Prestressed Concrete Structures

4th Class – Civil Engineering Department – College of Engineering – University of Mosul

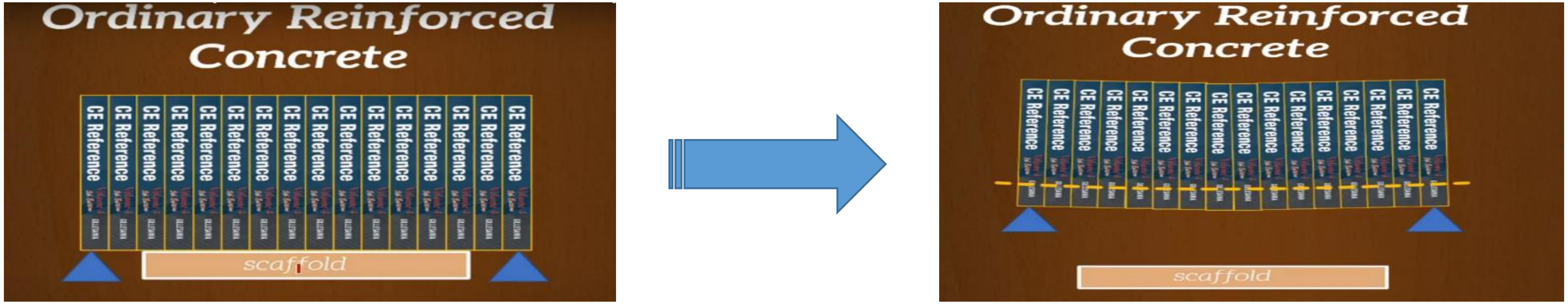
Course Code : CIV 417

2023-2024

(Spring Semester)

Prestressed Concrete

- Prestress can be defined in general as a **pre loading** of a structure before application of the service load, so as to improve its performance in a specify way.
- The original concept of prestressing was to introduce sufficient **precompression** in a beam so that all the tension in the concrete is eliminated in the member at service load.


Ordinary Reinforced Concrete
CE Reference
scaffold
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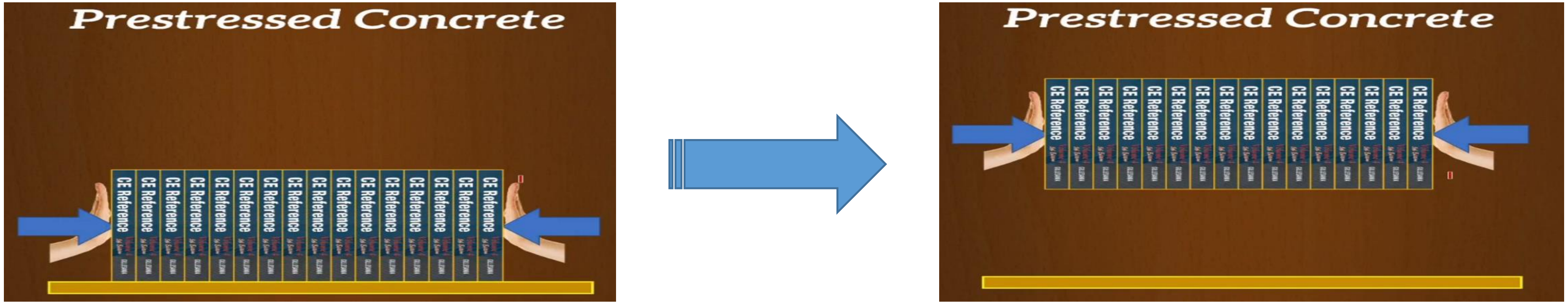
Prestressed Concrete
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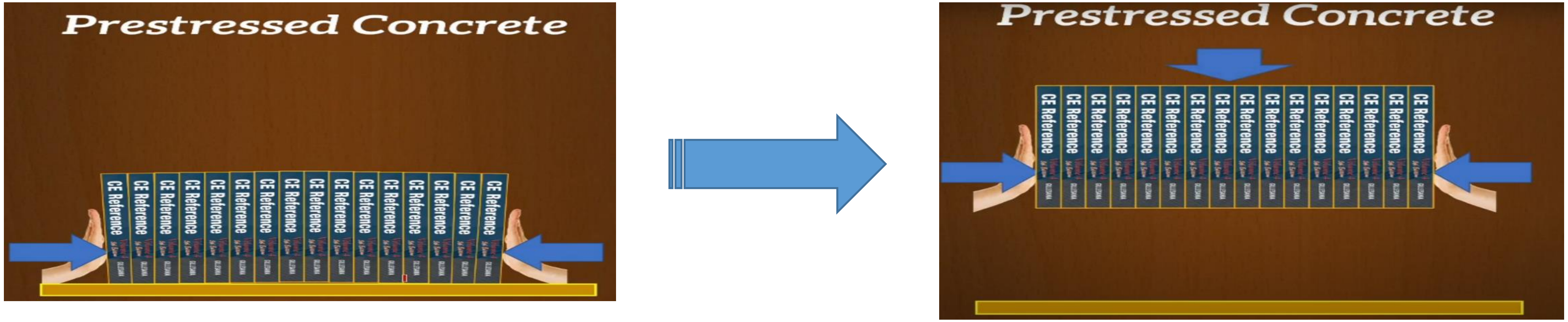
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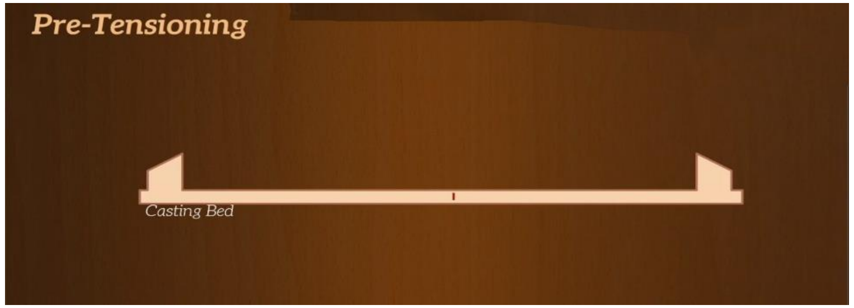
Method of Prestressing:

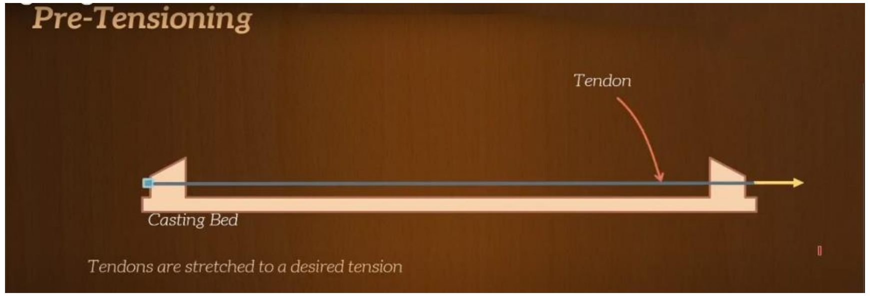
There are two methods of application of the prestress force on concrete:

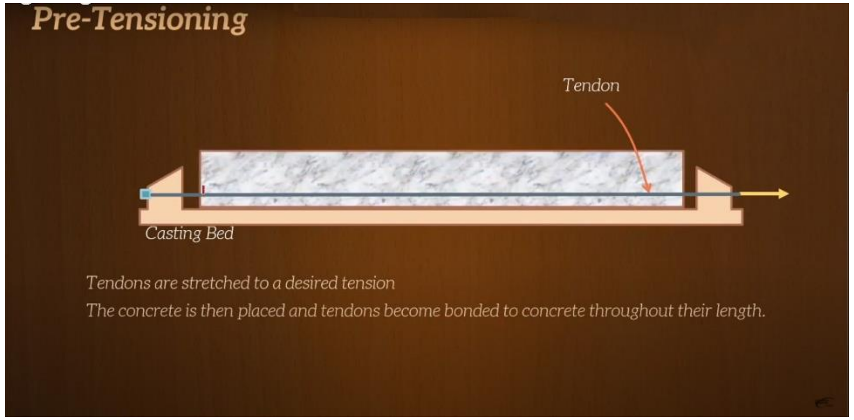
a) Pre-Tensioned prestressed concrete

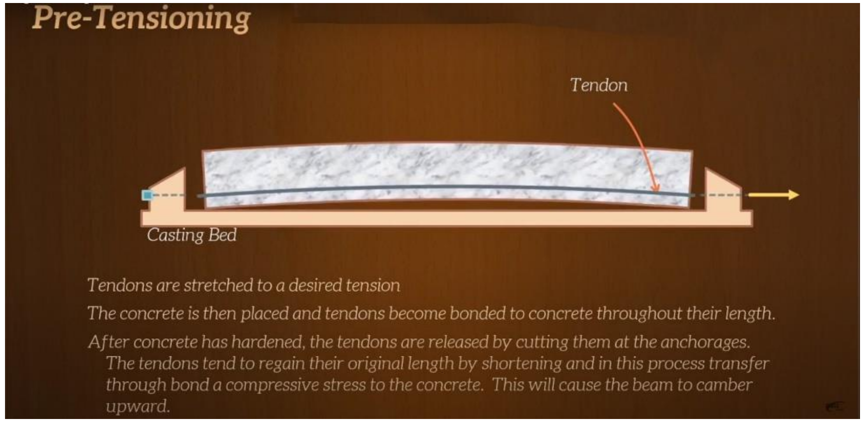
- Members are produced by stretching the tendon between external anchorages before the concrete is placed. After the concrete has hardens and reached a sufficient strength, the tendon force is released, and the force is transferred from the steel to the concrete by the bond developed between hardened concrete and prestress tendon.

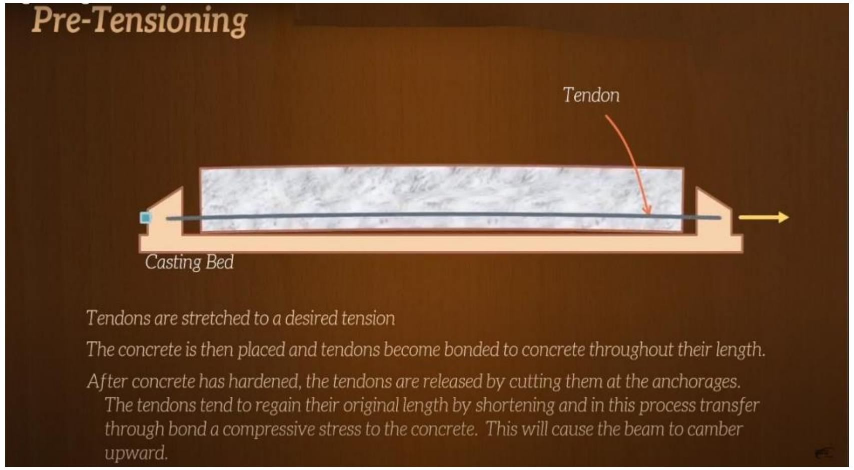
Method of Prestressing …. Cont.

