

## Hydraulic and air separations

Hydraulic and air separations are usually called hydraulic and air classifications. They utilize the differences in velocity of settling of particles. The use of “hydraulic” or “air” words before the term “classification” is very important as it allows to distinguish the two separations from a general term “classification” meaning, in this work, the way of analyzing the results of separation.

The product of hydraulic separation, containing fast settling particles, is usually called “underflow”, while the second product, containing slowly settling particles, is called “overflow”. Hydraulic and air separators can be used to remove very fine particles from the feed, separate larger and heavier particles from lighter and smaller, divide the feed into narrow size fractions, limit lower and upper range of particles size because of requirements by the applied technology, as well as to regulate the size reduction during grinding.

There are many ways of hydraulic and air separations. They can be carried out in stationary media, in the media moving vertically, horizontally, sideward, and in a pulsating or spiral stream. Fig. 5.1. shows schematically typical devices applied for hydraulic separation in the media.

The initial point for analyzing any hydraulic and air separation is the free vertical fall of particles due to gravity. The equation, which determines the relation between the velocity of settling of spherical particles  $v$  (m/s) and their density in the medium  $\Delta\rho$  (kg/m<sup>3</sup>) having diameter  $d$  (m) and resistance factor  $\zeta$  (dimensionless factor which characterizes the resistance of the medium to a moving particle) is as follows:

$$v \cong \sqrt{\frac{4 \Delta \rho d}{3 \zeta \rho_c}}, \quad (5.1)$$

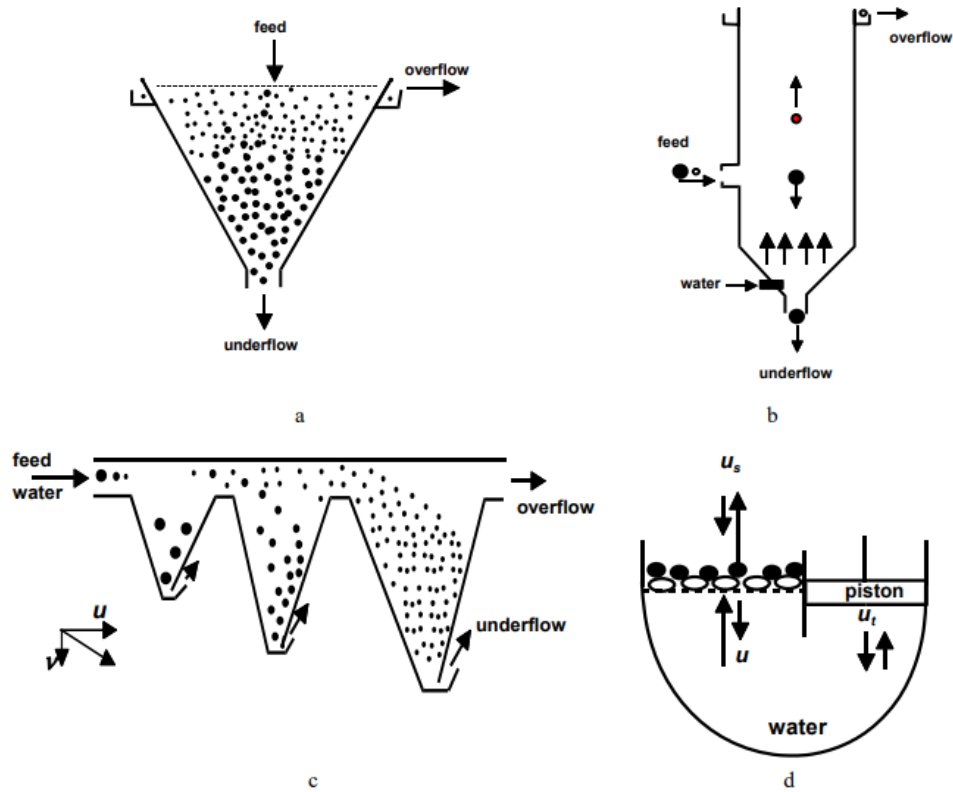
where:  $\Delta \rho$  - difference between densities of particle in vacuum  $\rho_z$  (kg/m<sup>3</sup>) and medium  $\rho_c$  (kg/m<sup>3</sup>)

حيث  $\Delta \rho$  - الفرق بين كثافات الجسيمات في الفراغ ( $\rho_z$  كجم / م<sup>3</sup>) والوسط ( $\rho_c$  كجم / م<sup>3</sup>)

i.e.  $\rho' = \Delta \rho = \rho_z - \rho_c$  (kg/m<sup>3</sup>).

