— University of Mosul —
College of Petroleum & Mining Engineering

Introduction and Errors Sources

Lecture No.1

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Numerical Analysis1

Chapter One
Introduction and Errors Sources

1.1 Introduction in Numerical Analysis

Numerical analysis involves the *study*, *development*, and *analysis* of algorithms for obtaining numerical solutions to various mathematical problems. Frequently, numerical analysis is called the *mathematics of scientific computing*.

Most computers deal with real numbers in the binary system, in contrast to the **decimal number** system that humans prefer to use. The **binary system** uses 2 as the base in the same way that the **decimal system** uses 10.

The representation of Fractions:

If x is a positive real number, then its **integral part** x_I is the largest integer less than or equal to x, while $x_F=x-x_I$ is its **fractional part**. The fractional part can always be written as ***decimal fraction***:

$$x_F = \sum_{k=1}^{\infty}\left(b_k \times 10^{-k}\right)$$

where each b_k is a nonnegative integer less than 10. If $b_k=0$ for all k greater than a certain integer, then the fraction is said to terminate. Thus $\frac{1}{4}=0.25=2\times10^{-1}+5\times10^{-2}$ is a terminating decimal fraction, while $\frac{1}{3}=0.333...=3\times10^{-1}+3\times10^{-2}+3\times10^{-3}+...$ is not.

Definition: (Rounding and Chopped)

We say that a number x is **chopped** to n digits or figures when all digits that follow the nth digit are discarded and none of the remaining n digits is changed. Conversely, x is **rounded** to n digits or figures when x is replaced by an n-digit number that approximates x with minimum error.

If x is rounded to that $\hat{x}$ is the n-digit approximation to x, then

$$|x - \hat{x}| \leq \frac{1}{2} \times 10^{-n}$$

If x is a decimal number, the chopped or truncated n-digit approximation to it is the number $\tilde{x}$ obtained by simple discarding all digits beyond the nth. For it, we have $|x - \tilde{x}| \leq 10^{-n}$

Definition: (Absolute and Relative Errors)

The **absolute error** is the magnitude of the difference between the exact value and its approximation. Given exact value x and its approximation x^*, the **error** is x-x^*. The **absolute error** is $e_x = |x - x^*|$ and the **relative error** is the absolute error divided by the magnitude of the exact value i.e. $\delta_x = \left|\frac{x - x^*}{x}\right|$.

<u>Example(1)</u>: Let p= 0.3×10^{-3} , $p^*=0.31\times10^{-3}$, find the values of absolute and relative errors.

<u>Solution</u>:

The absolute error is $e_p = |\, p-p^* | = |\, 0.3\times10^{-3} - 0.31\times10^{-3} \,| = 0.1\times10^{-4}$

And the relative error is

$$\delta_p = \left|\frac{p - p^*}{p}\right| = \frac{e_p}{|p|} = \frac{0.1\times10^{-4}}{0.3\times10^{-3}} = 0.3333333\times10^{-1}$$

1.2 Errors Sources

Often the error in an approximate solution to a problem is the accumulation of many errors caused by:

1- Inherent errors

It is the error resulting from the entered data values resulting from the imprecision of measurements, such as the readings of some devices in a laboratory experiment.

2- Formulation errors

It is the error resulting from analyzing a specific problem and converting it into a mathematical question, since the mathematical model rarely gives the true formula of the phenomenon, i.e. neglecting some factors and effects in this case, then the results are loaded with formulation errors.

3- Computation errors

a) Truncation errors

This is the error caused by replacing an infinite process with a finite process for example:

$$f(x) = \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$f(2) = \sin(2) = 2 - \frac{(2)^3}{3!} + \frac{(2)^5}{5!} - \cdots$$

b) Round of errors

This error results from rounding decimal fractions with numerical decimal places to numbers with decimal places commensurate with the nature of the problem and the required accuracy, for example:

$$\frac{1}{3} = 0.333333 \approx 0.3333$$

$$3.1415926 \approx 3.14159$$

$$0.5235724 \approx 0.5236$$

4- Accumulation errors

It is the error resulting from the repetition of a set of calculations with successive steps and increases depending on the approximate values calculated from the previous steps.

$x_0, x_1, \dots, x_n.$