recall the familiar fluid compressibility definition, which applies to any fluid at constant temperature:

$$c_f = -\left(\frac{1}{V}\right)\left(\frac{\partial V}{\partial P}\right)_T.$$

Equally familiar is the gas equation, which for an ideal gas is:

$$pV = nRT,$$

and for a real gas includes the deviation factor, Z:

$$pV = nZRT\,.$$

These descriptive equations for the fluids are frequently used in reservoir engineering applications. However, for more general purposes, such as in reservoir simulation models, we normally use either so-called *Black Oil* fluid description, or *compositional* fluid description. Below, we will review the Black Oil model.

The standard Black Oil model includes *Formation Volume Factor*, *B*, for each fluid, and *Solution Gas-Oil Ratio*, R_{so}, for the gas dissolved in oil, in addition to viscosity and density for each fluid. A modified model may also include oil dispersed in gas, r_s, and gas dissolved in water, R_{sw}. The definitions of formation volume factors and solution gas-oil ratio are:

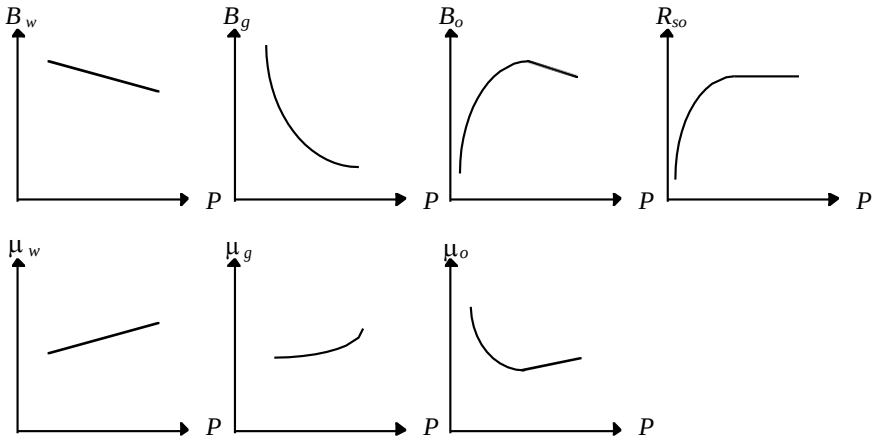
$$B = \frac{\text{volume at reservoir conditions}}{\text{volume at standard conditions}}$$

$$R_{so} = \frac{\text{volume of gas evolved from oil at standard conditions}}{\text{volume of oil at standard conditions}}$$

The density of oil at reservoir conditions is then, in terms of these parameters and the densities of oil and gas, defined as:

$$\rho_o = \frac{\rho_{oS} + \rho_{gs} R_{so}}{B_o}.$$

Typical pressure dependencies of the standard Black Oil parameters are:



Simple form of the flow equation and analytical solutions
In the following, we will briefly review the derivation of single phase, one dimensional, horizontal flow equation, based on continuity equation, Darcy's equation, and compressibility definitions for rock and fluid, assuming constant permeability and viscosity.

Let us substitute Darcy´s equation into the continuity equation derived above:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\partial}{\partial t}(\rho\phi)$$

The right hand side (RHS) of the equation may be expanded as:

$$\frac{\partial}{\partial t}(\rho\phi)=\rho\frac{\partial}{\partial t}(\phi)+\phi\frac{\partial}{\partial t}(\rho)$$

Since porosity and density both are functions of pressure only (assuming temperature to be constant), we may write:

$$\frac{\partial}{\partial t}(\phi)=\frac{d\phi}{dP}\frac{\partial P}{\partial t}$$

and

$$\frac{\partial}{\partial t}(\rho)=\frac{d\rho}{dP}\frac{\partial P}{\partial t}.$$

From the compressibility expressions we may obtain the following relationships:

$$\frac{d\rho}{dP}=\rho c_f \text{ and } \frac{d\phi}{dP}=\phi c_r.$$

By substituting these expressions into the equation, we obtain the following form of the right hand side of the flow equation:

$$\frac{\partial}{\partial t}(\rho\phi)=\phi\rho(c_f+c_r)\frac{\partial P}{\partial t}.$$

