



— University of Mosul —
College of Petroleum & Mining Engineering



Errors in Calculations

Lecture No.2

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1.3 Errors in Calculations

Let x^*, y^* are approximation values to the exact values x, y respectively, with the absolute errors e_x, e_y and the relative errors δ_x, δ_y

1- The addition

$$e_{x+y} = |(x + y) - (x^* + y^*)| = e_x + e_y$$

$$\delta_{x+y} = \frac{e_{x+y}}{|x + y|} = \frac{e_x + e_y}{|x + y|} = \frac{|x| \delta_x + |y| \delta_y}{|x + y|}$$

2- The subtraction

$$e_{x-y} = |(x - y) - (x^* - y^*)| = e_x - e_y$$

$$\delta_{x-y} = \frac{e_{x-y}}{|x - y|} = \frac{e_x - e_y}{|x - y|} = \frac{|x| \delta_x - |y| \delta_y}{|x - y|}$$

3- The multiplication

$$\begin{aligned} e_{xy} &= |(xy) - (x^* y^*)| \\ &= xy - (x - e_x)(y - e_y) \\ &= xy - (xy - ye_x - xe_y + e_x e_y) \\ &= ye_x + xe_y - e_x e_y \\ &= ye_x + xe_y \end{aligned}$$

$$\begin{aligned} \delta_{xy} &= \frac{e_{xy}}{|xy|} = \frac{ye_x + xe_y}{xy} \\ &= \frac{ye_x}{xy} + \frac{xe_y}{xy} \\ &= \delta_x + \delta_y \end{aligned}$$

4- The division (**Home work**)

$$e_{x/y} = \frac{x}{y} \left(\frac{e_x}{x} - \frac{e_y}{y} \right)$$

$$\delta_{x/y} = \frac{e_{x/y}}{|x/y|} = \delta_x - \delta_y$$

Example(2): Find the upper limit of absolute error in **addition** $x+y=12.23+3.12$

Solution:

$$e_x = \frac{1}{2} \times 10^{-n}, \quad n=2$$

$$e_x = \frac{1}{2} \times 10^{-2} = \frac{1}{200} = 0.005$$

Also

$$e_y = \frac{1}{2} \times 10^{-2} = \frac{1}{200} = 0.005$$

Then

$$e_{x+y} = e_x + e_y = 0.005 + 0.005 = 0.01$$

Exercise:

Find the upper limit of absolute error in multiplication $x.y=(2.23).(12.3)$

Chapter Two

Solutions of Equations in one Variable

Definition: If $f(p)=0$ then p is a root of the equation $f(x)=0$.

Note: If p is the exact root of the equation $f(x)=0$ then p_n is the approximation root and $|p - p_n| \leq \varepsilon$ or $|f(p_n)| \leq \gamma$ where ε and γ are small values accuracy of the solution.

2.1 Determination of roots positions

To locate the position of roots of the function (equation) $f(x)=0$, let $f(x)$ be continuous function and the roots are real

1- $f(x)$ is polynomial

If the value p is a root of the function

$$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$$

$$\text{Then } |p| \leq 1 + \max |a_i| \leq 1 + \lambda \quad \text{and} \quad \lambda = \max_{\forall i} |a_i|$$

Example(1): Find the approximate location of the root for the function

$$f(x)=x^3-2x^2+3x+1=0$$

Solution: By using $|p| \leq 1 + \lambda$ and $\lambda = \max_{\forall i} |a_i|$ we have

$$-(1 + \lambda) \leq p \leq 1 + \lambda ,$$

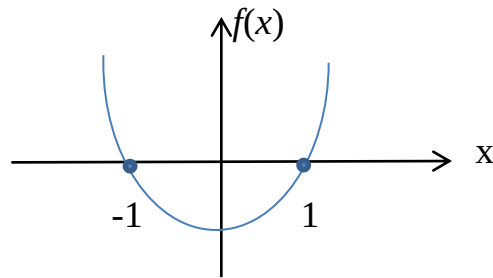
$$\lambda = \max_{\forall i} |a_i| = \max\{|1|, |-2|, |3|, |1|\} = 3$$

$$-4 \leq p \leq 4$$

2- Set the location of the roots using the graph

- a- When drawing $y=f(x)$, the points of intersection of the function curve with the x-axis are the roots of the equation $f(x)=0$.