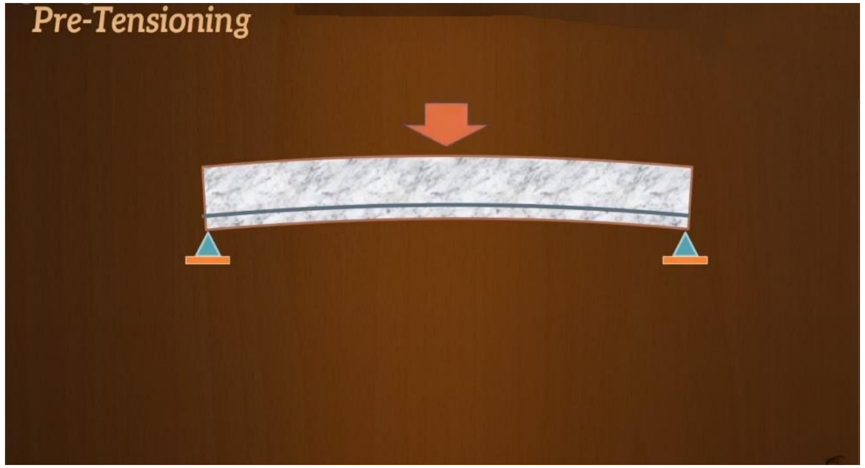












Method of Prestressing …. Cont.

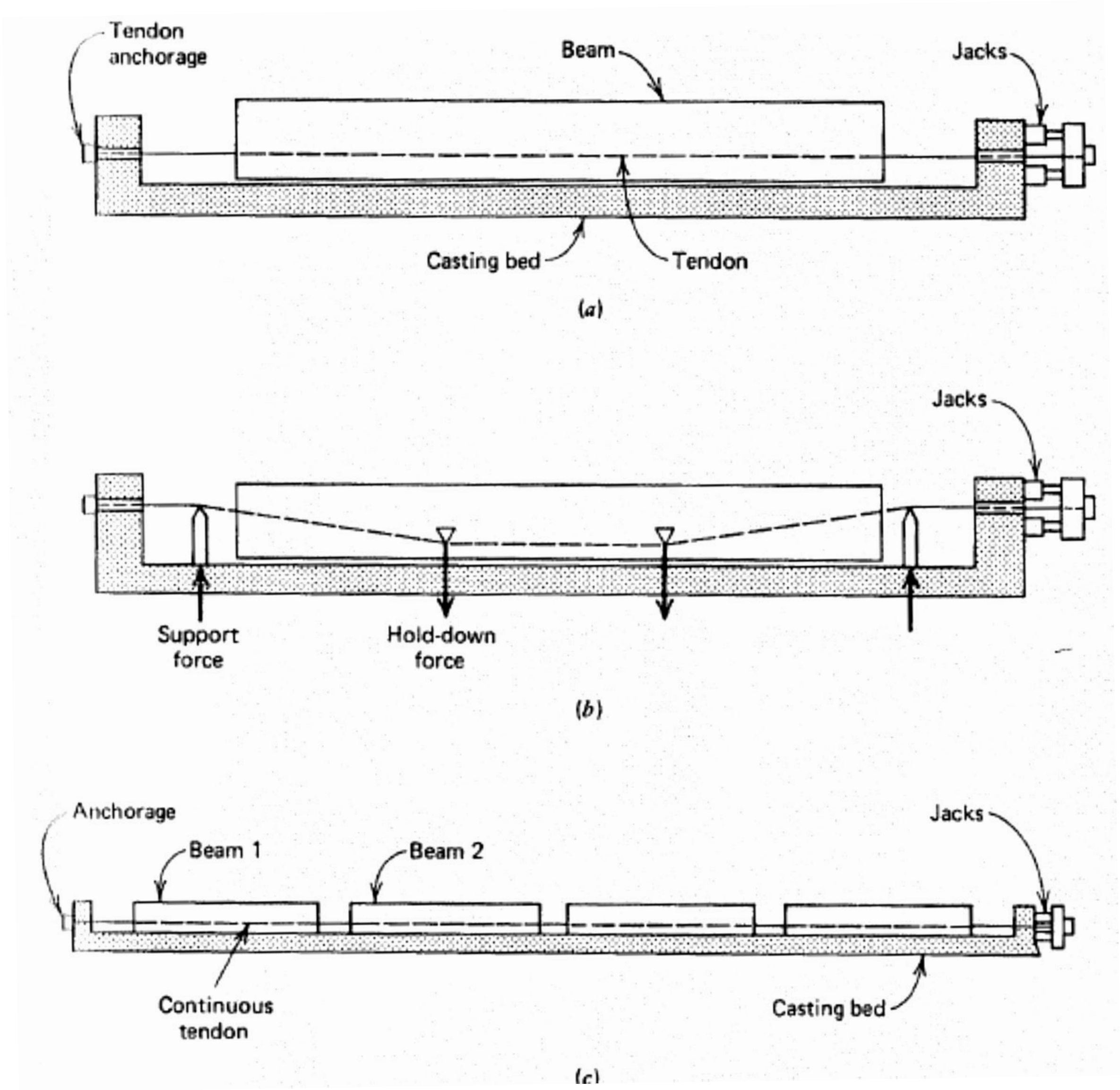
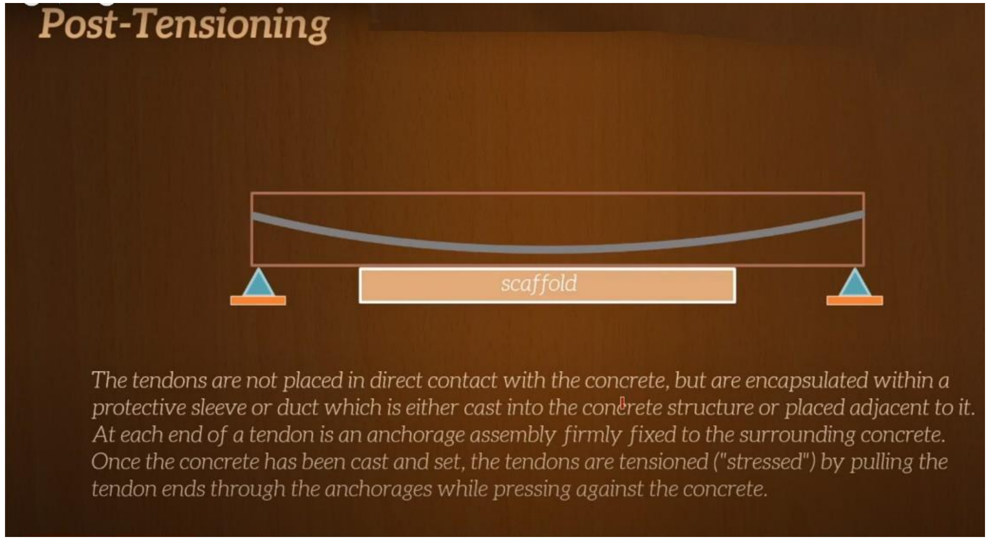
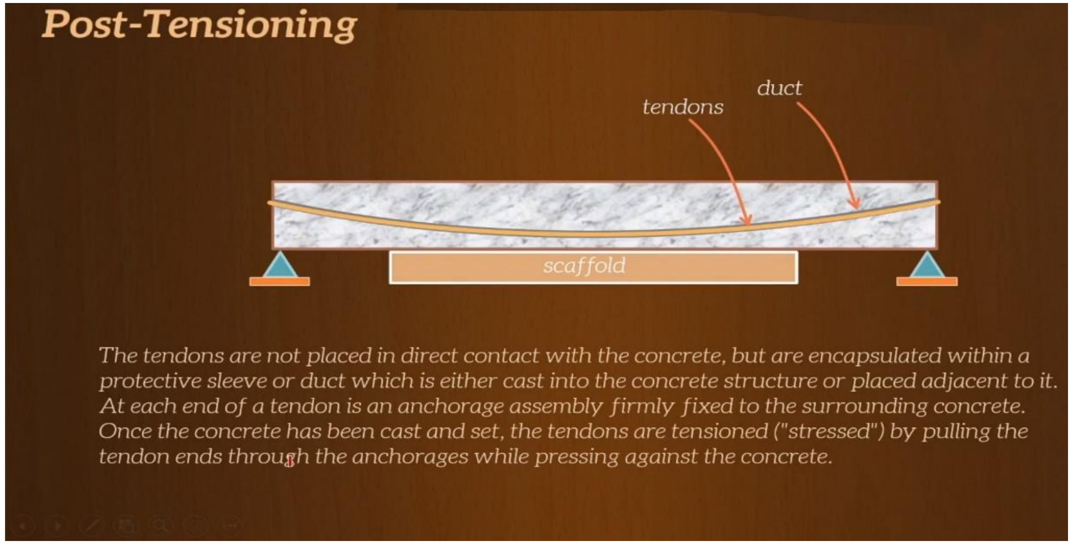


Figure (1) Method of Pretrensioning: (Beam with straight Tendon. (b) Beam with Variable Tendon. (C) Long-line stressing and casting.

b) Post-Tensioning Method

- In post tensioning method, the tendon may be placed in a hollow conduit and the tendon is stressed after the concrete has hardened and achieved a sufficient strength by jacking the tendon against the member itself.


Post-Tensioning
scaffold
The tendons are not placed in direct contact with the concrete, but are encapsulated within a protective sleeve or duct which is either cast into the concrete structure or placed adjacent to it. At each end of a tendon is an anchorage assembly firmly fixed to the surrounding concrete. Once the concrete has been cast and set, the tendons are tensioned ("stressed") by pulling the tendon ends through the anchorages while pressing against the concrete.