The left hand side of the flow equation may be expanded as follows:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right)+\frac{k}{\mu}\frac{\partial P}{\partial x}\frac{\partial}{\partial}(\rho)=\rho\frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right)+\frac{k}{\mu}\frac{\partial P}{\partial x}\frac{d\rho}{dP}\frac{\partial P}{\partial x}$$

For now, let us assume that k=constant and μ=constant. Let us also substitute for $\frac{d\rho}{dP}=\rho c_f$. The LHS may now be written as:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right)=\frac{\rho k}{\mu}\left[\frac{\partial^2 P}{\partial x^2}+c_f\left(\frac{\partial P}{\partial x}\right)^2\right]$$

Since c_f is small, at least for liquids, and the pressure gradient is small for the low velocity flow we normally have in reservoirs, we make the following assumption:

$$c_f\left(\frac{\partial P}{\partial x}\right)^2 << \frac{\partial^2 P}{\partial x^2}.$$

Then, our LHS simplifies to:

$$\frac{\partial}{\partial x}\left(\rho\frac{k}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\rho k}{\mu}\frac{\partial^2 P}{\partial x^2}.$$

The complete partial differential flow equation (PDE) for this simple rock-fluid system then becomes:

$$\frac{\partial^2 P}{\partial x^2} = \left(\frac{\phi\mu c}{k}\right)\frac{\partial P}{\partial t},$$

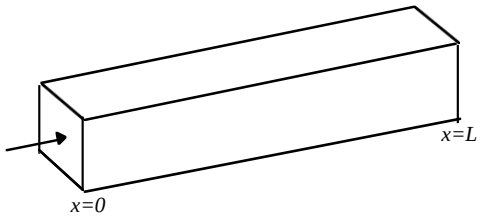
where c is the sum of the rock and fluid compressibilities.

Assumptions made in the derivation of the above PDE:

1. One dimensional flow
2. Linear flow
3. Horizontal flow
4. One phase flow
5. Darcy´s equation applies
6. Small fluid compressibility (liquid)
7. Permeability and viscosity are constants

Initial and boundary conditions

In order to solve the above equation, we need to specity one initial and two boundary conditions. The initial condition will normally specify a constant initial pressure, while the boundary conditions will either specify pressures or flow rates at two positions of the system. For our simple horizontal rod of porous material, these conditions may be specified as:



Initial condition (IC):

$$P(x,t=0) = P_i$$

Normally, the initial pressure of a horizontal system such as the one above is constant, but in principle it could be a function of position (x).

Boundary conditions (BC´s):

Pressure conditions (Dirichlet conditions) would typically be specified as:

$$P(x=0,t) = P_L$$
$$P(x=L,t) = P_R$$

The other commonly used BC´s are rate specifications (Neumann conditions). Using Darcy´s equation, flow rates would typically be specified as:

$$q_L = -\frac{kA}{\mu}\left(\frac{\partial P}{\partial x}\right)_{x=0}$$

$$q_R = -\frac{kA}{\mu}\left(\frac{\partial P}{\partial x}\right)_{x=L}$$

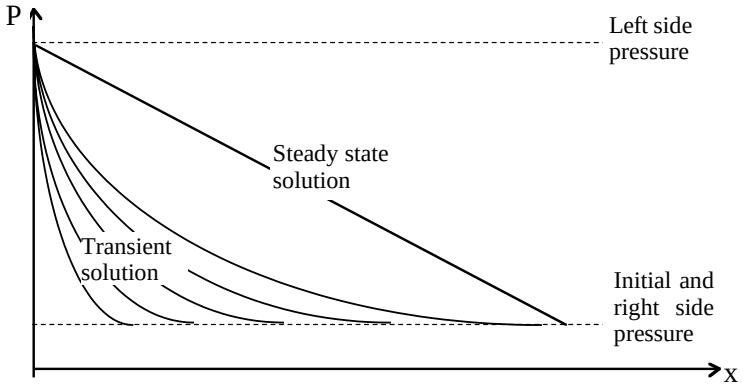
Analytical solution to the simple, linear PDE
Using the following set of initial and boundary conditions:

$$P(x,t=0) = P_i,\ P(x=0,t) = P_L \text{ and } P(x=L,t) = P_R,$$

we may obtain the following analytical solution of the transient pressure development in the porous rod above:

$$P(x,t) = P_L + (P_R - P_L)\left[\frac{x}{L} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2}{L^2}\frac{k}{\phi\mu c}t\right)\sin\left(\frac{n\pi x}{L}\right)\right]$$

This solution is depicted graphically in the figure below.