Example(2): Let $f(x)=x^2-1=0$

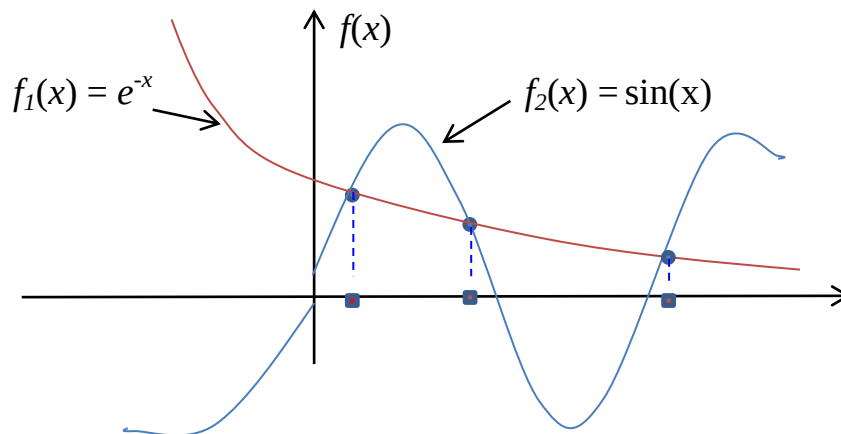


- b- If it is difficult to draw the function $y=f(x)$, then we write $f(x)$ as the formula $f_1(x) = f_2(x)$ where $f_1(x)$, $f_2(x)$ everyone can draw easily so that the points of the intersection of the two curves on the axis are the roots of the equation $f(x)=0$.

Example(3): Let $f(x)=e^x \sin(x)-1=0$

we write $f_1(x) = f_2(x)$, i.e.:

$$e^x \sin(x) - 1 = 0 \longrightarrow e^x \sin(x) = 1 \longrightarrow e^{-x} = \sin(x)$$



3- Set roots locations by changing the function signal

let $f(x)$ be continuous function on the interval $[a,b]$, we divide the interval $[a,b]$ into n subintervals $a = x_0 < x_1 < \dots < x_{n-1} < x_n = b$ where

$$x_i = a + ih, i = 0, 1, \dots, n; \quad h = \frac{b-a}{n}.$$

If $f(x_i) \times f(x_{i+1}) < 0$ for any $0 \leq i \leq n$, then there exists c , $a < c < b$ for which $f(c) = 0$.

Example(4): Find the approximate location of the roots for the functions:

1. $f(x) = x^4 - 7x^3 + 3x^2 + 26x - 10 = 0$ on the interval $[-8, 8]$ with $n=4$ and $n=8$.

2. $f(x) = x^3 + 4x^2 - 10 = 0$ on the interval $[1, 2]$ with $n=5$.

Solution:

1- Let $n=4$, $h = \frac{b-a}{n} = \frac{8 - (-8)}{4} = 4$

X	-8	-4	0	4	8
f(x)	+	+	-	-	+

There is a root between $(-4, 0)$ and $(4, 8)$.

If $n=8$, $h=2$:

X	-8	-6	-4	-2	0	2	4	6	8
f(x)	+	+	+	+	-	+	-	+	+

There is a root between $(-2, 0)$, $(0, 2)$, $(2, 4)$ and $(4, 6)$.

2- Let $n=5$, $h=0.2$

X	1	1.2	1.4	1.6	1.8	2
f(x)	-	-	+	+	+	+

There is a root between $(1.2, 1.4)$.