



— University of Mosul —
College of Petroleum & Mining Engineering



False-Position method (Regula falsi method):

Lecture No.4

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(2) False-Position method (Regula falsi method):

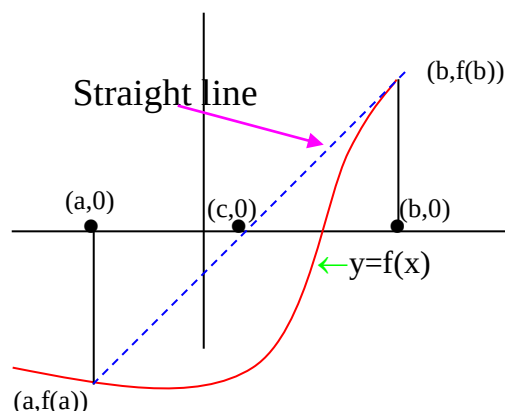
Suppose a continuous function f defined on the interval $[a,b]$ is given with $f(a)$ and $f(b)$ of opposite sign (i.e. $f(a) \times f(b) < 0$). To derive a formula for false-position method, approximate the graph of the function $f(x)$ by a straight line on $[a,b]$ connecting $(a, f(a))$ and $(b, f(b))$ which intersect x -axis at $(c,0)$ where c is more approximate to the exact (actual) root than a and b .

To obtain a formula for c , we use the slop equality:

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(b) - y}{b - x} \Rightarrow \frac{f(b) - f(a)}{b - a} = \frac{f(b) - 0}{b - c} \\ &\Rightarrow (f(b) - f(a))(b - c) = f(b)(b - a) \\ &\Rightarrow c = b - \frac{f(b)(b - a)}{f(b) - f(a)} \end{aligned}$$

To find another approximation:

$$\text{If } f(a) \times f(c) \begin{cases} < 0 & \text{there is a root between } a, c \Rightarrow d = c - \frac{f(c)(c - a)}{f(c) - f(a)} \\ > 0 & \text{there is a root between } c, b \Rightarrow d = b - \frac{f(b)(b - c)}{f(b) - f(c)} \\ = 0 & c \text{ is exact root ((Stop))}. \end{cases}$$



Example (7):

Find an approximate root of the equation $x\sin(x) - 1 = 0$ in the interval $[0,2]$ by using False position method.

Solution: It is possible to use False position method because $f(x)$ is continuous on $[0,2]$ and $f(a)=f(0)=-1$; $f(b)=f(2)=0.81859 \Rightarrow f(a) \times f(b) < 0$.

$$\Rightarrow c_1 = b - \frac{f(b)(b-a)}{f(b)-f(a)} = 2 - \frac{f(2)(2-0)}{f(2)-f(0)} = 2 - \frac{(0.81859)(2)}{0.81859-(-1)} = 1.0997502$$

$$f(c_1)=f(1.0997502)=-0.0200192$$

$$\Rightarrow f(c_1) \times f(a) > 0 \Rightarrow \text{there is a root between } c_1 \text{ and } b \Rightarrow [c_1, b]$$

$$\Rightarrow c_2 = 2 - \frac{f(2)(2-1.0997502)}{f(2)-f(1.0997502)} = 2 - 0.878759 = 1.1212407$$

$$f(c_2)=f(1.1212407)=0.0098345$$

$$\Rightarrow f(c_2) \times f(c_1) < 0 \Rightarrow \text{there is a root between } c_1 \text{ and } c_2 \Rightarrow [c_1, c_2]$$

$$\Rightarrow c_3 = 1.1141612$$

$$f(c_3)=f(1.1141612)= 0.0000056$$

$$\Rightarrow f(c_3) \times f(c_1) < 0 \Rightarrow \text{there is a root between } c_1 \text{ and } c_3 \Rightarrow [c_1, c_3]$$

$$\Rightarrow c_4 = 1.1141571$$

$$f(c_4)=f(1.1141571)= 0$$

Stop iteration with $c_4=1.1141571$ is the root.

Algorithm (False-Position method)

Input: a, b, ε , δ , f(x)

Step(1): If $f(a) \times f(b) > 0$ then stop (does not exist root).

Step(2): Put $i=1$ compute $dx = \frac{f(b)(b-a)}{f(b)-f(a)}$ and Set $c = b - dx$, and find $f(c)$.

Step(3): If $f(c)=0$ then print c is a root and stop.

Step(4): else if $f(a) \times f(c) > 0$ then $a=c$ and $f(a)=f(c)$

Step(5): If $f(a) \times f(c) < 0$ then $b=c$ and $f(b)=f(c)$

Step(6): Evaluate if $|dx| > \varepsilon$ or $|f(c)| > \delta$ then go to step(2) with $i=i+1$.

Step(7): Print c

Home work:

Find an approximate root of $f(x)=x \ln(x)-1=0$ in the interval $[1,2]$ by using False-Position method with error $\leq \varepsilon = 10^{-3}$.

Properties of the False-Position method

خواص الطريقة

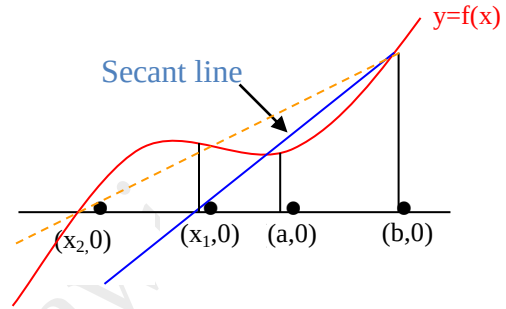
- 1- هذه الطريقة تتقارب اسرع من طريقة التنصيف (Bisection)
- 2- نلاحظ عدم استخدام مقياس التوقف $|b_i - a_i| < \varepsilon$ بسبب اذا كانت $b_i - a_i$ تقترب من الصفر فان المقياس الثاني $|f(c_i)| \neq 0$.
- 3- مضمونة الوصول الى الجذر ولهذا فهي ذات نوع اقترابي شمولي (Global convergent).
- 4- سرعة اقتراب الطريقة خطي (Linear).