Transient vs. steady state flow
The partial differential equation above includes time dependency through the right hand side term. Thus, it can describe transient, or time dependent flow. In the figure illustrating the solution, the system will first have a time dependent, or transient, period, where the pressure will gradually penetrate the porous material. Then, after some time, the flow reaches a state where it is no longer time dependent, and the pressure distribution is described by the straight line denoted steady state solution.

We could have reduced the partial differential equation directly to a steady state equation by setting the time dependent term on the right hand side to zero. Then the equation becomes an ordinary differential equation (ODE):

$$\frac{d^2P}{dx^2} = 0$$

By integrating this equation twice, and using the two boundary conditions to determine the integration constants, we obtain the steady state solution:

$$P(x,t) = P_L + (P_R - P_L)\frac{x}{L}.$$

which is a straight line connecting the two end pressures. As can be seen, the transient solution will reduce to this steady state expression as time becomes large.

General form of the one-phase, one-dimensional, horizontal PDE

Above we derived and solved the simplest forms of the PDE, using fluid compressibility definition as a constitutive fluid equation, and assuming constant viscosity and permeability. Generally, the Black Oil form of the fluid model is used, and the two parameters are not constants. Recall the Black Oil definition of oil density:

$$\rho_o = \frac{\rho_{oS} + \rho_{gs} R_{so}}{B_o}.$$

For undersaturated oil, the solution gas-oil ratio, R_{so}, is constant. Thus, the oil density may be written:

$$\rho_o = \frac{\text{constant}}{B_o}.$$

Similar expressions may be written for single phase gas and single phase water. Substitution of this fluid model into the continuity equation with Darcy´s equation yields a general Black Oil form of the single phase, one-dimensional, horizontal flow equation:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$$

Multiphase flow

A continuity equation may be written for each fluid phase flowing:

$$-\frac{\partial}{\partial x}\left(\rho_l u_l\right) = \frac{\partial}{\partial t}\left(\phi \rho_l S_l\right), l = o,w,g,$$

and the corresponding Darcy equations for each phase are:

$$u_l = -\frac{k k_{rl}}{\mu_l}\frac{\partial P_l}{\partial x}, l = o,w,g,$$

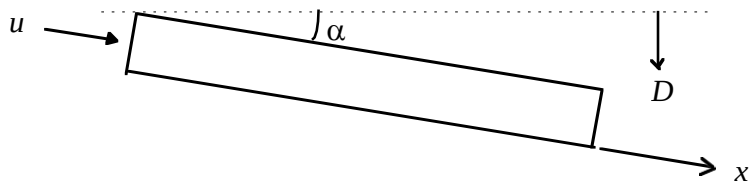
where

$$P_{cow} = P_o - P_w$$
$$P_{cog} = P_g - P_o$$
$$\sum_{l=o,w,g} S_l = 1.$$

The continuity equation for gas has to be modified to include solution gas as well as free gas, and the one for oil to include dispersed oil in gas, if any.

Non-horizontal flow

For one-dimensional, inclined flow, as shown in the following figure:



the Darcy equation becomes:

$$u = -\frac{k}{\mu}\left(\frac{\partial P}{\partial x} - \rho g \frac{dD}{dx}\right),$$

or, in terms of dip angle, α, and hydrostatic gradient:

$$u = -\frac{k}{\mu}\left(\frac{\partial P}{\partial x} - \gamma \sin(\alpha)\right),$$

where $\gamma = \rho g$ is the hydrostatic gradient of the fluid.