Equation (5.1) is based on the balance of the forces taking part in the process, that is the weight of particle in the medium  $F_c$ , resistance force  $F_o$  which opposes particle falling down and the force of inertia  $F_b$ . At any moment of settling the sum of forces should equal zero:

$$F_c + F_o + F_b = 0$$



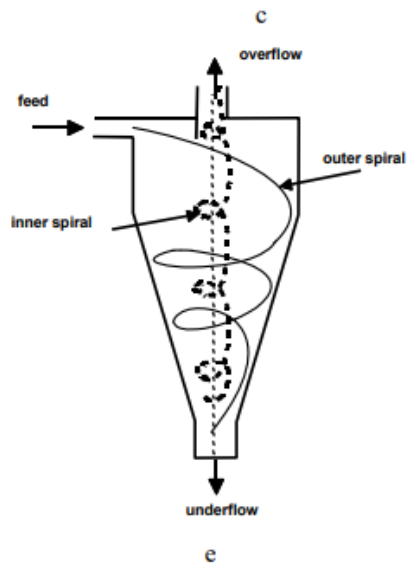


Fig. 5.1. Modes of separation in media:  
 a – stationary (cone classifier)  
 b – vertical (elutriator),  
 c – horizontal (multi-product cone-type sedimentation tank),  
 d – pulsating (jig),  
 e – spiral movement of the medium (hydro-cyclone)

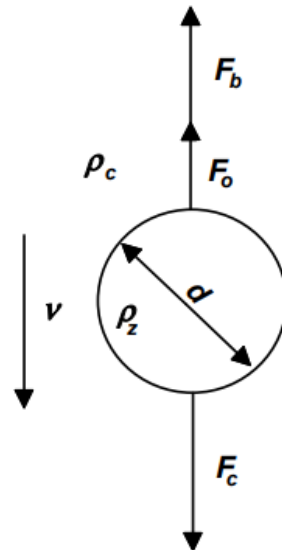


Fig. 5.2. Forces acting on a particle settling in a medium

The forces are determined by the relations:

$$F_c = V_d \Delta \rho g, \quad (5.3)$$

$$F_o = 0.5 \zeta \rho_c v^2 A, \quad (5.4)$$

$$F_b = V_d (\rho_z + a_d \rho_c) dv/dt, \quad (5.5)$$

where: A – surface of the largest projection of the cross section of dispersed elements in the direction perpendicular to particle movement, m<sup>2</sup>

V<sub>d</sub> – particle volume, m<sup>3</sup>

a<sub>d</sub> – acceleration factor (dimensionless, 0.5 for spheres and 0.75 for a cylinders)

t – time, s

g – acceleration due to gravity, m/s<sup>2</sup>.

After rearrangement, one obtains a general equation for particles of any shape:

$$V_d \Delta \rho g + 0.5 \zeta \rho_c v^2 A + V_d (\rho_z + a_d \rho_c) \frac{dv}{dt} = 0. \quad (5.6)$$

For spherical particles Eq. (5.6) can be simplified by introducing the relation:

$$A = \pi d^2/4, \quad V_d = \pi d^3/6 \quad \text{and} \quad a_d = 0.5.$$

After the initial period of acceleration, the velocity of settling particles reaches constant value, and then  $dV/dt = \text{const}$ . The equation, determining the velocity of spherical particles falling down as a function of their density in the medium and particle size, takes the form shown in Eq. (5.1)

There are other forms of Eq. (5.1). For instance, it can be written as a combination of the dimensionless Reynolds (Re) and Archimedes (Ar) numbers (Koch and Noworyta, 1992):

$$4/3Ar = \zeta Re^2, \quad (5.7)$$

where:

$$Re = \frac{vd \rho_c}{\eta_c}, \quad (5.8)$$

$$Ar = \frac{d^3 \Delta \rho \rho_c g}{\eta_c^2} \quad (5.9)$$

and  $\eta_c$  denotes liquid viscosity. Particle resistance factor  $\zeta$  depends on hydrodynamics of the system characterized by the Reynolds number. Table 5.1 shows the expressions for the resistance factor which can be applied to Eq. (5.1) in order to calculate the velocity of particle settling. With reasonable accuracy Eq. (5.1) can be also be used for the determination of settling velocity of non-spherical particles. It requires however, introducing an appropriate function for the resistance factor which depends not only on the Reynolds number but also on the shape of the particle. Selected relations for  $\zeta$  for non-spherical particles are shown in Table 5.1.

