University of Mosul

College of Petroleum & Mining Engineering

# Properties of the Fixed-point method

## Lecture No.7


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**Properties of the Fixed-point method** **خواص الطريقة**

1− سرعة الاقتراب في طريقة Fixed-point خطية (Linear convergent) ولكن هي افضل من طريقة التنصيف.

2− مشتقة الدالة g(x) يجب ان تحقق الشرط $|g'(x)| \le k < 1$ عند نقطة البداية $x_0$ لكي نحصل على تكرارات مستقرة ومتقاربة الى الجذر, وايضا" كلما كانت قيمة k صغيرة وقريبة من الصفر كلما كان الاقتراب اسرع للجذر.

3− ان صيغة الحد الاعلى للخطأ في طريقة النقطة الصامدة هي

$$e_i = |x_i - P| \le \frac{k^i}{1-k}|x_1 - x_0| \quad \text{for all } i \ge 1$$

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**(6) Aitken Method:**

To find an approximate root of f(x)=0 by using Aitken method, we need an initial approximate root $x_o$ such that $x_{i+1}=g(x_i)$, i=0, 1, …which slowly convergence to the root P and , by using Aitken's formula:

$$\hat{x}_{i+1} = x_{i+2} - \frac{(x_{i+2} - x_{i+1})^2}{x_{i+2} - 2x_{i+1} + x_i}$$

The sequence $\{\hat{x}_i\}_{i=0}^{\infty}$ can be used to speed up convergence of any sequence that is linearly convergent such that $\{x_i\}_{i=0}^{\infty}$ generate from Fixed-point formula.

**Example (15):** Find the approximate root of the equation $x^2-x-3=0$ by using Aitken's method with take $x_0=2.5$ .

**Solution:**

1- $f(x) = x^2 - x - 3 = 0 \rightarrow x = 1 + \frac{3}{x} = g_1(x)$

2- $x^2 - x - 3 = 0 \rightarrow x = x^2 - 3 = g_2(x)$

3- $x^2 - x - 3 = 0 \rightarrow x = \frac{9x - x^2 - 3}{8} = g_3(x)$

نلاحظ ان $g_1(x)$ وايضا" $g_3(x)$ تحققان شرط التقارب في طريقة fixed-point وهو $|g'(x)| \le k < 1$

بينما $g_2(x)$ لا تحقق شرط التقارب عند نقطة البداية, عليه نكمل الحل باستخدام $g_3(x)$ وكالاتي":

$$x_{i+1} = g_3(x_i) = \frac{9x - x^2 - 3}{8}$$

$$\left.\begin{array}{l} x_0 = 2.5 \\ x_1 = g_3(x_0) = \frac{9x_0 - {x_0}^2 - 3}{8} = 2.40625 \\ x_2 = g_3(x_1) = \frac{9x_1 - {x_1}^2 - 3}{8} = 2.35828 \end{array}\right\} \text{find } \hat{x}_1$$

$$x_3 = g_3(x_2) = g_3(2.35828) = \frac{9x_2 - {x_2}^2 - 3}{8} = 2.33288$$

$$x_4 = g_3(x_3) = g_3(2.33288) = \frac{9x_3 - {x_3}^2 - 3}{8} = 2.31920$$

$$x_5 = g_3(x_4) = g_3(2.31920) = \frac{9x_4 - {x_4}^2 - 3}{8} = 2.31176$$

$\vdots$

Now we find $\hat{x}_1$

$$\hat{x}_1 = x_2 - \frac{(x_2 - x_1)^2}{x_2 - 2x_1 + x_0} = 2.35828 - \frac{(2.35828 - 2.40625)^2}{2.35828 - 2(2.40625) + 2.5} = 2.30802$$

$$\left.\begin{array}{l} x_1 \\ x_2 \\ x_3 \end{array}\right\} \longrightarrow \hat{x}_2$$

$$\hat{x}_2 = x_3 - \frac{(x_3 - x_2)^2}{x_3 - 2x_2 + x_1} = 2.33288 - \frac{(2.33288 - 2.35828)^2}{2.33288 - 2(2.35828) + 2.40625} = 2.30430$$

$$\left.\begin{array}{l} x_2 \\ x_3 \\ x_4 \end{array}\right\} \longrightarrow \hat{x}_3 = x_4 - \frac{(x_4 - x_3)^2}{x_4 - 2x_3 + x_2} = 2.30323$$

$$\left.\begin{array}{l} x_3 \\ x_4 \\ x_5 \end{array}\right\} \longrightarrow \hat{x}_4 = x_5 - \frac{(x_5 - x_4)^2}{x_5 - 2x_4 + x_3} = 2.30289$$

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**<u>Home work:</u>**

Find the approximate root of the equation $x^2$-2x-3=0 by using Aitken's method with take $x_0$=4, (Find only three iteration) .