Multidimensional flow

The continuity equation for one-phase, three-dimensional flow in cartesian coordinates, is:

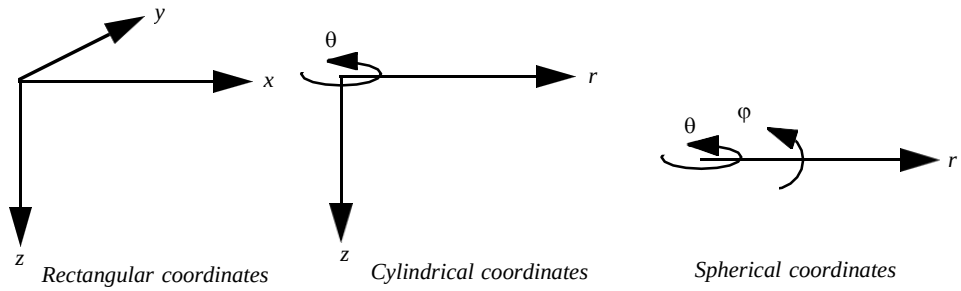
$$-\frac{\partial}{\partial x}(\rho u)_x - \frac{\partial}{\partial y}(\rho u)_y - \frac{\partial}{\partial z}(\rho u)_z = \frac{\partial}{\partial t}(\phi\rho),$$

and the corresponding Darcy equations are:

$$u_x = -\frac{k_x}{\mu}\left(\frac{\partial P}{\partial x} - \gamma \frac{dD}{dx}\right)$$
$$u_y = -\frac{k_y}{\mu}\left(\frac{\partial P}{\partial y} - \gamma \frac{\partial D}{\partial y}\right)$$
$$u_z = -\frac{k}{\mu}\left(\frac{\partial P}{\partial z} - \gamma \frac{\partial D}{\partial z}\right).$$

Coordinate systems

Normally, we use either a rectangular coordinate system, or a cylindrical coordinate system in reservoir engineering



Rectangular coordinates *Cylindrical coordinates* *Spherical coordinates*

In operator form, the continuity and the Darcy equations for one-phase flow may be written:

$$-\nabla\cdot\left(\rho\vec{u}\right)=\frac{\partial}{\partial t}\left(\phi\rho\right)$$

$$\vec{u}=-\frac{K}{\mu}\left(\nabla P-\gamma\nabla D\right),$$

where the operators are defined as:

rectangular coordinates

$$\nabla\cdot(\ )=\frac{\partial}{\partial x}(\ )+\frac{\partial}{\partial y}(\ )+\frac{\partial}{\partial z}(\ ) \qquad \text{(divergence)}$$

$$\nabla(\ )=\hat{i}\frac{\partial}{\partial x}(\ )+\hat{j}\frac{\partial}{\partial y}(\ )+\hat{k}\frac{\partial}{\partial z}(\ ) \qquad \text{(gradient)}$$

cylindrical coordinates

$$\nabla\cdot(\ )=\frac{1}{r}\frac{\partial}{\partial r}\left(r(\ )\right)+\frac{1}{r}\frac{\partial}{\partial\theta}(\ )+\frac{\partial}{\partial z}(\ )$$

$$\nabla(\ )=\hat{i}\frac{\partial}{\partial r}(\ )+\frac{\partial}{\partial\theta}(\ )+\hat{k}\frac{\partial}{\partial z}(\ )$$

spherical coordinates

$$\nabla\cdot(\ )=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(\ )\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left((\ )\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\ )$$

$$\nabla(\ )=\hat{i}\frac{\partial}{\partial r}(\ )+\frac{\partial}{\partial\theta}(\ )+\hat{k}\frac{\partial}{\partial\varphi}(\ )$$

Reservoir Discretization and Multidimensional flow in Cartesian coordinates

2.1 Reservoir discretization

Reservoir discretization means that the reservoir is described by a set of grid-blocks (or gridpoints) whose properties, dimensions, boundaries, and locations in the reservoir are well defined. It also deals with reservoirs discretized using a block-centered grid, and furthermore discusses reservoirs discretized using a point-distributed grid. Fig. 2.1 shows reservoir discretization in the *x*-direction as one focuses on block *i*.