Table 5.1. Dimensionless resistance factor for particles of different shapes and for various modes of settling

Mode of particle settling	Reynolds number (for particles)	Resistance factor, $\zeta$	
		spherical particles	non-spherical particles
Laminar (Stokes)	$Re < 0.2$	$24/Re$	$28.46/[Re \lg(\Phi/0.065)]^*$
Intermediate (Allen)	$0.2 < Re < 5 \cdot 10^2$	$18.5Re^{-0.6}$	–
Turbulent Newton (Rittinger)	$5 \cdot 10^2 < Re < 3 \cdot 10^5$	0.44	0.44–1.9**

\*  $\Phi$  is the sphericity of particle

\*\* Values for different shapes and mode of settling are given in Koch and Noworyta (1992)

The relationship between the resistance factor and the Reynolds number for  $Re < 3 \cdot 10^5$  is given by the Yilmaz equation (Koch and Noworyta, 1992):

$$\zeta = \frac{24}{Re} + \frac{3.73}{\sqrt{Re}} - \frac{0.00483 \sqrt{Re}}{1 + 3 \cdot 10^{-6} Re^{1.5}} + 0.49. \quad (5.10)$$

The equations for the velocity of settling presented so far do not directly show the relation between the velocity of settling and particle density and its size. It can be done separately for each range of the Reynolds numbers shown in Table 5.1. For  $Re < 0.2$ , i.e. for spherical particles with diameter approximately from 0.1 mm to 0.1 mm (exact range of particle size depends on their density and shape) one can use the equation:

$$v = 0.0556 \frac{\Delta\rho g d^2}{\eta} = f(\Delta\rho^{0.5} d). \quad (5.11)$$

This is the so-called Stokes range of particle settling, also known as laminar settling.

For the Reynolds numbers  $0.2 < Re < 5 \cdot 10^2$  (also called the Allen or transition range) the resistance factor vs. the Reynolds number is  $\zeta = 18.5 Re^{-0.6}$  (Table 5.1). The presentation of a simple formula describing the settling velocity in the Allen range is not an easy task. It is so because the settling velocity depends on the Reynolds number, which, in turn, depends on the same velocity we use for calculations. Various approximations have been used in different works. They usually are incomplete as they contain the constants for which the values are not provided (Laskowski and Luszczykiewicz 1989; Bogdanov, 1972; Malewski, 1981). The best seems to be diagrams, presented for instance by Kelly and Spottiswood (1982). It should be added that when it is not important to have the equation but only the

values of the settling velocity, they can be calculated by introducing the expression  $\zeta = 18.5 \text{ Re}^{-0.6}$  to Eq. (5.1). To approximately write the relationship between velocity of falling and particle density and its size for the Allen range, i.e. for the particles of diameter from 0.1 mm to 1 mm, one can make use of the formula (Laskowski and Luszczykiewicz (1989):

بالنسبة لأرقام رينولدز  $0 < \text{Re} < 5 \cdot 10^2$  وتسمى أيضا ألين أو النطاق الانتقالي (، فإن عامل المقاومة مقابل رقم رينولدز هو  $\zeta = 18.5 \text{Re}^{-0.6}$  الجدول (5.1) إن تقديم صيغة بسيطة تصف سرعة الترسيب في نطاق ألين ليس بالمهمة السهلة. ذلك لأن سرعة الترسيب تعتمد على رقم رينولدز، والذي بدوره يعتمد على نفس السرعة التي نستخدمها في العمليات الحسابية. تم استخدام تقديرات تقريبية مختلفة في أعمال مختلفة. وعادة ما تكون غير مكتملة لأنها تحتوي على الثوابت التي لا يتم توفير القيم لها (Laskowski and Luszczykiewicz 1989; بوغدانوف، 1972؛ ماليوسكي، 1981) يبدو أن الأفضل هو الرسوم البيانية، التي قدمها على سبيل المثال Kelly and Spottiswood (1982). يجب إضافة أنه عندما لا يكون من المهم الحصول على المعادلة ولكن فقط قيم سرعة الترسيب، يمكن حسابها عن طريق إدخال التعبير  $\zeta = 18.5 \text{ Re}^{-0.6}$  إلى Eq. (5.1) لكتابة العلاقة تقريبا بين سرعة السقوط وكثافة الجسيمات وحجمها لنطاق ألين، أي بالنسبة للجسيمات التي يتراوح قطرها من 0.1 مم إلى 1 مم، يمكن للمرء الاستفادة من الصيغة (Laskowski and Luszczykiewicz (1989):

$$v = k_A d \sqrt[3]{\frac{\Delta \rho^2}{\eta \rho_c}} = f(\Delta \rho^{2/3} d), \quad (5.12)$$

where  $k_A$  is a dimensionless constant. This is an approximate formula as it has been derived by assuming that  $\zeta$  is proportional to  $\text{Re}^{-0.5}$  instead of  $\text{Re}^{-0.6}$ . For a turbulent movement of the liquid around a settling particle (the Rittinger or Newton range of settling for  $5 \cdot 10^2 < \text{Re} < 3 \cdot 10^5$  and approximate particle diameter  $d > 1$  mm) the particle sedimentation velocity is:

$$v = 1.74 \sqrt{\frac{\Delta \rho d g}{\rho_c}} = f(\Delta \rho d). \quad (5.13)$$