Post-Tensioning
duct
tendons
scaffold
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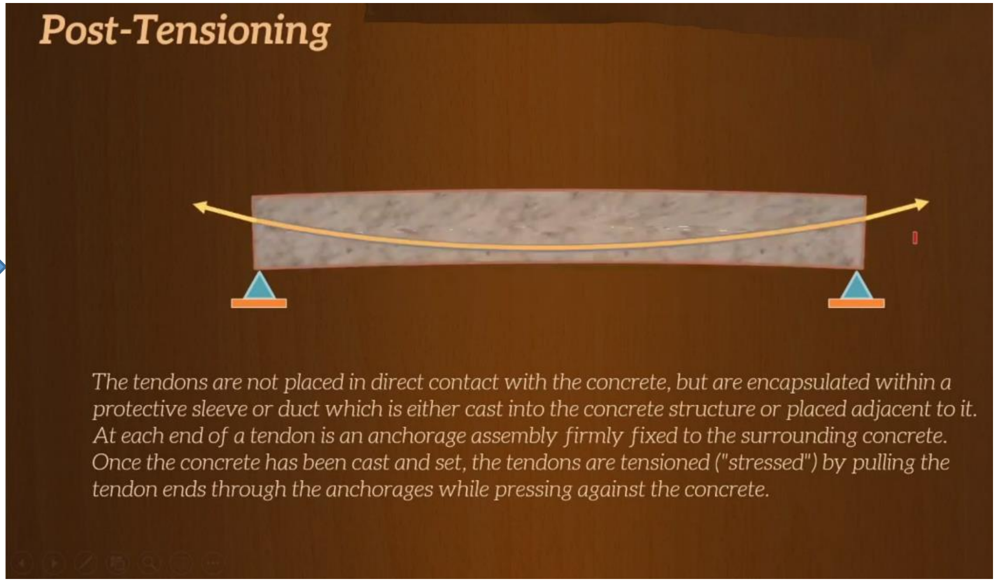
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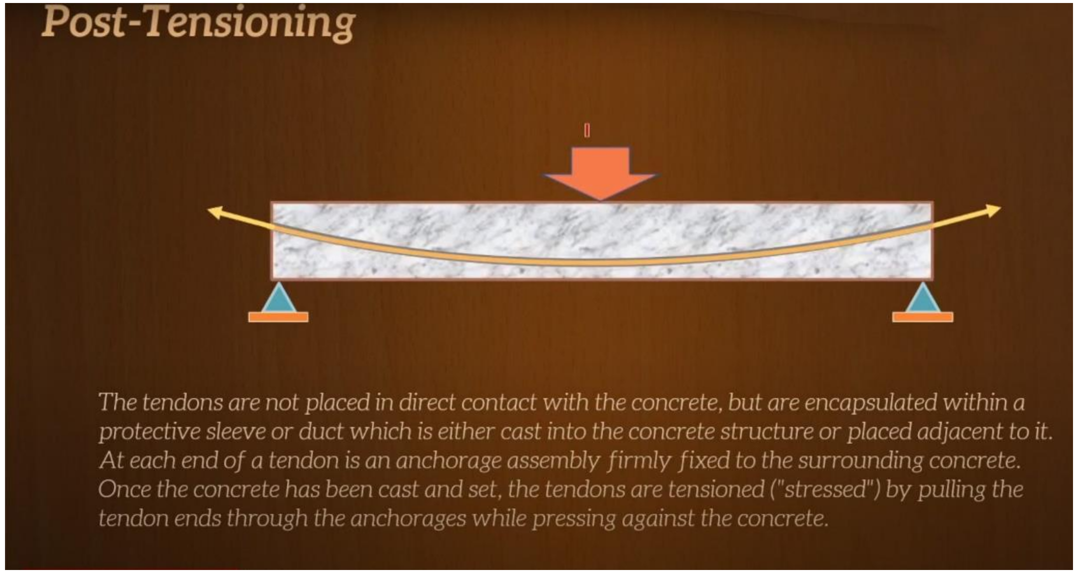
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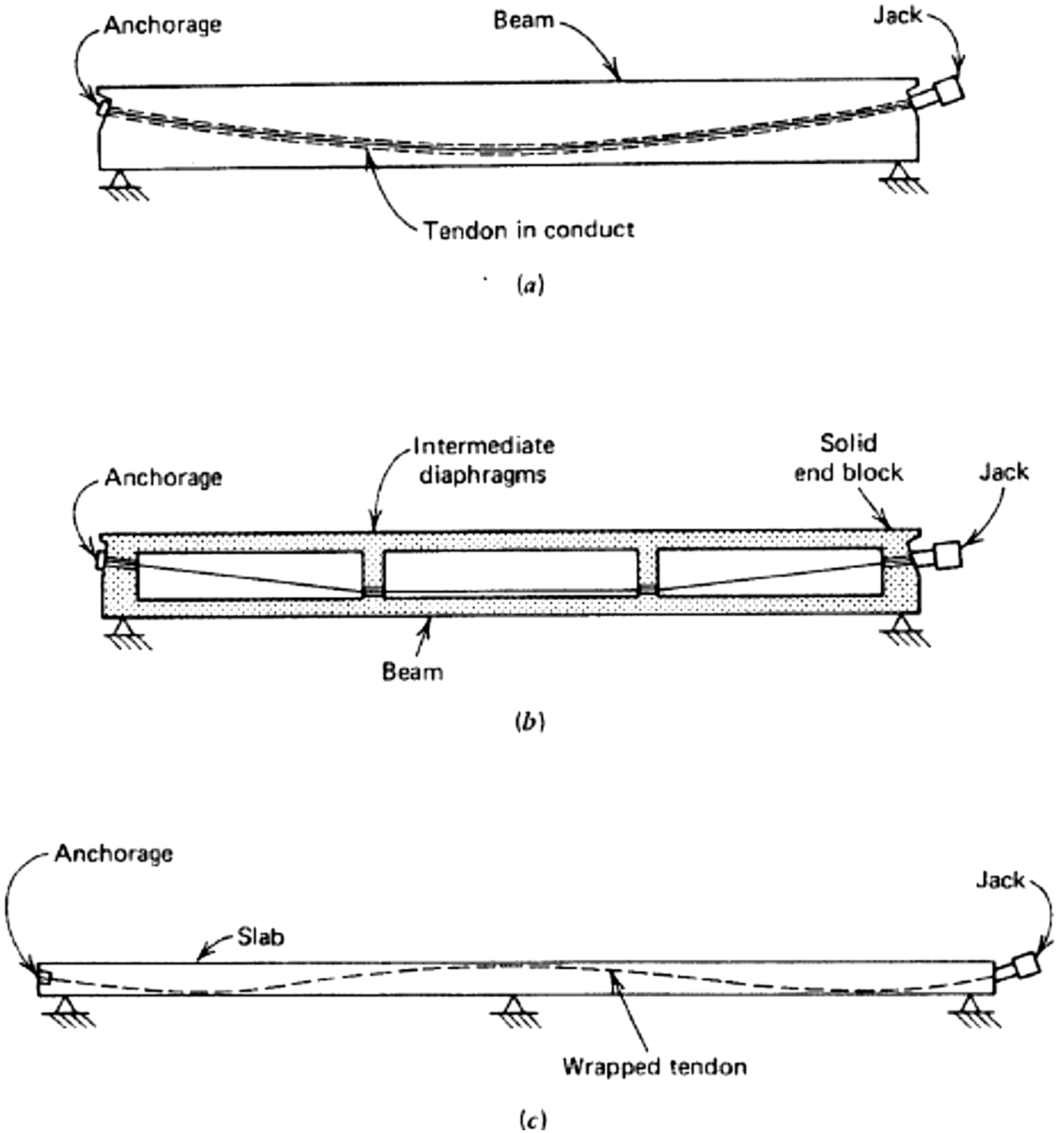


Figure (2) Method of Prost-trensioning: (1) Beam with hollow conduit embedded in concrete . (b)Hollow cellular Beam with intermediate diaphragms. (C) Continuous slab with plastic-sheathed tendons .

Materials of Prestressing Concrete:

a) High tensile steel.

- 1 – Steel wires (5 –7mm) diameter
- 2 – Strands consist of number of wires (6 – 16 mm) usually seven wire strands.
- For wires and strands the ultimate strength is between (1720 –1860N/mm^2). Tendons usually consist of (8 – 52) wires.
- 3 – Alloy steel bars:
- Diameter (16 – 36 mm), ultimate strength *fpu* (1000 – 1100 N/mm^2).

b) Ordinary Steel

- Non prestressed steel usually used in non prestressed concrete (member) supplementary reinforcement where used in region of high local compressive stress at anchorages of post tension members also used as longitudinal bars to control shrinkage and shear cracks and temp. Changes, also used for over hanged T and I section. Finally may used to increase the flexural strength (ultimate capacity) of prestress member as a supplementary longitudinal bar reinforcement (unstressed).

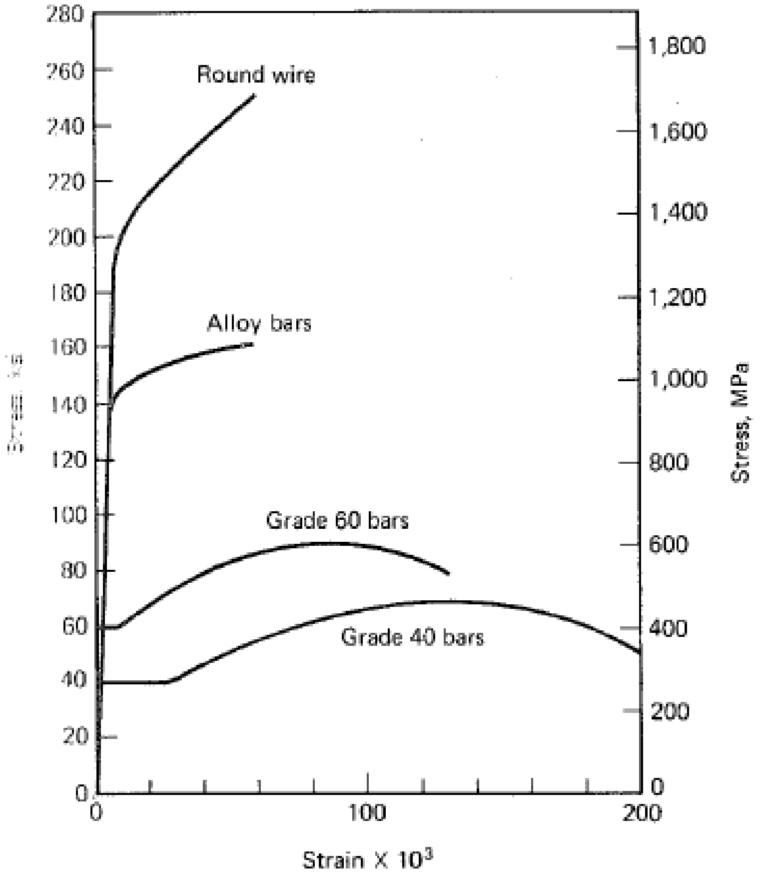


Figure (3) Comparative Stress Strain Curves for reinforced concrete steel and prestress steel

c) Concrete

- For prestress member, it is required to use a higher concrete strength usually between (30 – 55 MPa) and up to (83 MPa) been used.
- High strength concrete has a higher elastic modulus Ec, has a high bearing stress, has a higher bond stress also high compressive strength result in a high flexure and tensile stress so that the formation of the tension and diagonal crack is delayed.

