



Lecture title: Statistics: Measures of Central Tendency

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Summary:

▪ **Measures of Central Tendency**

1. **Arithmetic Mean \bar{X}** : this is the summation of all observations divided by the number of observations.

$$\bar{X} = \frac{\sum xi}{n}$$

Example: what is the mean (average) of the following data:

12,16,18,21,40,32,27.5

$$\bar{X} = (12+16+18+21+40+32+27.5)/7= 23.5$$

We can also calculate the mean from the frequency table by using the following:

$$\bar{Xf} = \frac{\sum fi xi}{\sum fi}$$

no.	class	Class mid-points(xi)	Class boundaries	Frequency(fi)	fi xi
1	31-40	35.5	30.5-40.5	1	35.5
2	41-50	45.5	40.5-50.5	2	91
3	51-60	55.5	50.5-60.5	5	277.5
4	61-70	65.5	60.5-70.5	15	982.5
5	71-80	75.5	70.5-80.5	25	1887.5
6	81-90	85.5	80.5-90.5	20	1710
7	91-100	95.5	90.5-100.5	12	1146

$$\sum fi = 80 \quad \sum fi xi = 6130$$

$$\bar{Xf} = 6130/80 = 76.6$$



Characteristics of the Mean:

- $\sum (xi - \bar{X}) = 0$
- The sum of the square of the deviations of the items from the arithmetic mean is minimum, that is less than the sum of squared deviations of items from any other value.
 $\sum (xi - \bar{X})^2 = \text{lowest amount}$
- If we add a constant (k) to each value of a data set then the new mean will be:
New mean = old mean + k
- If we multiply each value of a data set by a constant then the new mean will be:
New mean = old mean * k
- The mean of the sum of two variables equals to the sum of two variables.

2. Weighted mean \bar{X}_w

$$\bar{X}_w = \frac{\sum wi xi}{\sum wi}$$

Example: The following are grades of one student in all exams in statistics subject. Find the weighted mean.

exam	degree xi	weight wi	xi*wi
1st	80	15	1200
2nd	55	30	1650
3rd	70	15	1050
4th	50	40	2000

$$\sum wi = 100 \quad \sum xi wi = 5900$$

$$\bar{X}_w = 5900/100 = 59$$

3. **Median:** the middle in a set of observations arranged in order.
 \bar{Me} = middle observation.



If the number of observations is odd then $\overline{Me} = X_{\frac{n+1}{2}}$

If the number of observations is even then $\overline{Me} = (X_{\frac{n}{2}} + X_{\frac{n}{2}+1})/2$

Example: find the median \overline{Me} for the two following sets of data:

1) 80,82,76,87,84

First order 76,80, **82**,84,87 $\overline{Me} = X_{\frac{n+1}{2}}$

$$\overline{Me} = X_{\frac{5+1}{2}} = X_3 = 82$$

2) 2,3,4,5,7,8,9,12

$$\begin{aligned}\overline{Me} &= (X_{\frac{8}{2}} + X_{\frac{8}{2}+1})/2 \\ &= (X_4 + X_5)/2 \\ &= (5+7)/2 = 6\end{aligned}$$

4. **Mode: \overline{Mo}** = the most commonly occurring value in a set of the values. May be there is one or more mode or no mode.

Example: find \overline{Mo} for the following:

1) 1,5,7,6,5,9,4,6 $\overline{Mo} = 5$ and 6

2) 2,7,5,6,4,9,5,9,8,9 $\overline{Mo} = 9$

3) 7,2,6,9,12,3,15,20 $\overline{Mo} = \text{no mode}$

5. **Mid-Range (M.R)** = $\frac{X_{min} + X_{max}}{2}$

Example: find M R for the following: 9,5,4,10,8,2

$$M.R = (2+10)/2 = 6$$