The figure shows how the blocksare related to each other—block *i* and its neighboring blocks (blocks $i-1$ and $i+1$)—block dimensions (Δx_i, Δx_{i-1}, Δx_{i+1}),

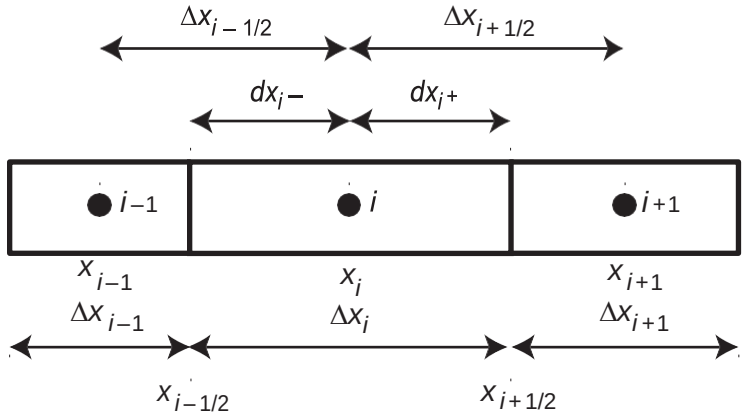


FIG. 2.1 Relationships between block *i* and its neighboring blocks in 1-D flow.

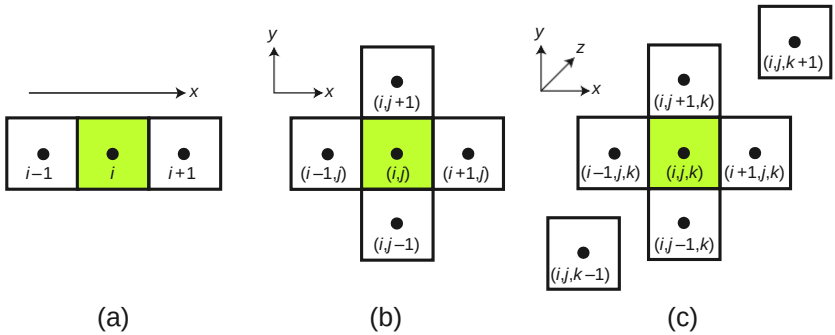


FIG. 2.2 A block and its neighboring blocks in (a) 1-D, (b) 2-D, and (c) 3-D flow using engineering notation.

block boundaries ($x_{i-1/2}$, $x_{i+1/2}$), distances between the point that represents the block and block boundaries (δx_{i-},δx_{i+}), and distances between the points representing the blocks ($\Delta x_{i-1/2}$, $\Delta x_{i+1/2}$). The terminology presented in Fig. 2.1 is applicable to both block-centered and point-distributed grid systems in 1-D flow in the direction of the *x*-axis. Reservoir discretization in the *y*- and *z*-directions uses similar terminology. In addition, each gridblock (or gridpoint) is assigned elevation and rock properties such as porosity and permeabilities in the *x*-, *y*-, and *z*-directions. The transfer of fluids from one block to the rest of reservoir takes place through the immediate neighboring blocks. When the whole reservoir is discretized, each block is surrounded bya set (group) of neighboring blocks. Fig. 2.2a showsthatthere are two neighboring blocks in 1-D flow along the *x*-axis, Fig. 2.2b shows that there are four neighboring blocks in 2-D flow in the *x*-*y* plane, and Fig. 2.2c shows that there are six neighboring blocks in 3-D flow in *x*-*y*-*z* space.

It must be made clear that once the reservoir is discretized and rock properties are assigned to gridblocks (or gridpoints), space is no longer a variable and functions that depend on space, such as interblock properties, become well defined. In other words, reservoir discretization removes space from being a variable in the formulation of the problem. More elaboration follows in Section 2.6.2.

2.2 Basic engineering concepts

The basic engineering concepts include mass conservation, equation of state, and constitutive equation. The principle of *mass conservation* states that the total mass of fluid entering minus the fluid leaving a volume element of the reservoir, shown in Fig. 2.3 as block *i*, must equal the net increase in the mass of the fluid in the reservoir volume element, that is,

$$m_i - m_o + m_s = m_a \tag{2.6}$$

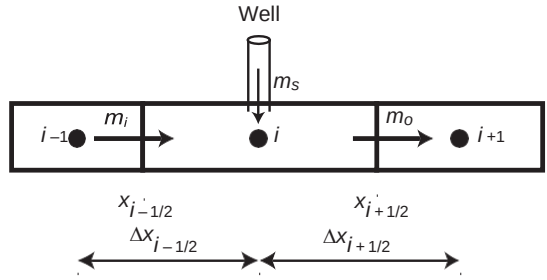


FIG. 2.3 Block *i* as a reservoir volume element in 1-D flow.

where m_i = the mass of fluid entering the reservoir volume element from other parts of the reservoir, m_o = the mass of fluid leaving the reservoir volume element to other parts of the reservoir, m_s = the mass of fluid entering or leaving the reservoir volume element externally through wells, and m_a the mass of excess fluid stored in or depleted from the reservoir volume element over a time interval.