Flexural Analysis of Prestress Members:

- In flexural analysis concrete and steel dimensions and the magnitude and line of action of effective prestress force are given, if the load are given, therefore it is required to calculate the fiber stresses at different load stages and compare the obtained fiber stresses with the permissible stresses according to the ACI – Code, or can calculate the maximum load can be applied without exceeding the fiber stresses (permissible stresses).
- In flexural design, the applied load and the concrete strength and steel ultimate strength are known. It is required to calculate the beam section and dimensions and the effective prestress force and its line of action without exceeding the permissible stresses.

The prestressing processes will be as follow

Jacking Force

Pj (Applying jacking force)

After instantaneous losses (Anchorage slip, elastic shortening and friction post tension)

Initial Force

Pi (Initial pre force)

After time depended losses (shrinkage, sreep of concrete and relaxation of prestress steel).

Effective Force

Pe (Effective prestress force)

Effective ratio ***R = (Pe/Pi) < 1.0***

Where; (Pi – Pe) / Pi = (1 – R) **Time depended losses**.

Prestressing Stages:

Both flexural analysis and design of prestress concrete required to consider the following load stages:-

- 1) ***Initial prestress*** immediately after prestress transfer (*Pi alone*).
- 2) ***Initial prestress force*** + self weight of beam (Pi + Moment due to self-weight). [pi + Mo].
- 3) ***Initial prestress force*** + full dead load (*Pi + Mo + Md*). [*Pi +* M_d *total*].
- 4***) Effective prestress force*** after losses with total load. (Pe + Mtotal).
- *M total = Mo + Md + ML*
- *Pe = R . Pi*
- *Where; R* < 1.0 (Reduction Factor Considering Losses)
- 5) Ultimate load, when the expected service load is increased by load factor and the member is at edge to failure.
- Load factor = (Ultimate strength / Service strength)
- Load factor = (Ultimate at collapse / Service load) >= 1.2

The following stages ***at mid-span*** will be considered:

1) Stage (1) Pi alone

If the member is subjected only to the initial prestressing force P_i, it has just been shown that the compressive resultant acts at the steel centroid. The concrete stress f_1 at the top face of the member and f_2 at the bottom face can be found by superimposing axial and bending effects:

$$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c}$$

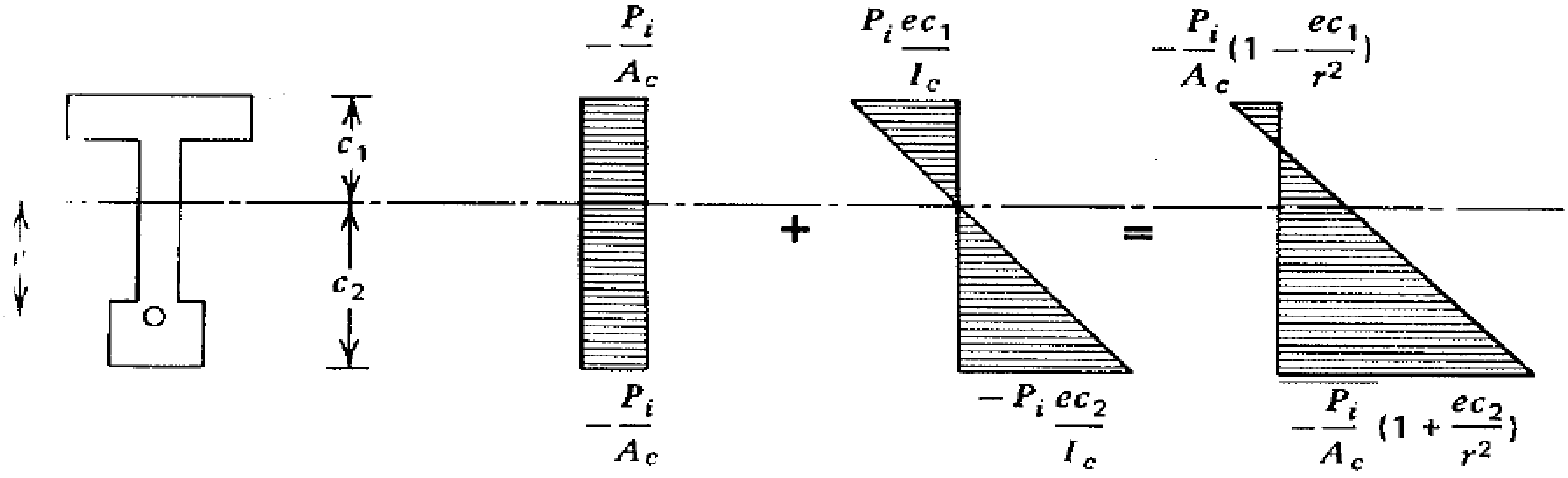
$$f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c}$$

where e is the tendon eccentricity measured downward from the concrete centroid, A_c is the area of the concrete cross section, and I_c is the moment of inertia of the concrete cross section. Other terms are as already defined. Substituting the radius of gyration $r^2 = I_c/A_c$, these equations can be written in the more convenient form:

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{e c_1}{r^2}\right)$$

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{e c_2}{r^2}\right)$$

The resulting stress distribution is shown in Figure 3.2(a)



(a)

The following stages ***at mid-span*** will be considered:

2) Stage (2) Pi + Mo

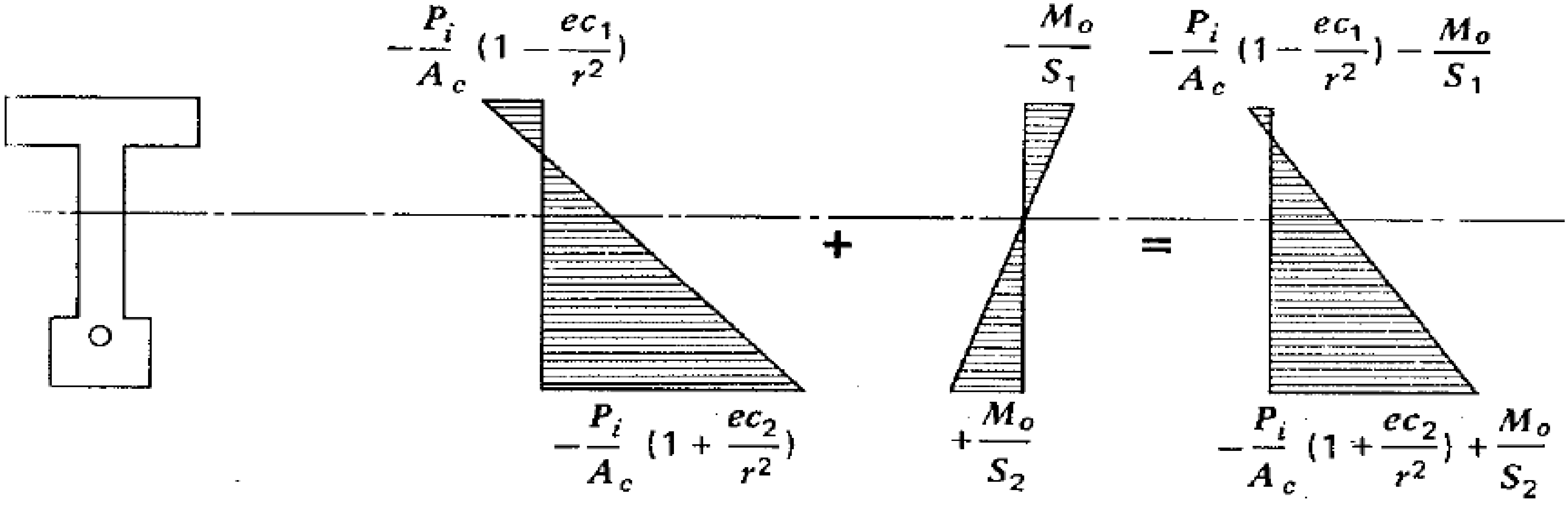
Almost never would the initial prestress P_i act alone. In most practical cases, with the tendon below the concrete centroid, the beam will deflect upward because of the bending moment caused by prestressing. It will then be supported by the formwork or casting bed essentially at its ends, and the dead load of the beam itself will cause moments M_o to be superimposed immediately. Consequently, at the initial stage, immediately after transfer of prestress force, the stresses in the concrete at the top and bottom surfaces are

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_o}{S_1}$$

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_o}{S_2}$$

where M_o is the bending moment resulting from the self-weight of the member, and $S_1 = I_c/c_1$ and $S_2 = I_c/c_2$ are the section moduli with respect to the top and bottom surfaces of the beam. The stress distribution at this load stage is shown in Fig. 3.2b.

The resulting stress distribution is shown in Figure 3.2 (b)



(b)

The following stages ***at mid-span*** will be considered:

3) Stage (3) Pe+M total

Superimposed dead loads (in addition to the self-weight) may be placed when the prestress force is still close to its initial value, that is, before time-dependent losses have occurred. However, this load stage would seldom, if ever, control the design, as can be confirmed by study of Fig. 3.2.

Superimposed live loads are generally applied sufficiently late for the greatest part of the loss of prestress to have occurred. Consequently, the next load stage of interest is the full service load stage, when the effective prestress P_e acts with the moments resulting from self-weight (M_o), superimposed dead load (M_d), and superimposed live load (M_l). The resulting stresses are

$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_t}{S_1}$$

$$f_2 = -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_t}{S_2}$$

where the total moment M_t is

$$M_t = M_o + M_d + M_l$$

These *service-load stresses* are shown in Fig. 3.2*c*.

