



— University of Mosul —
College of Petroleum & Mining Engineering



LU Factorization Method

Lecture No.9

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(3) LU Factorization Method

3-1 Doolittle Factorization

To solve

$$A_{n \times n} X_{n \times 1} = B_{n \times 1} \quad \text{..... (1)}$$

by LU factorization method, this method is one of the direct methods, as it converts the coefficient matrix A into the product of two matrices of the same order: L (a lower triangular matrix) and U (an upper triangular matrix).

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 & \cdots & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 & \cdots & 0 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & 1 & 0 \\ \ell_{n1} & \ell_{n2} & \ell_{n3} & \ell_{n4} & \cdots & \ell_{n,n-1} & 1 \end{bmatrix}_{n \times n} \quad \text{and} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} & \cdots & u_{1,n-1} & u_{1n} \\ 0 & u_{22} & u_{23} & u_{24} & \cdots & u_{2,n-1} & u_{2n} \\ 0 & 0 & u_{33} & u_{34} & \cdots & u_{3,n-1} & u_{3n} \\ 0 & 0 & 0 & u_{44} & \cdots & u_{4,n-1} & u_{4n} \\ \vdots & \vdots & \vdots & \vdots & \cdots & u_{n-1,n-1} & u_{n-1,n} \\ 0 & 0 & 0 & 0 & \cdots & 0 & u_{n,n} \end{bmatrix}_{n \times n}$$

Then we solve two triangular systems

$$LY=B \text{ for } Y \text{ and } UX=Y \text{ for } X(\text{solution}) \text{ Where } X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

By using the Gauss elimination method without the partial pivoting process to factorize a matrix A into a product of two matrices L and U.

$$\ell_{ij} = \begin{cases} 0 & \text{if } i < j \\ 1 & \text{if } i = j \\ m_{ij} & \text{if } i > j \end{cases} \quad \text{and} \quad u_{ij} = \begin{cases} r_{ij} & \text{if } i < j \\ 0 & \text{if } i > j \end{cases}$$

$$\Rightarrow A = L.U$$

Example 6: Factorization the matrix A into L.U by using Doolittle Factorization.

$$A = \begin{bmatrix} 4 & -9 & 2 \\ 2 & -4 & 6 \\ 1 & -1 & 3 \end{bmatrix}$$

Solution: After using the elimination process in the Gaussian elimination method, we obtain an upper triangular matrix, which is U:

$$U = \begin{bmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 5 \\ 0 & 0 & -10 \end{bmatrix}$$

and the matrix L is

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix}, \text{ where } m_{21} = \frac{2}{4} = 0.5, \quad m_{31} = \frac{1}{4} = 0.25, \quad m_{32} = \frac{1.25}{0.5} = 2.5$$

$$A = \begin{bmatrix} 4 & -9 & 2 \\ 2 & -4 & 6 \\ 1 & -1 & 3 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 2.5 & 1 \end{bmatrix} \begin{bmatrix} 4 & -9 & 2 \\ 0 & 0.5 & 5 \\ 0 & 0 & -10 \end{bmatrix}$$

Example 7: H.W.

Use LU factorization to solve

$$3x_1 + 3x_3 = 0$$

$$-x_2 + 3x_3 = 1$$

$$x_1 + 3x_2 = 2$$

Exact solution is $x_1 = -\frac{5}{8}$, $x_2 = \frac{7}{8}$, $x_3 = \frac{5}{8}$