An *equation of state* describes the density of fluid as a function of pressure and temperature. For single-phase fluid,

$$B = \rho_{sc}/\rho \tag{2.7a}$$

for oil or water,

$$B_g = \frac{\rho_{gsc}}{\alpha_c \rho_g} \tag{2.7b}$$

for gas, where ρ and ρ_g fluid densities at reservoir conditions, ρ_{sc} and ρ_{gsc} fluid densities at standard conditions, and α_c the volume conversion factor.

A *constitutive equation* describes the rate of fluid movement into (or out of) the reservoir volume element. In reservoir simulation, Darcy's law is used to relate fluid flow rate to potential gradient. The differential form of Darcy's law in a 1-D inclined reservoir is

$$u_x = q_x/A_x = -\beta_c \frac{k_x}{\mu} \frac{\partial \Phi}{\partial x} \tag{2.8}$$

where β_c = the transmissibility conversion factor, k_x = absolute permeability of rock in the direction of the *x*-axis, μ = fluid viscosity, Φ = potential, and u_x = volumetric (or superficial) velocity of fluid defined as fluid flow rate (q_x) per unit cross-sectional area (A_x) normal to flow direction *x*. The potential is related to pressure through the following relationship:

$$\Phi - \Phi_{ref} = (p - p_{ref}) - \gamma (Z - Z_{ref}) \tag{2.9}$$

where Z = elevation from datum, with positive values downward.

Therefore,

$$\frac{\partial \Phi}{\partial x} = \frac{\partial p}{\partial x} - \gamma \frac{\partial Z}{\partial x} \tag{2.10}$$

and the potential differences between block i and its neighbors, block $i-1$ and block $i+1$, are

$$\Phi_{i-1} - \Phi_i = (p_{i-1} - p_i) - \gamma_{i-1/2}(Z_{i-1} - Z_i) \tag{2.11a}$$

and

$$\Phi_{i+1} - \Phi_i = (p_{i+1} - p_i) - \gamma_{i+1/2}(Z_{i+1} - Z_i) \tag{2.11b}$$

2.3 Multidimensional flow in Cartesian coordinates

2.3.1 Block identification and block ordering

Before writing the flow equation for a 1-D, 2-D, or 3-D reservoir, the blocks in the discretized reservoir must be identified and ordered. Any block in the reservoir can be identified either by engineering notation or by the number the block holds in a given orderingscheme. Engineering notation uses the order of the blockin the *x*-, *y*-, and *z*-directions, that is, it identifies a block as (*i*, *j*, *k*), where *i*, *j*, and *k* are the orders of the block in the three directions *x*, *y*, and *z*, respectively. The engineering notation for block identification is the most convenient for entering reservoir description (input) and for printing simulation results (output). Fig. 2.4 shows the engineering notation for block identification in a 2-D reservoir consisting of 4 5×blocks. Block ordering not only serves to identify blocks in the reservoir but also minimizes matrix computations in obtaining the solutionof linear equations.

There are many block-ordering schemes, including natural ordering, zebra ordering, diagonal (D2) ordering, alternating diagonal (D4) ordering, cyclic ordering, and cyclic-2 ordering. If the reservoir has inactive blocks within its external boundaries, such blocks will be skipped, and ordering of active blocks will continue (Abou-Kassem and Ertekin, 1992). For multidimensional

(1,5)	(2,5)	(3,5)	(4,5)
(1,4)	(2,4)	(3,4)	(4,4)
(1,3)	(2,3)	(3,3)	(4,3)
(1,2)	(2,2)	(3,2)	(4,2)
(1,1)	(2,1)	(3,1)	(4,1)

FIG. 2.4 Engineering notation for block identification.

reservoirs, natural ordering is the simplest to program but is the least efficient in solving linear equations, whereas D4 ordering requires complicated programming but is the most efficient in obtaining the solution when the number of blocks is large. If the number of blocks is very large, however, the zebra ordering scheme becomes twice as efficient as D4 ordering in obtaining the solution (McDonald and Trimble, 1977). Fig. 2.5 shows the various block-ordering schemes for the 2-D reservoir shown in Fig. 2.4. Given the engineering notation for block identification, block ordering is generated internally in a simulator. Any ordering scheme becomes even more efficient computationally if the ordering is performed along the shortest direction, followed by the intermediate direction, and finally the longest direction (Abou-Kassem and Ertekin, 1992).