It results from formula (5.13) that the particle density increasingly effects the settling velocity and the hydraulic or air separation, because as the Reynolds number increases the density exponent also increases. For the analysis of settling of free falling particles of different sizes and densities one can apply the so called free-settling ratio, ie. the ratio of particle size required for two minerals to fall at equal rates (Wills, 1985). They are obtained by dividing the expression for settling velocity for one kind of particles by velocity of another kind of particles. For example, for a turbulent flow and spherical particles, when the falling velocity of particle A and B are identical, it is possible to obtain a relation for free-settling ratio C.

ينتج عن الصيغة (5.13) أن كثافة الجسيمات تؤثر بشكل متزايد على سرعة الترسيب والفصل الهيدروليكي أو الهواء ، لأنه مع زيادة عدد رينولدز ، تزداد الكثافة الأس أيضا. لتحليل ترسب جزيئات السقوط الحر ذات الأحجام والكثافات المختلفة ، يمكن للمرء تطبيق ما يسمى بنسبة الترسيب الحر ، أي نسبة حجم الجسيمات المطلوبة لسقوط معدنين بمعدلات متساوية) ويلز ، (1985 يتم الحصول عليها بقسمة التعبير عن سرعة الاستقرار لنوع واحد من الجسيمات على سرعة نوع آخر من الجسيمات. على سبيل المثال ، بالنسبة للتدفق المضطرب والجسيمات الكروية ، عندما تكون سرعة سقوط الجسيم A و B متطابقة ، فمن الممكن الحصول على علاقة لنسبة الاستقرار الحر. C.

$$C = \frac{d_A}{d_B} = \frac{\Delta \rho_B}{\Delta \rho_A} . \quad (5.14)$$

The value of the free-settling ratio determines the susceptibility to separation. The main material parameter of separation in the media is the settling velocity of particle (Fig. 5.3), which depends, first of all, on particle density and size. If separation in the media involves materials of similar particle density or size, the main parameter of hydraulic or air separation becomes the other variable parameter.



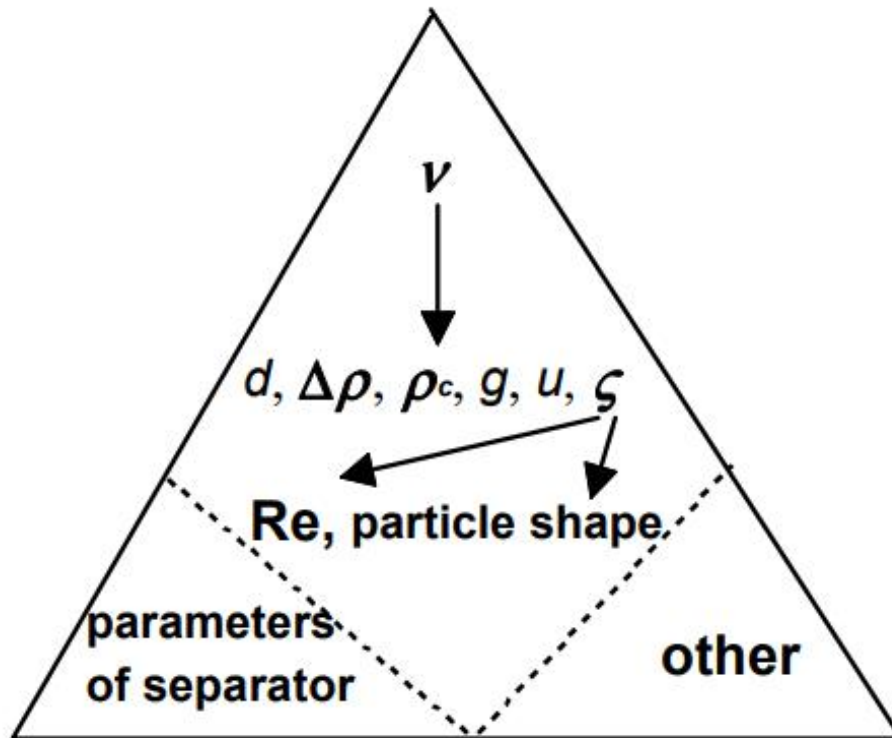


Fig. 5.3. General characteristics of hydraulic and air separations. The main material parameter is settling velocity  $v$ . Meaning of other parameters is given in the text

Additional obstacle in determination of particle movement in real suspensions is their hindered settling, occurring at high concentrations of the particles in the classifier. For such cases the presented above formula for free settling of particles must be modified. Appropriate formulas can be found in literature.