(3) Secant Method:

Suppose a continuous function f defined on the interval $[a,b]$ is given, but now we do not force the iterates to bracket a root. The graph of the function f is approximated by a secant line, we get

$$\frac{f(b) - f(a)}{b - a} = \frac{f(b) - y}{b - x} \Rightarrow \frac{f(b) - f(a)}{b - a} = \frac{f(b) - 0}{b - x_1}$$

$$\Rightarrow x_1 = b - \frac{f(b)(b - a)}{f(b) - f(a)} = \frac{af(b) - bf(a)}{f(b) - f(a)}$$



Similarly
$$x_2 = \frac{bf(x_1) - x_1 f(b)}{f(x_1) - f(b)}$$

In general
$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}; \quad i=1, \dots$$

x_{i+2} is an approximate root if $|x_{i+2} - x_{i+1}| < \varepsilon$ for any i .

Example (8):

Find an approximate root of the equation $x^3 - 3x + 2 = 0$ by using Secant method and take $x_0 = -2.6$, $x_1 = -2.4$, $\varepsilon = 5 \times 10^{-5}$.

Solution: we use the formula
$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}; \quad i=1, 2, \dots$$

$$\begin{aligned} i=1 \Rightarrow x_2 &= \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{(-2.6) f(-2.4) - (-2.4) f(-2.6)}{f(-2.4) - f(-2.6)} \\ &= \frac{(-2.6)((-2.4)^3 - 3(-2.4) + 2) - (-2.4)((-2.6)^3 - 3(-2.6) + 2)}{((-2.4)^3 - 3(-2.4) + 2) - ((-2.6)^3 - 3(-2.6) + 2)} = -2.106599 \end{aligned}$$

$$|x_2 - x_1| = |-2.106599 - (-2.4)| > \varepsilon = 5 \times 10^{-5}$$

$$x_0 = x_1 = -2.4 \quad \text{and} \quad f(x_0) = f(x_1)$$

$$x_1 = x_2 = -2.106599 \quad \text{and} \quad f(x_1) = f(x_2)$$

$$\begin{aligned} \mathbf{i=2} \Rightarrow x_3 &= \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{(-2.4) f(-2.106599) - (-2.106599) f(-2.4)}{f(-2.106599) - f(-2.4)} \\ &= -2.0226414 \end{aligned}$$

$$|x_3 - x_2| = |-2.0226414 - (-2.106599)| > \varepsilon = 5 * 10^{-5}$$

$$x_1 = x_2 = -2.106599 \quad \text{and} \quad f(x_1) = f(x_2)$$

$$x_2 = x_3 = -2.0226414 \quad \text{and} \quad f(x_2) = f(x_3)$$

$$\mathbf{i=3} \Rightarrow x_4 = -2.0015111$$

$$|x_4 - x_3| > \varepsilon = 5 * 10^{-5}$$

$$x_2 = x_3 = -2.0226414 \quad \text{and} \quad f(x_2) = f(x_3)$$

$$x_3 = x_4 = -2.0015111 \quad \text{and} \quad f(x_3) = f(x_4)$$

$$\mathbf{i=4} \Rightarrow x_5 = -2.0000225$$

$$|x_5 - x_4| > \varepsilon = 5 * 10^{-5}$$

$$x_3 = x_4 = -2.0015111 \quad \text{and} \quad f(x_3) = f(x_4)$$

$$x_4 = x_5 = -2.0000225 \quad \text{and} \quad f(x_4) = f(x_5)$$

$$\mathbf{i=5} \Rightarrow x_6 = -2$$

$$|x_6 - x_5| = 0.0000225 < \varepsilon = 5 * 10^{-5}$$

Stop iteration with the root $x_6 = -2$.

Note:

The method is not guarantee to converge, but it does so with a greater speed than the previous methods when it does converge.

Algorithm (Secant method)

Input: $x_0, x_1, \varepsilon, f(x)$

Step(1): Set $i=1$, compute $y_0=f(x_0)$, $y_1=f(x_1)$

Step(2): Evaluate $x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$

Step(3): If $|x - x_1| < \varepsilon$ then print x is a root and stop.

Step(4): else if set $i=i+1$

Step(5): Set $x_0 = x_1$ and $y_0 = y_1$, $x_1 = x$ and $y_1 = f(x)$ then go to step(2)

Home work:

Find an approximate root of equation $f(x)=x-0.2\sin(x)-0.5=0$ in the interval $[0.5,1]$ by using Secant method with $\varepsilon = \frac{10^{-4}}{2}$.

Answer : The root is $x_7=0.615470301$.

Properties of the Secant method

خواص الطريقة

- 1- سرعة الاقتراب في طريقة القاطع فوق الخطية (Supper Linear)
- 2- تحتاج الى حساب قيمة الدالة مرة واحدة لكل تكرار .
- 3- نوع الاقتراب محلي (Locally convergent).
- 4- صيغة الخطأ في طريقة القاطع هي $e_{i+1} = -\frac{e_i e_{i-1}}{2} \frac{f''(\eta_i)}{f'(\xi_i)}$ حيث كل من ξ_i , η_i تقع في الفترة التي تحتوي على الجذر الحقيقي والقيم x_i, x_{i-1} .