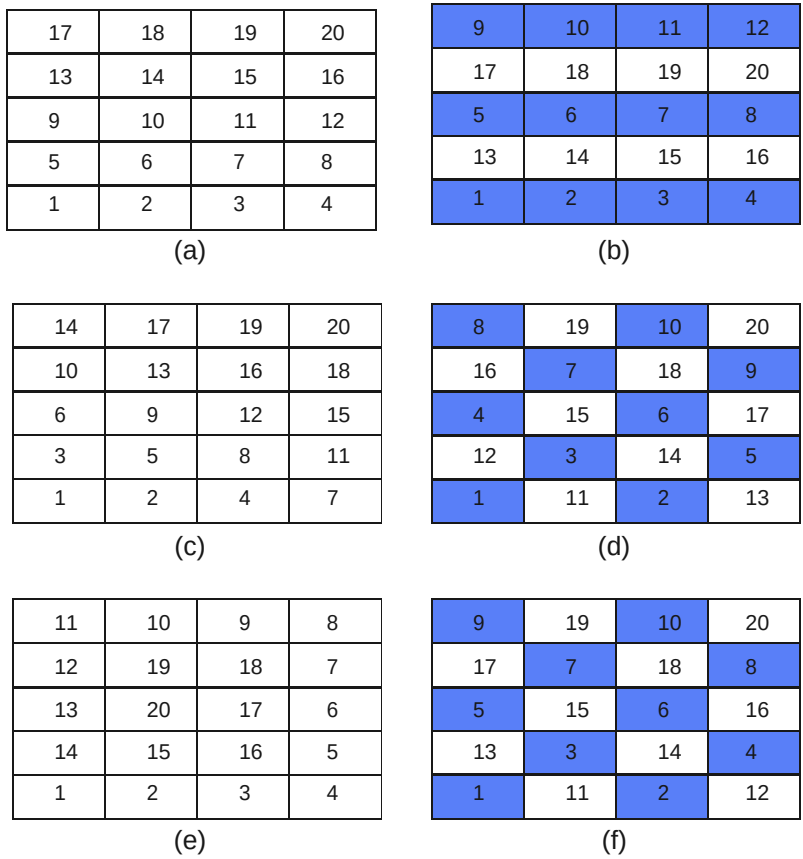


FIG. 2.5 Block-ordering schemes used in reservoir simulation. (a) Natural ordering, (b) zebra ordering, (c) diagonal (D2) ordering, (d) alternating diagonal (D4) ordering, (e) cyclic ordering, and (f) cyclic-2 ordering.

Details related to various ordering schemes and computational efficiency in solving linear equations are not discussed further in this book but can be found elsewhere (Woo et al., 1973; Price and Coats, 1974; McDonald and Trimble, 1977). The natural ordering scheme is used throughout this book because it produces equations that are readily solvable with handheld calculators and easily programmable for computer usage. The following three examples demonstrate the use of engineering notation and natural ordering to identify blocks in multidimensions.

Example 2.1 Consider the 1-D reservoir shown in Fig. 2.6a. This reservoir is discretized using four blocks in the *x*-direction as shown in the figure. Order the blocks in this reservoir using natural ordering.

Solution

We first choose one of the corner blocks (say the left corner block), identify it as block 1, and then move along a given direction to the other blocks, one block at a time. The order of the next block is obtained by incrementing the order of the previous block by one. The process of block ordering (or numbering) continues until the last block in that direction is numbered. The final ordering of blocks in this reservoir is shown in Fig. 2.6b.

Example 2.2 Consider the 2-D reservoir shown in Fig. 2.7a. This reservoir is discretized using 4×3 blocks as shown in the figure. Identify the blocks in this reservoir using the following:

1. Engineering notation
2. Natural ordering

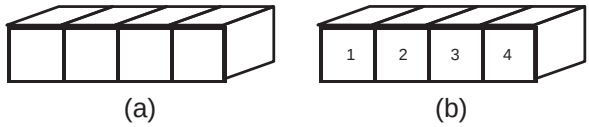


FIG. 2.6 1-D reservoir representation in Example 2.1. (a) Reservoir representation and (b) natural ordering of blocks.