The resulting stress distribution is shown in Figure 3.2 (c)

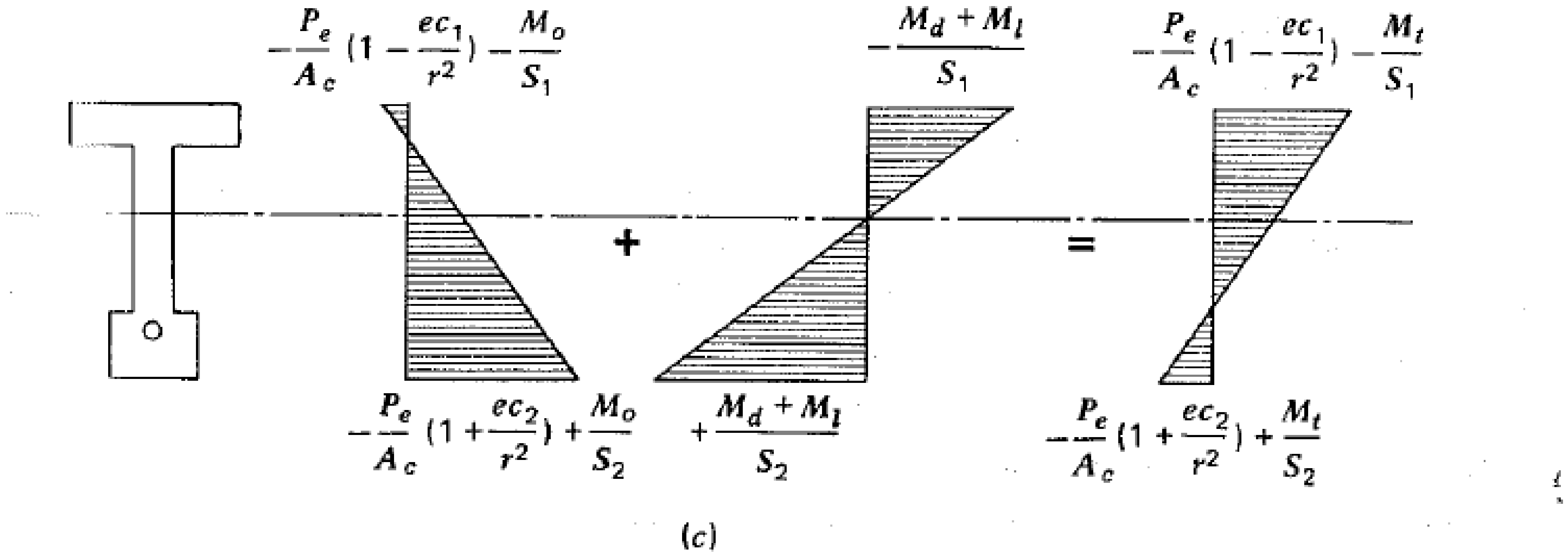
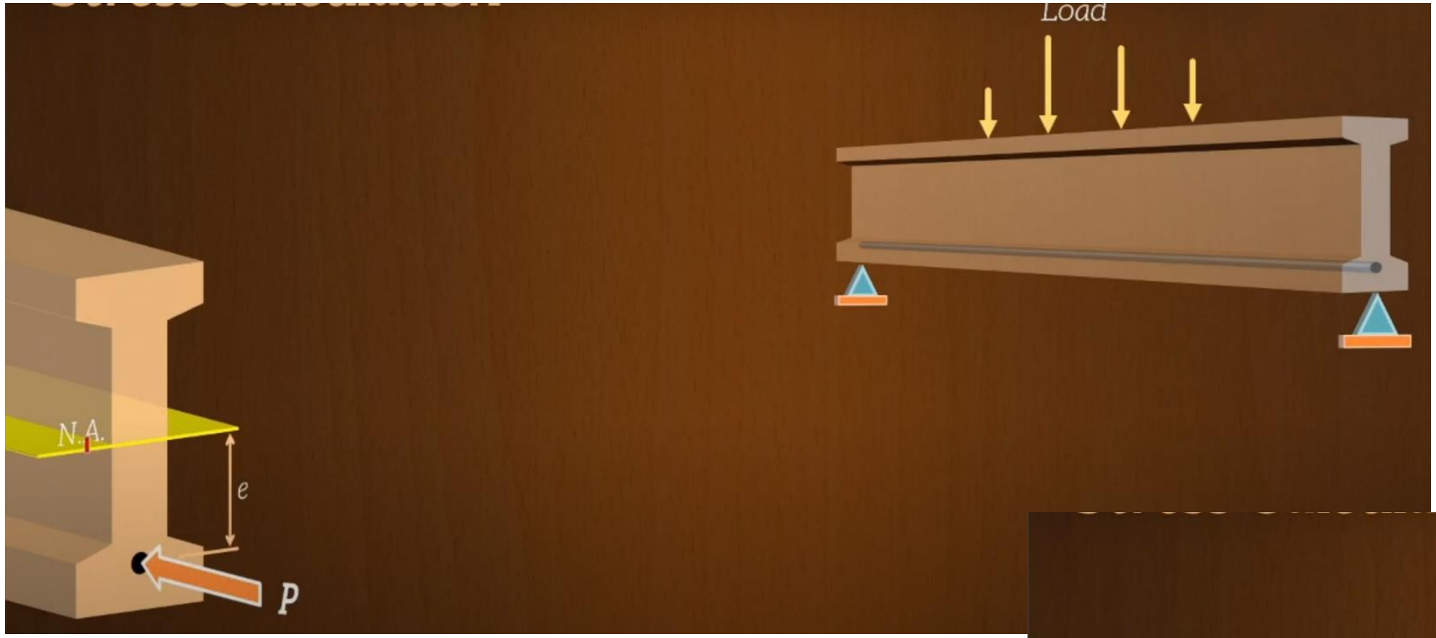
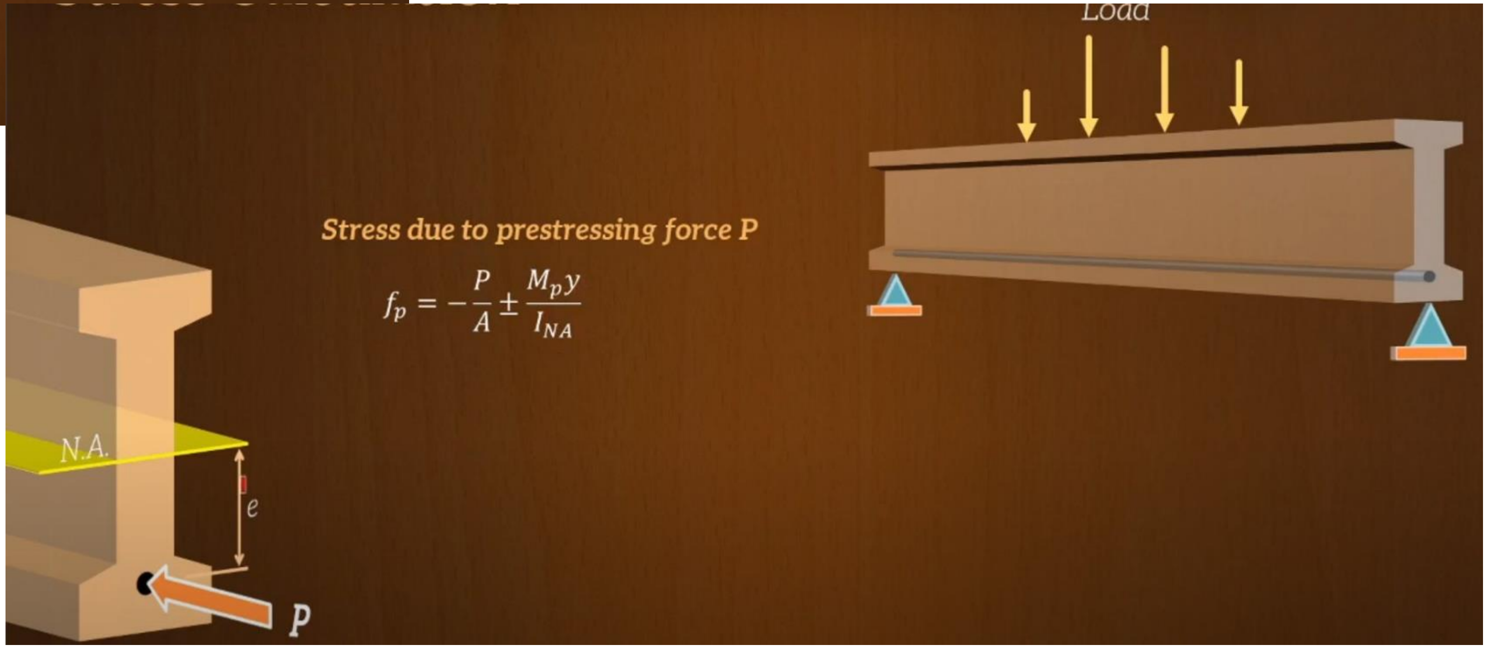


FIGURE 3.2 Elastic stresses in an uncracked prestressed beam. (*a*) Effect of initial prestress. (*b*) Effect of initial prestress plus self-weight. (*c*) Effect of final prestress plus full service load.

The resulting stress distribution at mid-span is shown also in Figure below





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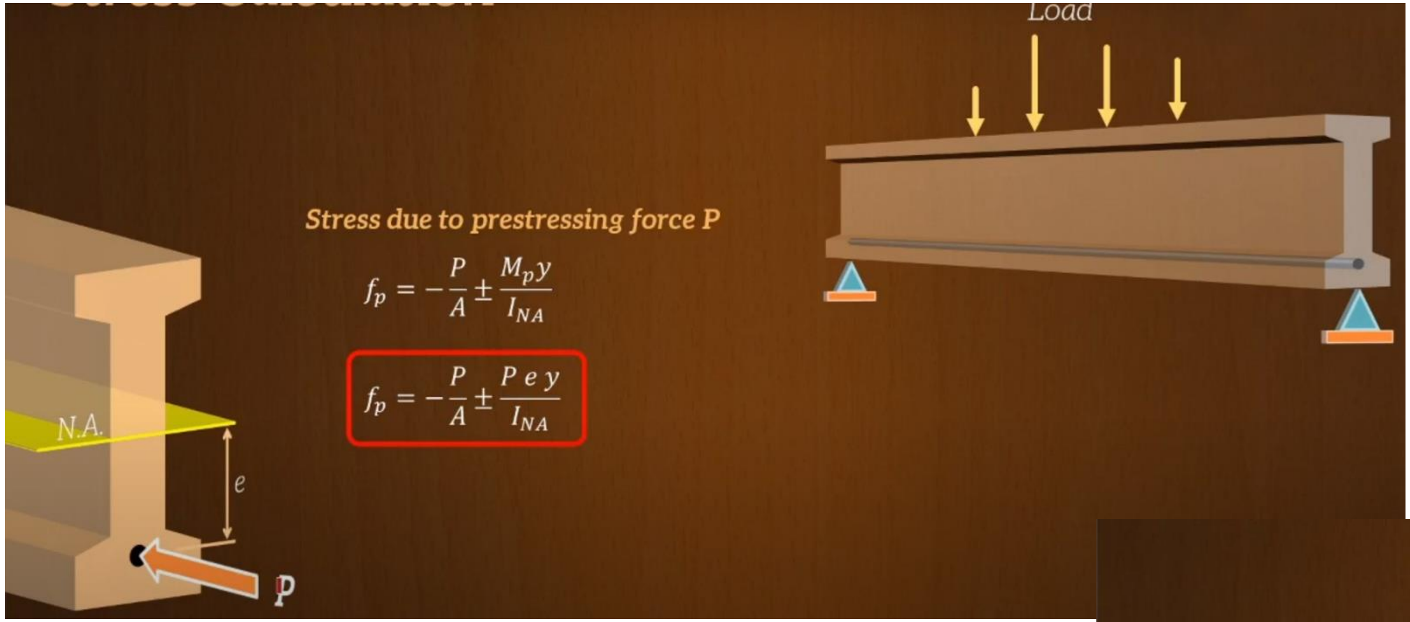