## 5.2. Classification by sedimentation

In sedimentation classifiers the feed is supplied from the upper part of the container and particles fall down in water or air vertically or nearly vertically. Particle separation takes place according to the difference in their velocity of settling. Fine and light particles, i.e. those which settle slowly, are removed from

the classifier together with water as overflow. The particles settling rapidly are removed either as an underflow at the bottom of the vessel, or with the use of appropriate mechanical devices. Therefore, sedimentation separators can be divided into mechanical and non-mechanical ones. Non-mechanical cone separator is shown in Fig. (5.1a), while Fig. (5.4) presents typical mechanical separator also called coil classifier. In these devices the particles settling rapidly are removed upwards from the container bottom with the use of a spiral while slowly settling particles are removed as an overflow in the upper part of classifier (Fig. (5.4)).

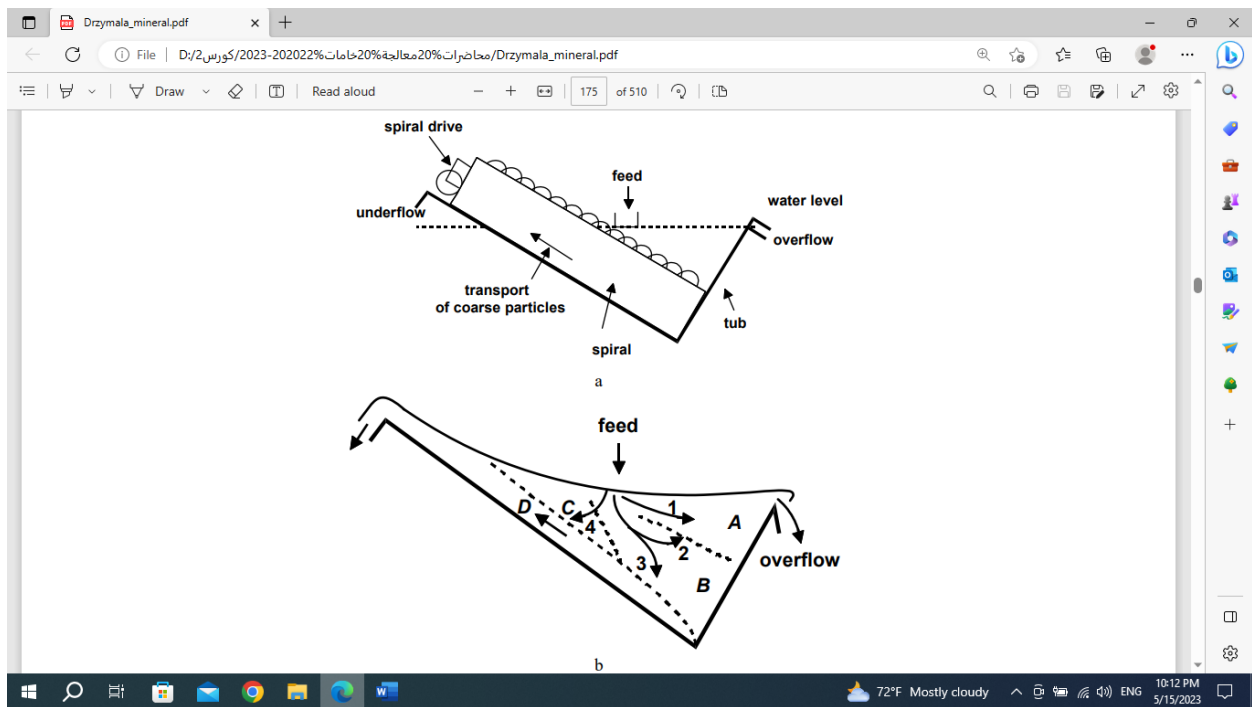


Fig. 5.4. Coil classifier: a – elements, b – regions and direction of particles movement (after Steward and Restarick, 1967)