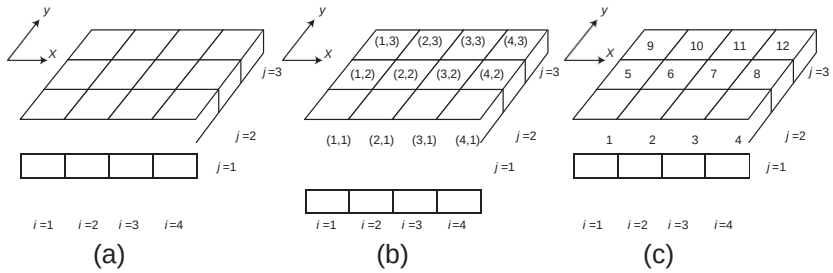


FIG. 2.7 2-D reservoir representation in Example 2.2. (a) Reservoir representation, (b) engineering notation, and (c) natural ordering of blocks.

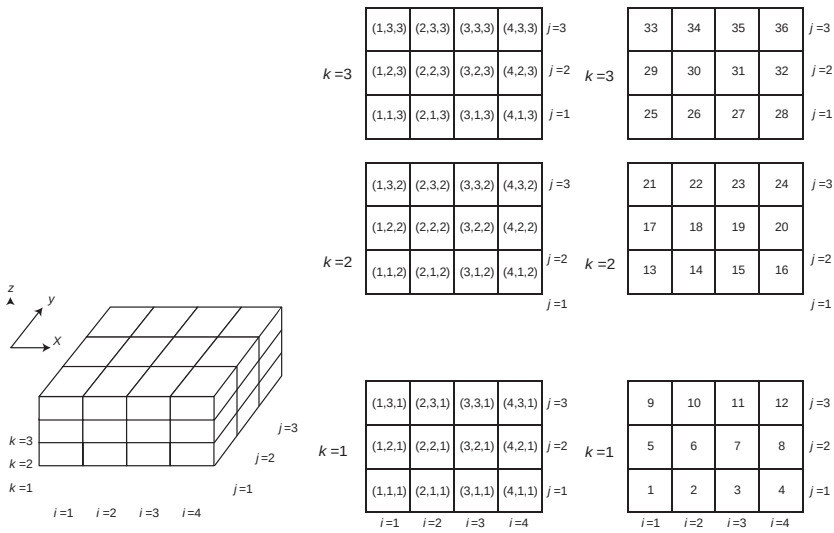


FIG. 2.8 3-D reservoir representation in Example 2.3. (a) Reservoir representation, (b) engineering notation, and (c) natural ordering of blocks.

Solution

1. The engineering notation for block identification is shown in Fig. 2.7b.
2. We start by choosing one of the corner blocks in the reservoir. In this example, we arbitrarily choose the lower-left corner block, block (1,1), and identify it as block 1. In addition, we choose to order blocks along rows. The rest of the blocks in the first row (j 1)¼are numbered as explained in Example 2.1. Block (1,2) in the first column (i 1) and¼second row (j 2) is nu¼mbered next as block 5, and block numbering along this row continues as in Example 2.1. Block numbering continues row by row until all the blocks are numbered. The final ordering of blocks in this reservoir is shown in Fig. 2.7c.

Example 2.3 Consider the 3-D reservoir shown in Fig. 2.8a. This reservoir is discretized into 4×3 ×3 blocks as shown in the figure. Identify the blocksin this reservoir using the following:

1. Engineering notation
2. Natural ordering.

Solution

1. The engineering notation for block identification in this reservoir is shown in Fig. 2.8b.
2. We arbitrarily choose the bottom-lower-left corner block, block (1,1,1), and identify it as block 1. In addition, we choose to order blocks layer by layer and along rows. The blocks in the first (bottom) layer (k¼1) are ordered as shown in Example 2.2. Next, block (1,1,2) is numbered as block 13, and the ordering of blocks in this second layer is carried out as in the first layer. Finally, block (1,1,3) is numbered as block 25, and the ordering of blocks