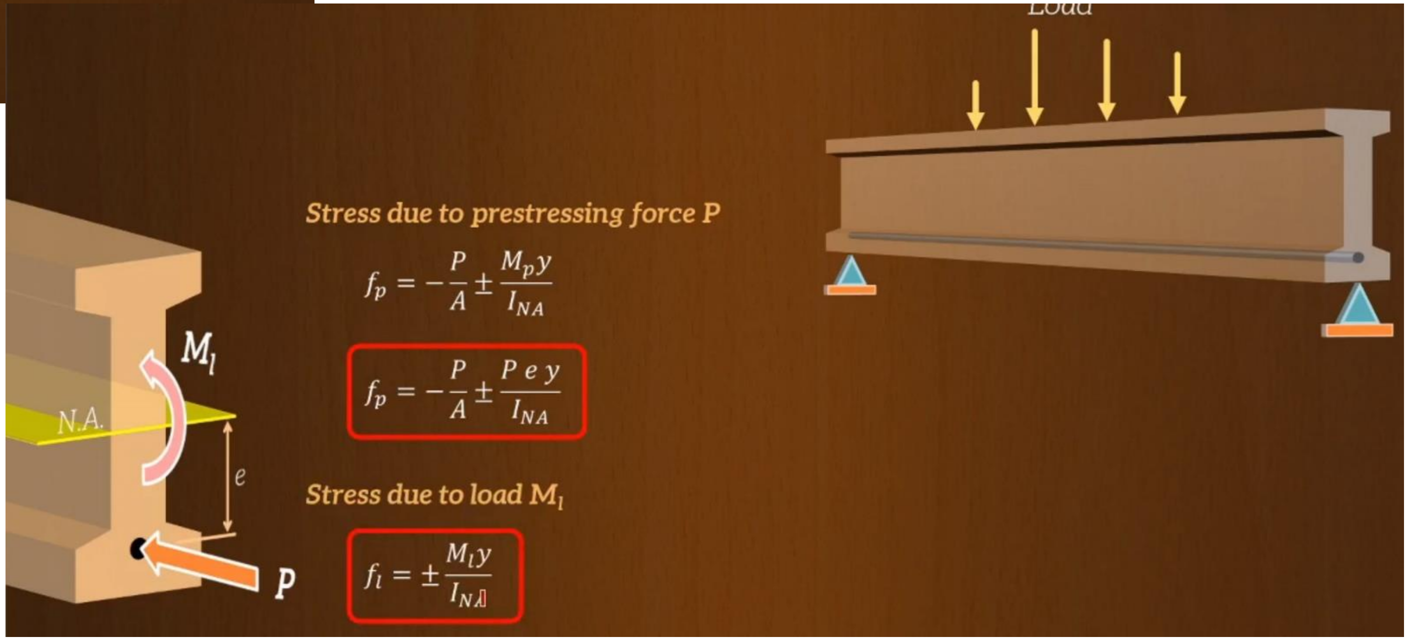

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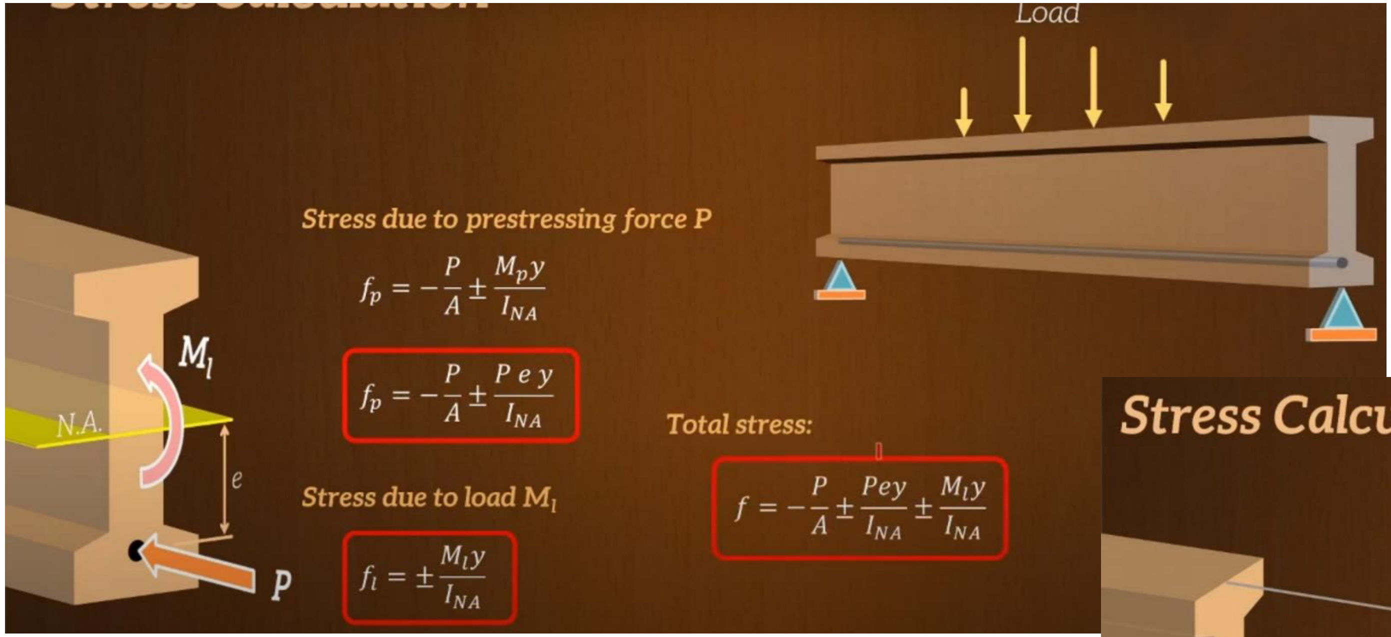
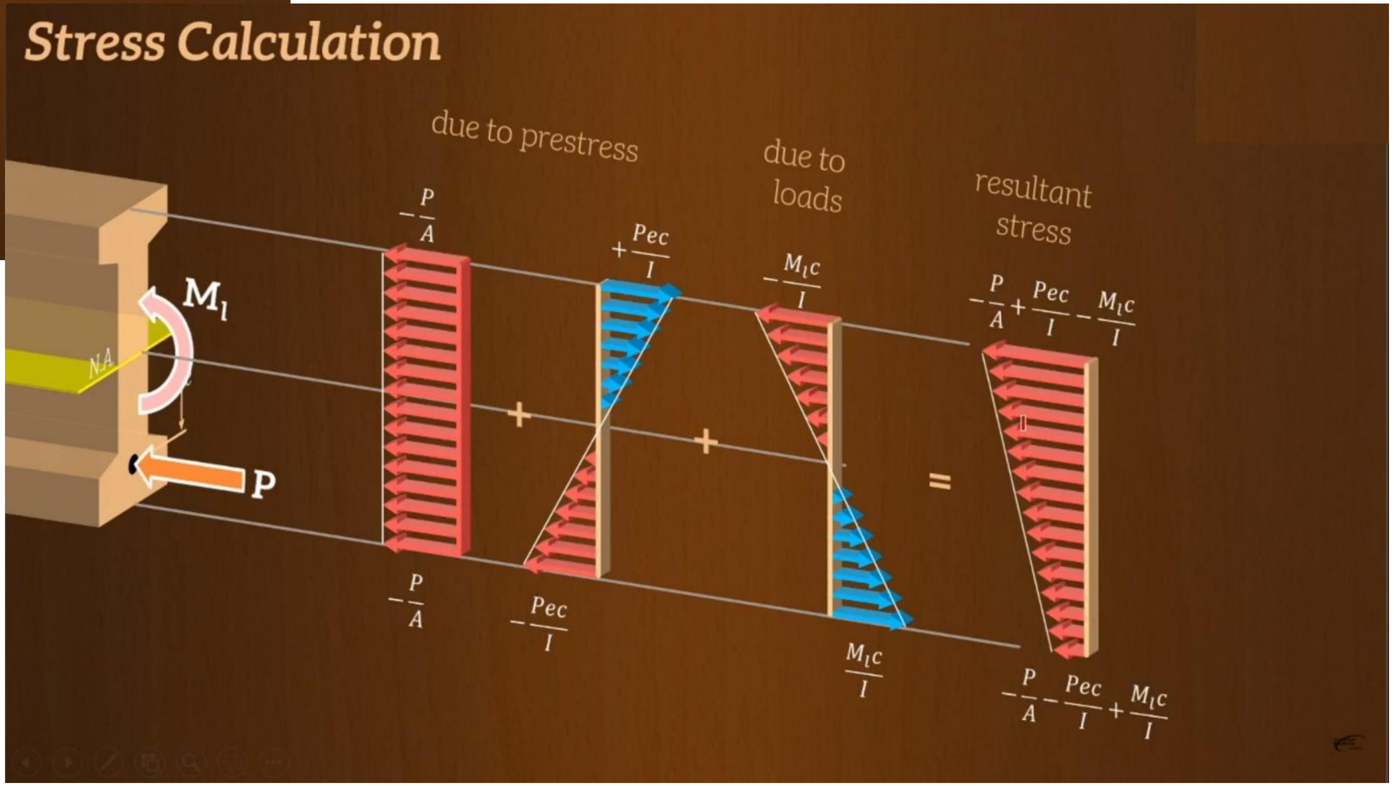




The following stages ***at Support*** will be:

Stages (1, 2, and 3)

Since, there are no moments at support the stresses for ***all stages*** ***at supports*** will be expressed as

$$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c}$$

$$f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c}$$

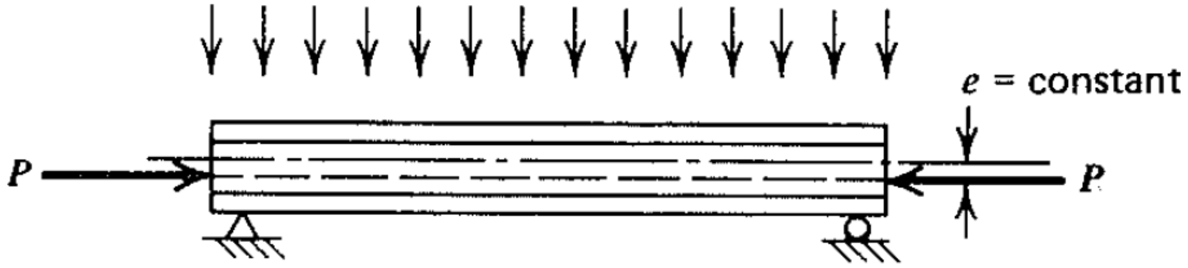
or in the form:

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right)$$

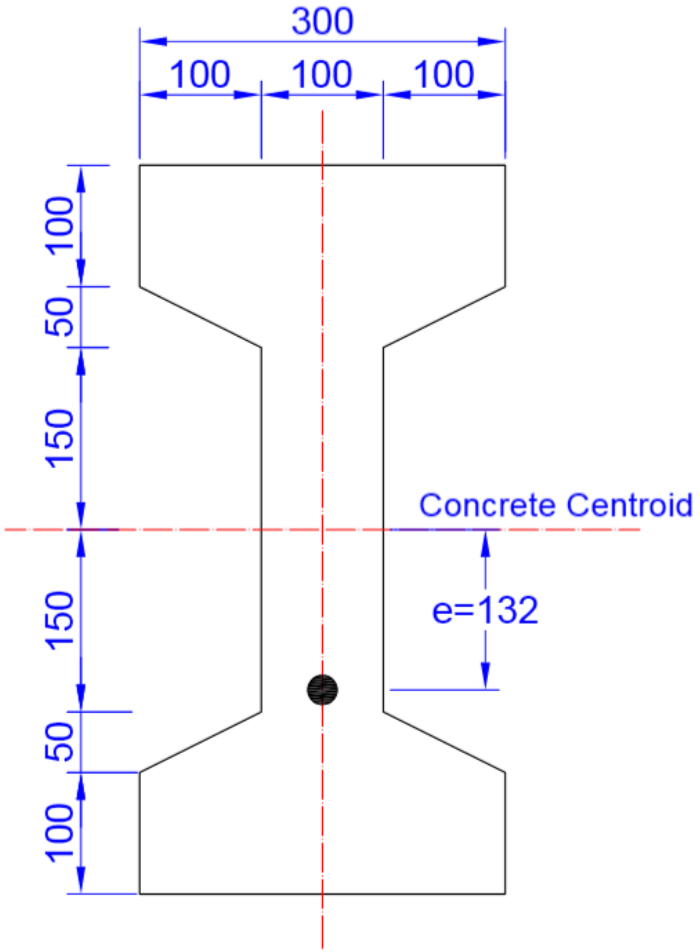
$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right)$$

Example 1:

The simply supported beam shown in cross section and elevation below to carry uniformly distributed surface dead load and live load totaling (*8 kN/m*) over a (*12.0m*) span, in addition to its own weight. Normal concrete will be used (*24 kN/m*^{3}). The beam will be pretensioned using multiple (7-*wires strands*), eccentricity is constant along the span *e=132mm*, *Pi=750 kN*, time dependent losses (*1-R=15%*). Find the concrete flexural stresses at midspan and at support section under initial and final conditions.



Beam Longitudinal View



Beam Cross Section

Serviceability requirements for Flexural members (ACI Code Permissible stresses)

A) Concrete

1- At Initial Stage: Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses):

(a) Extreme fiber stress in compression except as permitted in (b) shall not exceed....... 0.60*f'ci*.

(b) Extreme fiber stress in compression at ends of simply supported members shall not exceed................ 0.70*f'ci*.

(c) Extreme fiber stress in tension along the span of the members shall not exceed........ $0.25\sqrt{fc'i}$.

(d) Extreme fiber stress in tension at ends of simply supported members shall not exceed..$0.5\sqrt{fc'i}$

Where computed concrete tensile strength, ft , exceeds $0.50\sqrt{fc'i}$ .at ends of simply supported members, or $0.25\sqrt{fc'i}$ .at other locations, additional bonded reinforcement shall be provided in the tensile zone to resist the total tensile force in concrete computed with the assumption of an uncracked section.

Where; *f'ci*: is the specified compressive strength of concrete at time of transfer (MPa)

A) Concrete

2- At Service Load Stage: After allowance for all prestress losses stressed ***shall not exceed*** the following:

(a) Extreme fiber stress in compression due to prestress plus sustained load.......... *0.45f'c*

(b) Extreme fiber stress in compression due to prestress plus total load............... *0.60f'c*

Where; fc': is the specified compressive strength of concrete (MPa)

Tensile stresses

The computed extreme fiber stress in tension in the precompressed tensile zone calculated at service loads, as follows:

(a) Uncracked Section : $ft \leq 0.62 \sqrt{f c'}$.

(b) Transmission Section: $0.62\sqrt{f c'} < ft \leq 1.0\sqrt{f c'}$.

(c) Cracked Section: $ft > 1.0\sqrt{f c'}$

Where: *ft* is the tensile strength of concrete

B) Permissible stresses in prestressing steel:

Tensile stress in prestressing steel shall not exceed the following:

(a) Due to prestressing steel jacking force ***0.94fpy***

- but not greater than the lesser of 0.80fpu and

- the maximum value recommended by the manufacturer of prestressing steel or anchorage devices.

(b) Post-tensioning tendons, at anchorage devices and couplers, immediately after force transfer ***0.70fpu***

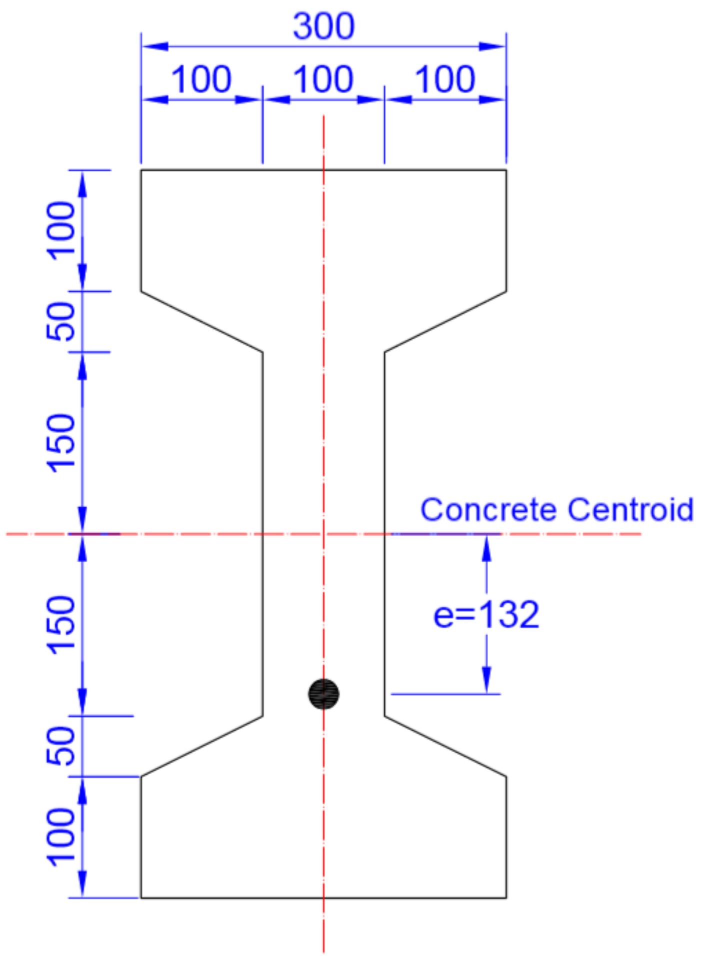
Where: ***fpy*** = the yield strength of prestressing steel (MPa)

fpu = the ultimate strength of prestressing steel (MPa)

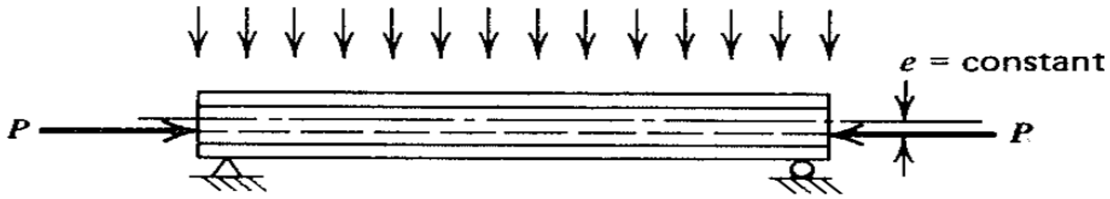
Example 2:

The simply supported beam shown in cross section and elevation below to carry uniformly distributed surface dead load and live load totaling (*8 kN/m*) over a (*12.0m*) span, in addition to its own weight. Normal concrete will be used (*24 kN/m*^{3}). The beam will be pretensioned using multiple (*7-wires strands*), eccentricity is constant along the span *e=132mm*, *Pi=750 kN*, time dependent losses (*1-R=15%*).

- Find the concrete flexural stresses at midspan and at support section under initial and final conditions.
- Check the fiber stresses with the permissible stresses at different load stages at midspan section and at support section *fc'=34 MPa, fc'i=25 MPa*