In a small model coil classifier (Steward and Restarick, 1967) the particles of the feed move in four directions, assigned with numbers 1-4, forming regions A-D. In region A, situated above the spiral, particle concentration is low and particles move towards the overflow. In region B, fine particles pass towards the overflow,

while the larger ones towards the underflow. In region C, most particles pass to the large particles fraction, commonly called the sandy fraction, while in region D large particles are transported with the use of a coil device (spiral) to the underflow. There are many types of mechanical sedimentation separators in which spirals are replaced with rakes of blades. In some classifiers rakes direct large particles to the underflow at the container bottom (Kelly and Spottiswool, 1982). One of the most important classification parameters (for a particular concentration of particles) is  $d_{50}$ . For low solids concentrations, when free sedimentation of particles takes place,  $d_{50}$  can be estimated from Eq. (5.1). When hindered settling occurs and also when there is some movement of the medium, determining  $d_{50}$  becomes more difficult. A full description of coil classifier operation does not exist due to considerable number of parameters effecting the process. Sztaba and Nowak (2000) defined and systematized a significant number of those parameters.

### 5.3. Fluidizing classification

In fluidizing classifiers (elutriators) (Fig. 5.1b) a stream of water beneath the tank is additionally introduced into the classifier with settling particles. Then, calculation of the forces balance and velocity of sedimentation require taking into account the flow of the medium. The rule of separation in those devices is simple. For ideal separation the particles whose sedimentation velocity is lower than the velocity of up going medium flow, are carried by the medium to the overflow, while a particle having higher sedimentation velocity is directed to the underflow (Fig. 5.5). The particle of size  $d_{50}$  has zero or close to zero velocity of settling in relation to the separator and therefore it has the same chance to report, after suitable period of time, either to underflow or overflow of the classifier.

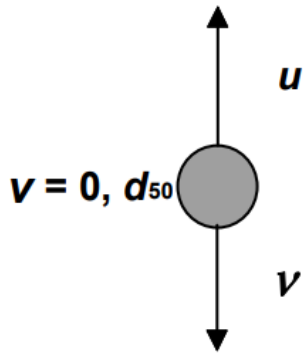


Fig. 5.5. In fluidized classifiers particle of size  $d_{50}$  has the same chance to move upward and downward:  $u$  – velocity of upward movement of medium,  $v$  – velocity of free falling of particle in medium,  $v = (v - u)$  – velocity of particle movement in relation to classifier

The  $d_{50}$  value for a particular separator can be estimated from the equations for free sedimentation. One of the equations taking into account the hindered settling is the relation (Kelly and Spottiswood, 1982):

$$v_h = v(\epsilon)^n, \quad (5.15)$$

where:

$v$  – particle velocity during free sedimentation

$v_h$  – velocity of hindered settling

$\epsilon$  – porosity of fluidized layer  $(\rho_z - \rho_m)/(\rho_z - \rho_c)$

$n$  – constant depending on the Reynolds number

$\rho_z$  – particle density

$\rho_m$  – suspension density

$\rho_c$  – medium density. Irregular flow of the medium in fluidizing columns can also considerably effect the change of separation  $d_{50}$ .

## 5.4. Classification in horizontal stream of medium

The feed consisting of medium and particles can be also supplied from one of the sides of the classifying vessel. In such a case additional horizontal force acting on particles, together with the remaining forces (gravity, buoyancy, resistance) create diagonal settling of particles.

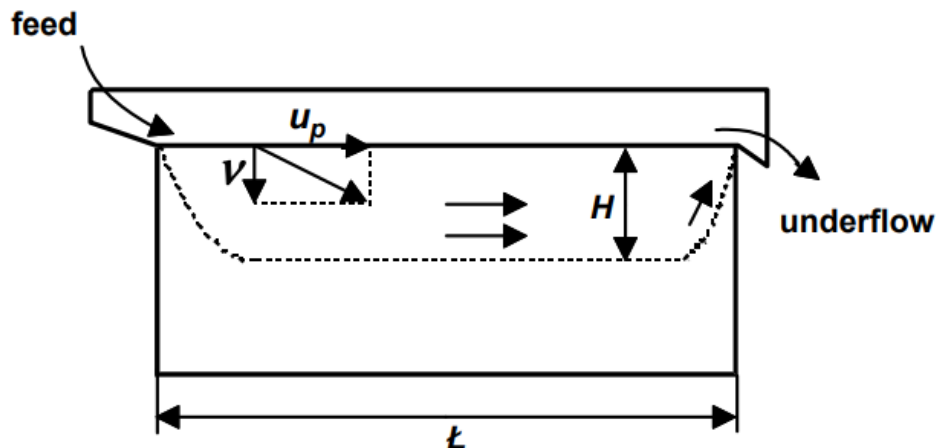


Fig. 5.6. Principle of particles separation of in a horizontal stream of liquid (after Laskowski et al., 1977)

In the separator, the feed is supplied from one side and the overflow is removed from the other side. The stream of liquid moving in the separators reaches  $H$  depth, the length of the separator is  $L$  and the stream of suspension moves at velocity  $u_p$ . The time necessary for horizontal movement of the suspension is:

$$t_{\text{vertical}} = L/u_p, \quad (5.16)$$

and the relation between vertical particle falling time  $t_{\text{pion}}$  at depth  $H$  takes the following form:

$$t_{\text{horizontal}} = H/v. \quad (5.17)$$

For the cut size particles ( $d_{50}$ ), which will be present at the corner of the classifier and still having a chance to report to the overflow horizontal =  $t_{\text{vertical}}$ . Thus:

$$v_{d50} = H \text{ up}/t. \quad (5.18)$$

There are many designs of hydraulic classifier utilizing horizontal movement of the medium stream. A scheme of a multiproduct classifier is shown in Fig. 5.1c.

## 5.5. Classification in pulsating stream

Classification with the use of pulsating medium stream is performed in jigs. The feed is supplied onto a jig screen and then it is treated with a pulsating stream of the medium, which is usually water. Water going upwards through a screen loosens the sediment, while a downward movement of the stream causes sedimentation and stratification of the sediment (Fig. 5.7). Medium pulsation is achieved due to the movement of a piston or air pressure, movement of membrane, movement of the screen surface, as well as due to other procedures.

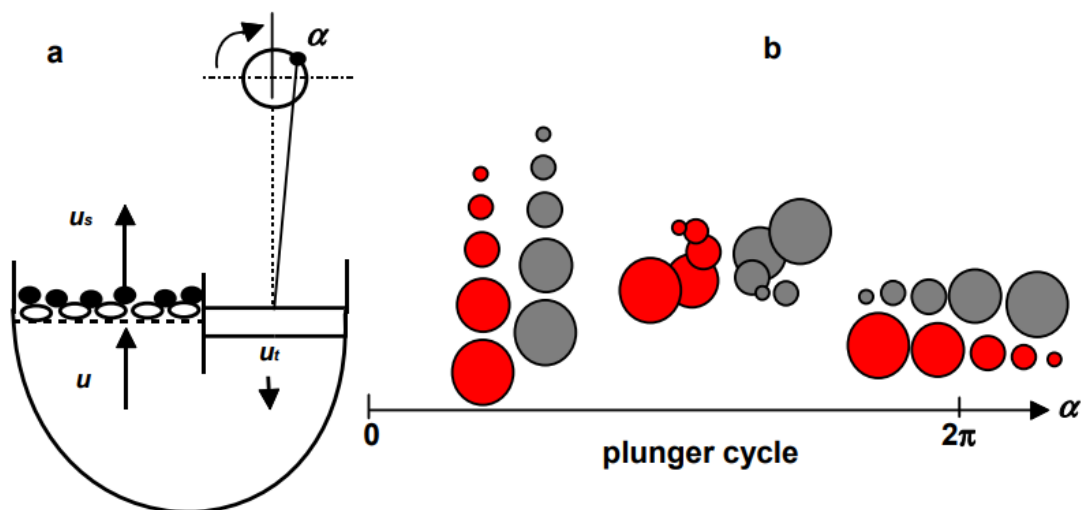


Fig. 5.7. Operation principles of jig:  $u_s$  – velocity of upward stream of water,  $u$  – velocity of water under screen surface,  $u_t$  – piston velocity

Jigs work continuously and they are used, first of all, for hydraulic separation of coarse particles according to the difference in their densities. Separation into the particles settling slowly and those settling fast is obtained due to appropriately situated dam and gate. More detailed description of jigs operation and the effect of different parameters on sedimentation can be found in scientific literature.

## 5.6. Hydrocyclones

Classification can be also conducted in a medium passing through the classifier with a spiral-type particle movement. This kind of the movement takes place in air cyclones, centrifuges, and hydrocyclones and it results from a cylindrical shape of the separator (Fig. 5.1e), as well as from pushing the feed stream tangent to the classifier wall. As a result of the spiral movement, the particles are subjected to centrifugal force which is the main separating force. The operation principle of a hydrocyclone and its design is shown in Fig. 5.8.