Beam Cross Section

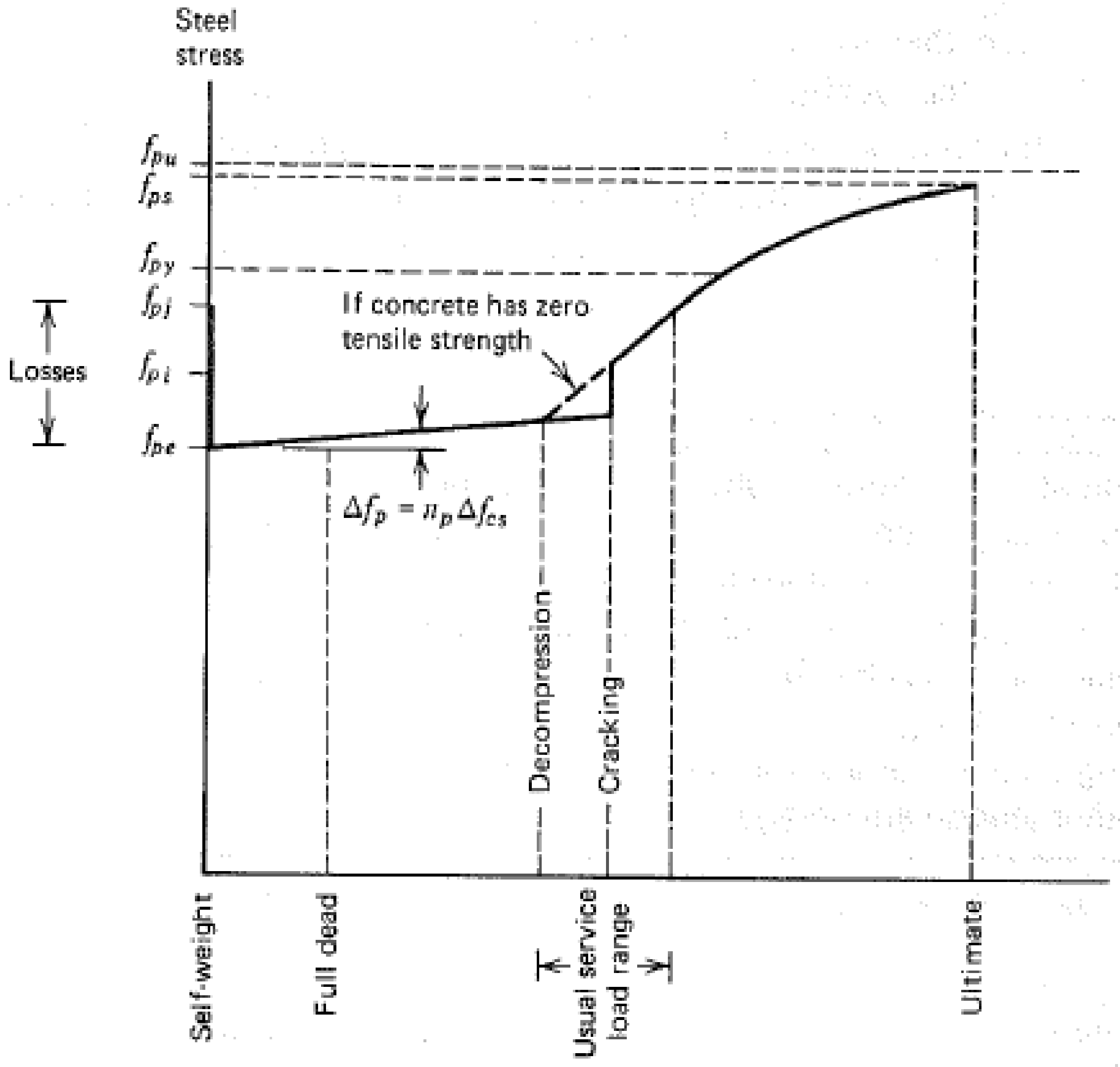


Beam Longitudinal View

Cracking of Concrete

The relation between applied load and steel stress is a typical well-bonded pre-tensioned beam and well-grouted pretensioned beam is shown in qualitative curve relation;

fpj : stress in steel at jacking load (N/mm^2)
fpi : stress in steel after instantaneous losses (friction, slip at end anchorage and elastic shortening)
fpe : effective stress in prestressing steel after all losses.
fp : increase the stress of prestressing steel.
np : modular ratio of prestress steel. np = (Ep / Ec)
Ep : Modulus of elasticity of prestressing steel.
Ec : Modulus of elasticity of concrete.
fcs : increase in areas of concrete at steel level.
- Decompression load: Is the load make the stress (f2) at bottom equal to zero.
- Cracking load: Is the load make the bottom tensile strength in concrete equal to rapture strength.
fps : stress in prestressing steel at failure of prestressed beam.



Variation of steel stress with load in bonded pretensioned beam

Cracking Load

It is necessary to predict the cracking load for the following reasons:

1 – Deflection is increased by the reduction in flexural rigidity of cracked prestressed member (EI/L).

2 – After cracking the prestressing steel will exposed to corrosion.

3 – The fatigue resistance of beam will reduced by cracking.

4 – Cracks are unlikely to appear.

5 – In case of liquid containment vessels, leaks are more likely after cracking.

In order to get cracking moment let f2=fr':

$$f_2 = \frac{-Pe}{Ac}\left(1+\frac{e.c_2}{r^2}\right)+\frac{Mcr}{S_2}$$

$$\frac{-Pc}{Ac}\left(1+\frac{e.c_2}{r^2}\right)+\frac{Mcr}{S_2} = fr'$$

$$Mcr = fr'.S_2 + Pe\left(\frac{r^2}{c_2}+e\right)$$

Where:

$$fr' = 0.62\sqrt{fc'}$$

Mcr is the cracking moment.

Mcr = Mo + MD + Fcr ML

However, for simply supported beam;

$$Mcr = \frac{Wcr.L^2}{8}$$

$$Wcr = \frac{8Mcr}{L^2}$$

Wcr = Wo +WD + Fcr WL

$$Fcr = \frac{Mcr - Mo - M_D}{M_L}$$

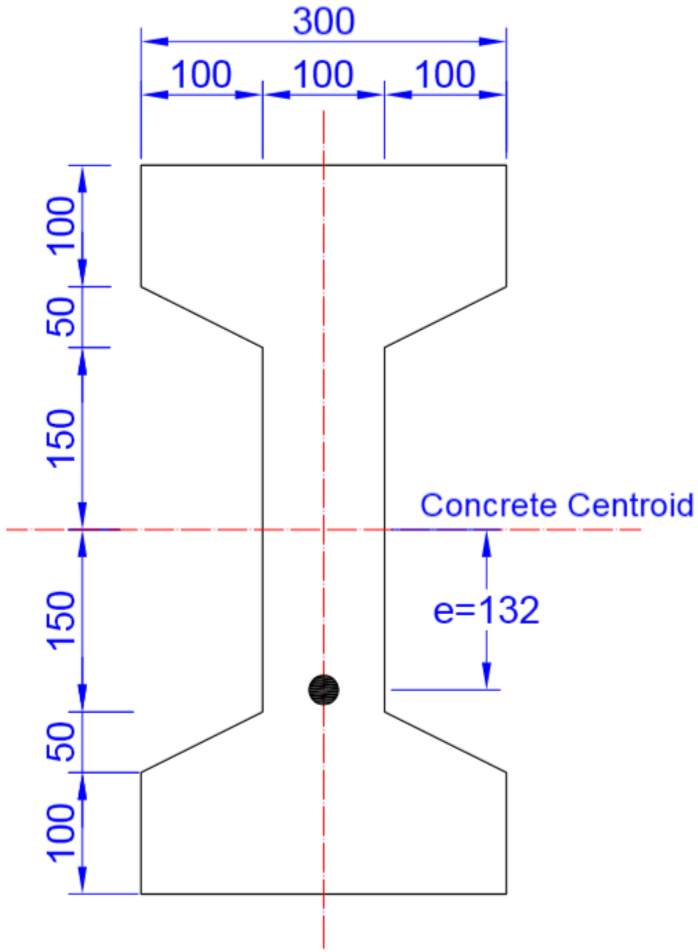
$$Fcr = \frac{Wcr - Wo - W_D}{W_L}$$

Where: Fcr is the factor of safety against cracking may be less than, equal to or greater than unity.

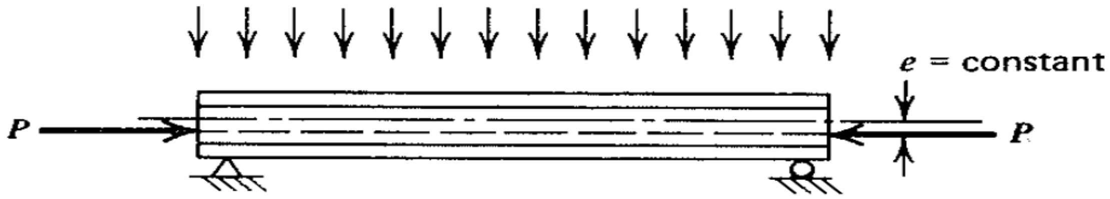
Example 3:

The simply supported beam shown in cross section and elevation below to carry uniformly distributed surface dead load and live load totaling (*8 kN/m*) over a (*12.0m*) span, in addition to its own weight. Normal concrete will be used (*24 kN/m*^{3}). The beam will be pretensioned using multiple (*7-wires strands*), eccentricity is constant along the span *e=132mm*, *Pi=750 kN*, time dependent losses (*1-R=15%*).

Calculate the cracking moment and find the factor of safety (FS) against cracking for the I-beam in the previous example. Use modulus of rapture fr'=2.4MPa.



Beam Cross Section



Beam Longitudinal View

Example 4:

A post tension prestress girder shown in Figure below has an eccentricity at mid-span e= 380mm and at ends e=0.0. The girder has a simple span 20mm, with the following data;

Ac=240X103 mm2

Wo=5.75 kN/m

Ic=27.3X109 mm4

Fc'=40 MPa.

fr=4 N/mm2

$ft \leq 0.62\sqrt{fc'}$

Pi=1850 kN

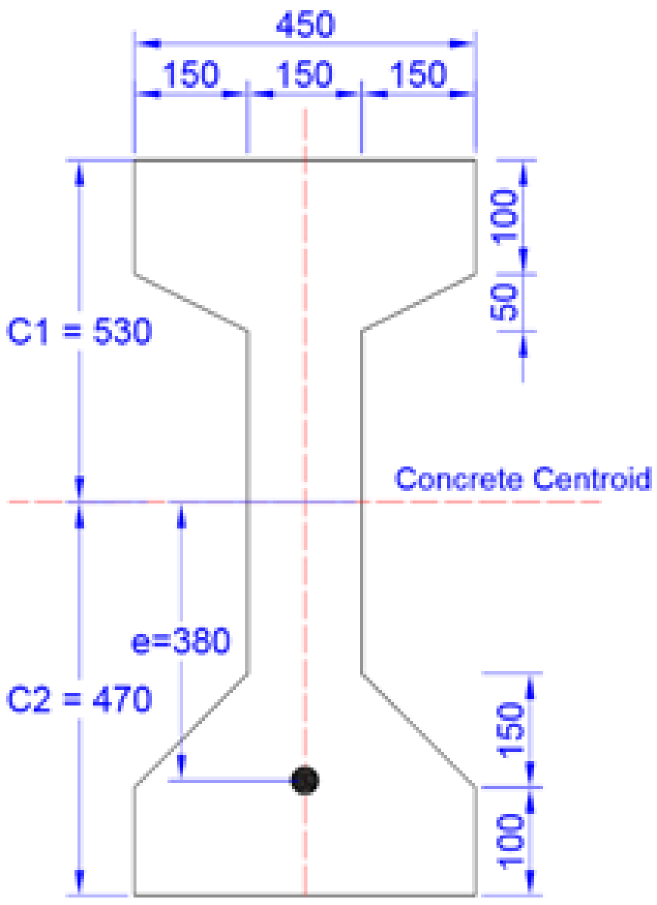
R=0.85

Find the uniformly distributed load that to produce the following cases:

1-Decompression load stage (f2=0)

2-Balancing load stage (zero deflection)

3-Cracking load.



Beam Cross Section