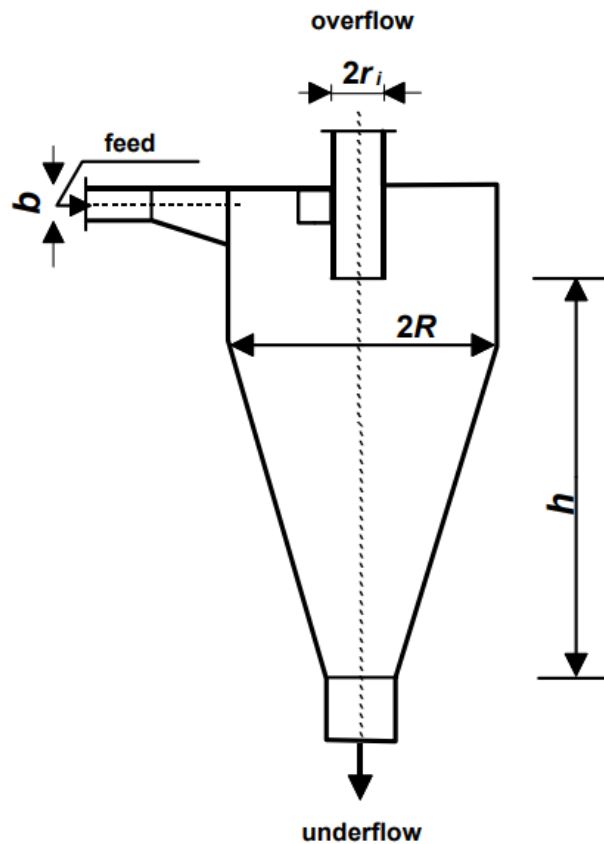


Fig. 5.8. Selected parameters of hydrocyclone

Particle movement in the hydrocyclone results from the force balance in the system. Resultant force  $F_w$  effecting the particles is the difference between the centrifugal force  $F_{cent.}$  causing particle movement towards hydrocyclone walls and medium resistance force  $F_R$  directed to the cyclone axis:

$$F_w = F_{cent.} + F_R. \quad (5.19)$$

After inserting physical expression for particular forces to Eq. (5.19), one obtains a general equation describing particles movement in a hydrocyclone:



$$m \frac{dv}{dt} = \frac{\pi d^3}{6} (\rho_z - \rho_c) \frac{v_t^2}{R} - \zeta \frac{\rho_c}{2} v_w^2 \frac{\pi d^2}{4}, \quad (5.20)$$

where:

$m$  – particle mass ( $\pi d^3 \rho_z / 6$ )

$t$  – time

$v$  – relative velocity of particle movement in the suspension in hydrocyclone

$v_t$  – liquid tangent velocity in the suspension depending on inlet pressure

$\rho_z$  – particle density

$\rho_c$  – liquid density

$R$  – hydrocyclone radius in the consideration site

$d$  – particle diameter

$\zeta$  – particle resistance factor.

As the description of particle movement is quite complicated, empirical relations are used for hydrocyclone design. Koch and Nowaryta (1992) provided, for example, an equation for determining the cut-size ( $d_{50}$ ) of the hydrocyclone:

$$d_{50} = \sqrt{\frac{18 \eta_c V_o R \rho_m}{2 \pi d_i h (\rho_z - \rho_c) \Delta p}}, \quad (5.21)$$

where:

$\eta_c$  – suspension viscosity

$V_o$  – suspension stream (volume per unit time)

$d_i = 2r_i$  – overflow outlet diameter

$h$  – hydrocyclone height from underflow to the overflow pipe

$g$  – acceleration due to gravity

$\rho_c$  – suspension density

$R$  – hydrocyclone radius

$\Delta p$  – pressure drop in hydrocyclone.

The principle of hydrocyclone operation was schematically shown in Fig. 5.1e. The suspension pumped tangent to hydrocyclone walls, due to its variable diameter, forms a spiral, directed downwards the stream around the walls of hydrocyclone. Since large particles are more strongly subjected to centrifugal force, the spiral carries mostly large particles. Specific design of hydrocyclone causes that around its axis there is an air core, close to which there is a second spiral stream of the liquid, directed upwards.

The secondary spiral stream carries fine particles to the overflow. There are many types of hydrocyclones. Generally, they can be divided into cylindrical, conic and cylindrical–conic hydrocyclones. They are applied in grinding circuits for removal of fine particle from the systems. The main advantage of hydrocyclones is simplicity of their operation, small size and low price. Hydrocyclones do not possess movable parts. They can work under different angles, although vertical position is recommended to create gravitation discharge of the underflow. Separation time in these classifiers is short. Hydrocyclones, however, have some disadvantages. One of them is not always precise particle classification. The devices are not suitable for extremely fine particles, especially of a micron size. They also require stable composition of feed.